

PRESSURE FIELD DISTRIBUTION IN A CYLINDRICAL GEOMETRY WITH ARBITRARY CROSS SECTION

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Abstract

This work presents analytical and numerical results for the pressure field distribution along the axis of tubular geometries with arbitrary axisymmetric cross sections in steady-state. Several areas of applied physics deal with problems in high-vacuum and ultra high-vacuum technology that present tubular form. In many cases one finds tubes with non-uniform cross sections, such as bellows, conical pipes and others, which are frequently present in particle accelerators, colliders, storage rings, gravitational antennas, and electron devices. This work presents and describes in detail the pressure field in tubes with arbitrary axisymmetric cross sections. Details of the mathematical and physical formulations are given; specific conductance and specific throughput are defined; and a detailed discussion about the boundary conditions is presented. These concepts and approach are applied to usual realistic cases, such as conical tubes, with typical laboratory dimensions.

1 INTRODUCTION

Several areas of applied and experimental physics deal with problems in high-vacuum and ultra high-vacuum technology which present tubular geometry, such as particle accelerators, colliders, storage rings, electron devices in general, and mass spectrometers. The systems cited above frequently use parts that present tubular geometry with variable cross section along the tube axis. The mathematical tools available to deal with this kind of geometry (variable cross section) are restricted to an approach that considers the vacuum system as composed by discrete parts. This approach simplifies the treatment, but at the price of giving only average values for the pressure of the system [1]. In this work we present an exact treatment, solving the differential equation for the case of azimuthal symmetry with variable cross section along the axis.

This work deals with a very common case in vacuum technology, the conic tube. From the overall dimensions and material employed, we define the punctual specific conductance and the degassing rate per unit length of the tube. With these we are able to derive the pressure field, as well as the throughput, along the length of the tube [2]. We assume that high-vacuum pumps with equal pumping speed are pumping both extremes of the tube. This assumption is instructive for it allows separating the intrinsic contribution of the tube in the gas transport

within the molecular flow regimen. We analyze the steady-state regimen, but the differential equation is suited to deal with the transient case, if a time-dependent gas source is given. This kind of situation occurs in an accelerator, for instance, when the beam hits the tube walls, causing an impulsive degassing. Another example is the situation where part of the vacuum system is vented, and then evacuated again.

In this work we determine the steady-state pressure field in a conic tube with a constant degassing rate per unit area.

2 PHYSICAL AND MATHEMATICAL MODELING

The adopted model assumes azimuthal symmetry, so that the tube surface can be defined as the revolution of a line around the axis of the tube. Once the tube geometry is defined, other relevant quantities can be defined as well. The specific conductance at each point along the x -axis of the tube may be defined as:

$$c(x) = 96.0 f^3(x) \quad (1)$$

where the function $f(x)$ defines a surface with cylindrical symmetry by revolution around the x -axis. The multiplicative constant depends on the gas and the temperature, and this value is valid for N_2 at 293 K. The unit of $c(x)$ is $\text{l.s}^{-1}.\text{cm}$. We will call it the punctual specific conductance. Analogously we can define the specific degassing rate per unit length, which is also a function of x , and can be defined as:

$$q(x) \equiv q_0 2\pi f(x) \left[1 + \left(\frac{df(x)}{dx} \right)^2 \right]^{\frac{1}{2}} \quad (2)$$

where q_0 is the degassing rate per unit area of the material. The unit of $q(x)$ is $\text{mbar.l.s}^{-1}.\text{cm}^{-1}$ [2]. The model assumes the molecular flow regimen, and the diffusion equation will be [2]:

$$\begin{aligned} c(x) \frac{\partial^2 p(x,t)}{\partial x^2} + \frac{dc(x)}{dx} \cdot \frac{\partial p(x,t)}{\partial x} = \\ = -q(x,t) + v(x) \frac{\partial p(x,t)}{\partial t} \end{aligned} \quad (3)$$

Since we will deal only with the steady-state case, the diffusion equation reduces to [2]:

$$c(x) \frac{d^2 p(x)}{dx^2} + \frac{dc(x)}{dx} \frac{dp(x)}{dx} = -q(x) \quad (4)$$

To solve this kind of equation one needs to define the boundary conditions. In general, this is done specifying the pressure and the throughput at a particular point of the tube. The discussion of the boundary conditions will be presented in section 3, with a detailed description of the geometry of the system.

We develop the concepts of punctual specific conductance and specific degassing rate per unit length at each point along the structure axis. In our treatment we consider two vacuum pumps with the same pumping speed, one at each extreme of the structure. The problem is treated in steady state, but we present a brief discussion of a situation where there is a gas burst, like the case where, in an accelerator, the beam hits the tube walls.

