A MODEL FOR DETERMINING DIPOLE, QUADRUPOLE, AND COMBINED FUNCTION MAGNET COSTS*

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Abstract

One of the most important considerations in designing large accelerators is cost. This paper describes a model for estimating accelerator magnet costs, including their dependences on length, radius, and field. The reasoning behind the cost model is explained, and the parameters of the model are chosen so as to correctly give the costs of a few selected magnets. A comparison is made with earlier formulae. Estimates are also given for other costs linearly dependent on length, and for 200 MHz superconducting RF.

INTRODUCTION

While no cost model for estimating magnet costs can be very reliable, such formulae allow the optimization of a machine design prior to engineering and real costing. M. Green [1] has provided two such formulae that have proved very useful in many applications. The first formula is only depends on the magnetic stored energy U (in MJ), and is, after correction for inflation,

$$\text{Cost} = 1.34 \times 0.844\, U^{0.459}\ \text{M\$}. \tag{1}$$

The expression was obtained from a fit of a number of dipoles, solenoids, and toroids on a cost vs. stored energy log-log plot. It's rms deviation from the dipole magnets used in the fit is rather poor: near a factor of 4. The second formula depends on the product Ω of field and volume in $\mathrm{T\,m^3}$, and, after correction for inflation, is

$$\text{Cost} = 1.34\ \times\ 0.77\ \Omega^{0.631}\ \text{M\$}. \tag{2}$$

The fit to the dipoles is better for this case: the log rms error of is about factor of 2. The 1.34 in both formulae is to correct for 2.5% inflation from the paper's publication date of 1992 to 2004.

There are, however, several known dependencies of magnet costs that are not represented in either of these formulae: a) short dipoles are more expensive per unit length than long ones, b) costs rise faster than linear for higher magnetic fields, c) costs do not go to zero as the aperture becomes small compared with the coil thickness, and d) mass produced magnets are cheaper than the cost of one or a few. Since such dependences can play a significant role in finding a cost minimum, it is useful to define a formula that attempts to include them. In addition, it is useful to extend the method to cover quadrupoles and combined function magnets.

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COST MODEL

This magnet model is designed to apply to superconducting magnets with circular apertures. The magnetic field profile in the midplane is assumed to be linear, but the magnet is allowed to have any combination of dipole (B_0) and quadrupole (B_1) fields. The largest field value on a circular aperture will be in the midplane, so we will use the fields at the aperture in the midplane. We specify that the beam itself requires an aperture of radius R. Because the field quality adjacent to the coils is expected to be poor, a buffer region must be built in beyond the radius that the beam requires. To allow for this, we define the radius of the inside edge of the coils to be $k_R R$. Thus

$$B_\pm = |B_0| \pm |B_1| k_R R \tag{3}$$

gives the field values at the inside edge of the magnet coils. The $\pm$ refers to the absolute maximum and minimum fields on the two sides. If a magnet has a finite field gradient B_1, and the coils have a finite thickness, then the maximum field in the coil will be larger than the field at the inside edge of the coil. The distance from the inside edge of the coil to the peak field in the coil will generally increase with coil thickness, and thus with the magnitude of the maximum field. We approximate the relationship between B_+ and the distance from the inside edge of the coil to the peak field to be linear, and call the constant of proportionality k_C. The maximum peak field in the coil is thus

$$\hat{B} = B_+ + |B_1|\, k_C B_+. \tag{4}$$

Also, because the coils must have a nonzero thickness which increases (we assume linearly) with the peak field, we can define a maximum radius:

$$\hat{R} = k_R R + k_M \hat{B}. \tag{5}$$

Our estimate of the cost of a single magnet out of run of n magnets will be written as a product of four factors:

$$\text{Cost} = f_B(\hat{B}) f_G(\hat{R}, L) f_S(B_-/B_+) f_N(n). \tag{6}$$

Here L is the reference length of the magnet. The first factor $f_B(\hat{B})$ gives the dependence on the magnetic field. A simple model is

$$f_B(\hat{B}) = C_0 + C_1 \hat{B}^{k_B}. \tag{7}$$

This allows one to have a power law behavior for high fields, and to take into account the fact that a magnet with zero field still has a finite construction cost.