3 DEFINITION OF THE GEOMETRY

We are dealing with an axis symmetric (azimuthal symmetry) tube with variable cross-section and we treat a conic tube as an example. The specific conductance changes along the axis, as well as the specific degassing rate per unit length. The gas source is considered to be due to the natural degassing of the materials of the structure, which we consider to be made of stainless steel. In this case the degassing rate per unit area is $q_{ss} = 10^{-9}$ mbar.l.s⁻¹.cm⁻². These values are typical for the initial conditions of this material [1]. A schematic drawing of the conic tube is presented in Fig. 1.

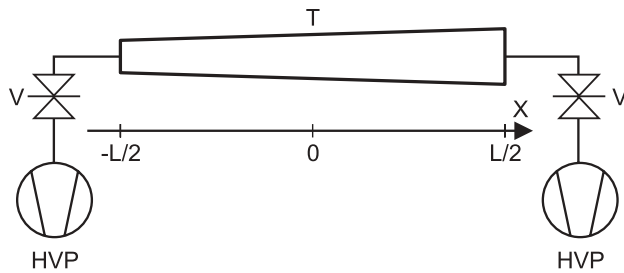


Figure 1: Schematic drawing of the system: T – conic tube; V – valve; and HVP – high-vacuum pump.

The function defining the line that, by revolution around the x-axis, defines the conic tube, is:

$$f(x) = \left(\frac{dM - dm}{2L} \right) \left(x + \frac{L}{2} \right) + \frac{dm}{2}$$

where L is the length; dm the smaller diameter; and dM the larger diameter of the tube. The examples that will be given adopt $L = 400$ cm, $dm = 3$ cm, and $dM = 6$ cm. The total throughput generated by the walls of the tube is $Q_T = 5.66 \times 10^{-6}$ mbar.l.s⁻¹. We can write the throughput at the extremities of the tube as:

$$Q_T = Q_L + Q_R = S_L p(-L/2) + S_R p(+L/2) \\ = c(-L/2) \frac{dp(x)}{dx} \Big|_{x=-L/2} - c(+L/2) \frac{dp(x)}{dx} \Big|_{x=+L/2}$$

The physically acceptable solutions are found imposing boundary conditions. We consider that the left and the right extremities are being pumped with the same effective pumping speed, and treat three different cases: $S_L = S_R = 5$ l.s⁻¹ (dotted lines in Figs. 3 and 4); $S_L = S_R = 50$ l.s⁻¹ (dashed lines in Figs. 3 and 4); and $S_L = S_R = 500$ l.s⁻¹ (solid lines in Figs. 3 and 4). Those values were chosen in order to show the influence of the pumping speed in the pressure at the extremes of the tube, because they depend on both the specific conductance and the throughput from the walls, which change with the position along the tube.

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4 RESULTS AND DISCUSSION

The solution was obtained using both analytical and numerical procedures. Figure 2 shows the specific conductance along the x-axis, from -200 cm $\leq x \leq +200$ cm. The specific degassing rate increases linearly along the x-axis, since it depends on the perimeter of the tube.

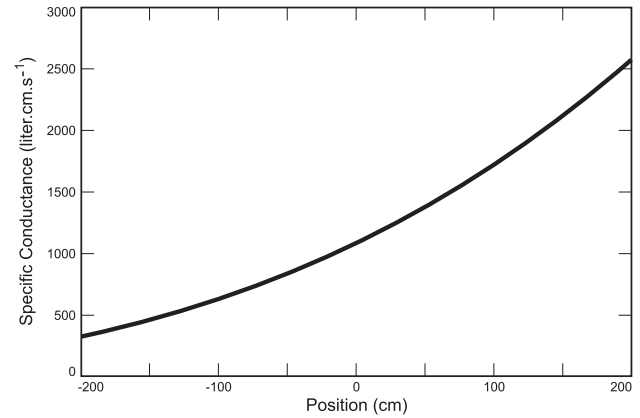


Figure 2: Specific conductance along the axis of the conic tube.

Once the characteristics of the system are specified and the boundary conditions adopted, we can solve equation 4 and find the steady-state pressure field along the tube. Figure 3 shows the pressure field distribution along the x-axis, while Fig. 4 shows the throughput along the same axis.

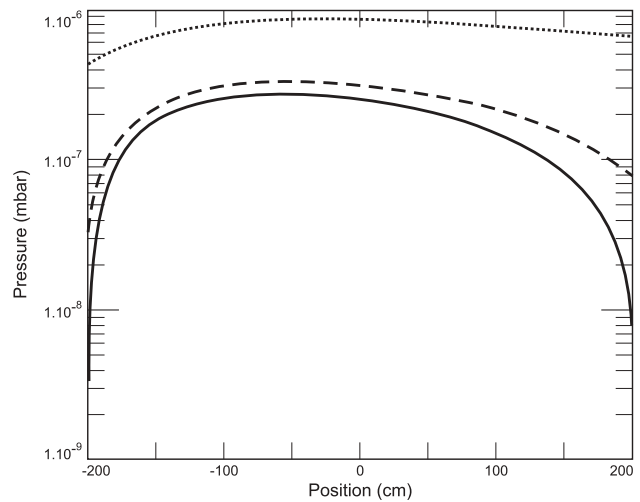


Figure 3: Pressure field along the axis of the conic tube. $S_L = S_R = 5$ l.s⁻¹ (dotted line); $S_L = S_R = 50$ l.s⁻¹ (dashed line); $S_L = S_R = 500$ l.s⁻¹ (solid line).

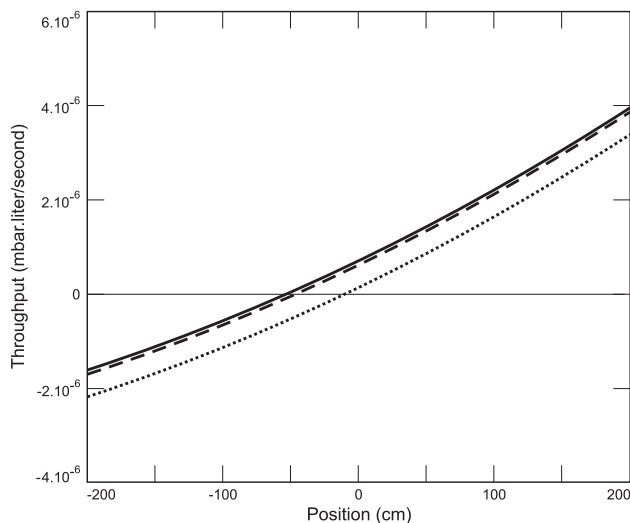


Figure 4: Throughput along the axis of the conic tube. $S_L = S_R = 5 \text{ l.s}^{-1}$ (dotted line); $S_L = S_R = 50 \text{ l.s}^{-1}$ (dashed line); $S_L = S_R = 500 \text{ l.s}^{-1}$ (solid line).

Several effects contribute to produce an asymmetric pressure field distribution as shown in Fig. 3. The first is that the specific conductance changes along the tube. In addition one must also consider the change in throughput caused by the change in diameter of the tube.

Figure 3 shows the pressure fields for the three pumping speeds considered. For the lowest pumping speed (5 l.s^{-1} at both ends, dotted line) the pressure does not change markedly along the tube, while the pressure gradient changes slowly. This happens because the pumping speed is much smaller than the specific conductance presented at any point along the tube, so that the pumping process is limited by the pumping speed and not by the specific conductance. The point of maximum pressure occurs at $x = -13.4 \text{ cm}$.

For the other two cases presented, the situation changes. For $S = 50 \text{ l.s}^{-1}$ (dashed line in Fig. 3) one can see that the pressure varies more steeply (higher gradient in absolute value) at the left hand side of the tube, next to the end, since the specific conductance is lowest in this region. The point of maximum pressure occurs at $x = -48.6 \text{ cm}$. For the case with $S = 500 \text{ l.s}^{-1}$ this effect is even more evident, and the point of maximum pressure occurs at $x = -55.4 \text{ cm}$. For these two cases the specific conductance dominates the pumping process: one can see this by the little difference present by the two respective curves in Fig. 3. The differences are more noticeable close

to the extremes of the tube, where the effects of the conductance are not dominant.

Figure 4 shows the results for the throughput: one can see that, in absolute value, the throughput pumped by the pump at the left hand side added to the throughput pumped by the pump at the right hand side equals the total throughput of the system. From Fig. 4 one can also see that, for the highest pumping speeds, the throughput changes very little, showing the dominance of the specific conductance in those situations.

This kind of analysis can be very useful in the design of vacuum systems, in order to optimize the pumping speed and number of pumps along the vacuum line, since increasing the pumping speed does not ensure a significant reduction of the pressure in the tube (except close to the extremes).

A component also widely used in vacuum systems is the bellows, which could be treated in a similar manner, but paying attention to some important differences. In this case, the radial specific conductance must be considered explicitly, since its magnitude is comparable to the axial specific conductance. This increases the complexity of the calculation, but it can be done within the same approach. This case will be presented in a forthcoming article.

5 ACKNOWLEDGMENTS

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6 REFERENCES

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