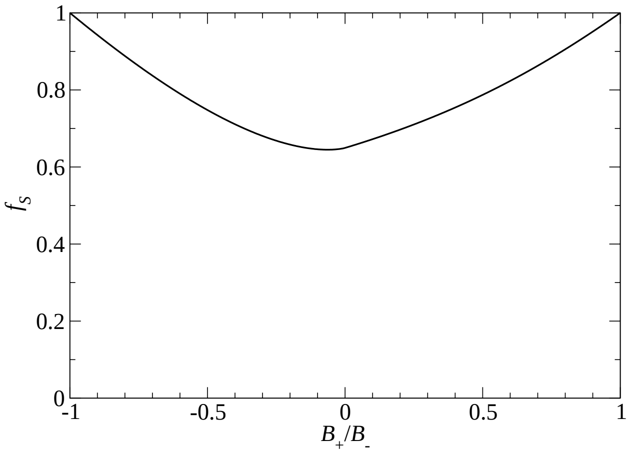

Figure 1: The function $f_S(B_-/B_+)$.

The factor $f_G(\hat{R}, L)$ reflects the costs associated with the size of the magnet. We will use

$$f_G(\hat{R}, L) = \hat{R}(L + k_G\hat{R}). \tag{8}$$

For very long magnets, this factor says that the cost is nearly proportional to the length of the magnet and its radius. But for very short magnets, the length is nearly $k_G\hat{R}$; this reflects the cost of magnet ends and other unit costs, which are assumed proportional to the radius of the magnet.

For the same $\hat{B}$, $\hat{R}$, and L, it is assumed that the cost of a quadrupole ($B_- = -B_+$) is equal to that of a dipole ($B_- = B_+$), since they both require the same amount of superconductor. But for combined function magnets, less conductor is required, and we assume that the cost is correspondingly less. This factor is given by $f_S(B_-/B_+)$. Defining

$$D = \frac{1}{2}\left(1 + \frac{B_-}{B_+}\right) \tag{9}$$

$$Q = \frac{1}{2}\left(1 - \frac{B_-}{B_+}\right) = 1 - D, \tag{10}$$

then

$$\begin{aligned} f_S(B_-/B_+) &= \frac{\int_0^\pi |D\cos\theta + Q\cos 2\theta|\, d\theta}{\int_0^\pi |\cos\theta|\, d\theta} \\ &= \frac{1}{4\sqrt{2}}\left[\frac{\left(\sqrt{D^2+8Q^2}+3D\right)^{3/2}}{\left(\sqrt{D^2+8Q^2}+D\right)^{1/2}}\right. \\ &\quad \left. + \begin{cases} 0 & Q \leqslant D \\ \dfrac{\left(\sqrt{D^2+8Q^2}-3D\right)^{3/2}}{\left(\sqrt{D^2+8Q^2}-D\right)^{1/2}} & Q > D \end{cases}\right]. \end{aligned} \tag{11}$$

Note that this factor is 1 for either a pure dipole or a pure quadrupole, but is just under 0.65 when the field is zero on one side of the magnet (the minimum is slightly lower than this). Figure 1 shows a plot of this function.

Finally, $f_N(n)$ accounts for the dependence of the per-magnet cost on the number of magnets being built. This is postulated to be a power law

$$f_N(n) = \left(\frac{n_0}{n}\right)^{k_N}, \tag{12}$$

where n_0 is the number of magnets used to define the constants, and n is the number of magnets required.

Table 1: Constants used

k_R	1.3		
k_C	2.47 mm/T	k_M	2 mm/T
C_0	101 k\$/m^2	C_1	16.78 k\$/T$^{1.5}$/m^2
k_B	1.5	k_G	36
k_N	1/3	n_0	300

Table 2: Costs and parameters of fitted magnets

	n	L	$k_R R$	B_0, (B_1)	cost
		m	m	T, (T/m)	k\$
RHIC Q	300	1.10	0.040	(91)	36.25
RHIC	300	10.00	0.040	4.30	143.0
Willen	300	18	0.02	5.6	193.0
LHC	300	2×15.0	0.028	8.30	708.0

Constants

The constants in this model are given in Tab. 1. They have been determined based on cost data from a few specific magnet designs whose relevant parameters and average magnets costs are given in Tab. 2. All costs have been escalated to 2004 at 2.5% per year. The RHIC quadrupole cost given is the quoted cold mass cost [3] plus 25% to include a cryostat. The RHIC dipole cost [2] required no correction. The ”Willen” example is for a design based on RHIC costing experience, but designed to yield the lowest cost per Tesla meter. The LHC magnets have two bores, and two coil packages, within a single yoke and cryostat. To give a representation to the savings achieved, the magnet length is entered as the total integral length in both bores. The number of magnets listed (300) is the number made by each of three manufacturers in different countries, and the cost is a word of mouth average of their somewhat differing costs. With the parameters given, the formula gives the listed costs to within 0.1%. However, the costs themselves, given differing information sources, currencies, inflation, and definition of what exactly is included, etc. are not better than 15%. The application of the formula to other magnets is probably not better than about 30%.

COMPARISON WITH EARLIER MODEL

In order to make a comparison with M. Green's formulae, we have to assume a particular length, radius, and number of magnets made. We arbitrarily pick a length L of 3 m,

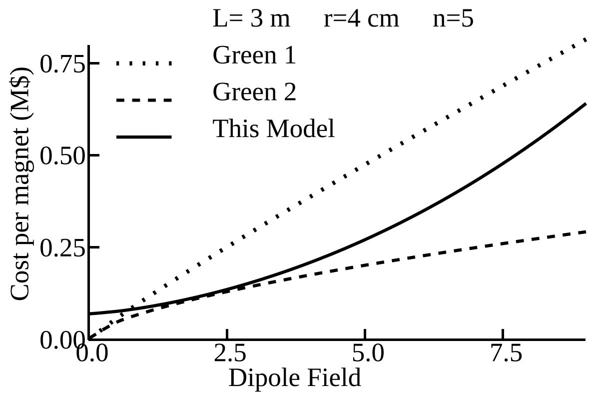

Figure 2: Dipole Magnet costs vs Field for

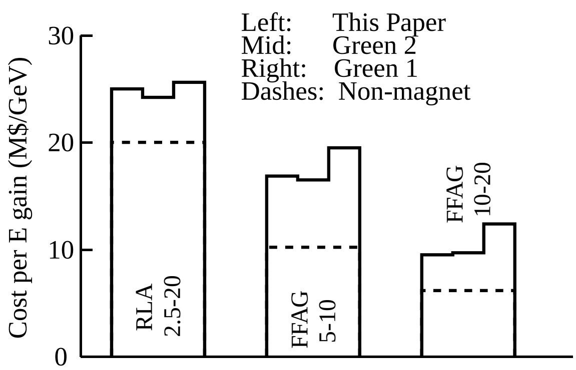

Figure 3: Cost comparison using different models.

a coil inside radius $k_R \times R$ of 4 cm, and a number of magnets n of 5. The relatively small number n was chosen for the comparison because the magnets used in Green's analysis were made in relatively small numbers [4]. Figure 2 shows the cost prediction by Green's formulae and the model given here. All three are seen to predict similar costs, with the new prediction falling between those of the two earlier formulae. Since the second of these formulae has the better fit, more attention should be paid to the comparison of this second formula with the new estimate. It is seen that, for magnets in the 2 to 4 T range, they agree well in both magnitude and slope. This range roughly corresponds to the fields of most of the magnets in the fit [4]. The increased cost at higher fields is expected as the current carrying capacity of NbTi superconductor falls.

Figure 3 shows the costs of three full machines (including costs other than magnets) according to all three formulas. This figure demonstrates that the relative costs of different accelerator designs are still independent of the model chosen. This is unlikely to be true for all cases, so it would be interesting to find two machine designs where one was more cost effective according to one cost model and the other was more cost effective according to the other cost model.

OTHER COSTS

To properly model the cost of accelerator systems, costs in addition to magnet costs must be considered. Such costs are here broken up into RF costs and linear costs. Linear costs are assumed proportional to the length of the accelerator. They take into account tunneling, access, cabling, diagnostics, and many other things. A good coefficient of proportionality to use is 25 k$/m. For RF costs, there are two components: cavity costs and RF Power supply costs. Cavity costs will be proportional to the length of the linac, and thus proportional to the voltage required (V) and inversely proportional to the gradient G. The RF power cost is assumed proportional to the peak power required, which in turn is proportional to V and to G. Thus, the RF cost can be written as

$$\text{RF Cost} = \frac{C_C V G_0}{G} + \frac{C_P V G}{G_0}. \qquad (13)$$

For 200 MHz superconducting (SC) and room temperature (NC) RF, the constants were found [5, 6] to be:

	SC	NC
C_C (M$/GV)	30	10
C_P (M$/GV)	26.8	150
G_0 (MV/m)	16	16

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