PRINCIPLE OF SKEW QUADRUPOLE MODULATION TO MEASURE BETATRON COUPLING*

Y. Luo, F. Pilat, T. Roser, D. Trbojevic, J. Wei
Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

The measurement of the residual betatron coupling via skew quadrupole modulation is a new diagnostics technique that has been developed and tested at the Relativistic Heavy Ion Collider (RHIC) as a promising method for the linear decoupling on the ramp. By modulating the strengths of different skew quadrupole families the two eigentunes are precisely measured with the phase lock loop system. The projections of the residual coupling coefficient onto the skew quadrupole coupling modulation directions are determined. The residual linear coupling could be corrected according to the measurement. An analytical solution for skew quadrupole modulation based on the Hamiltonian perturbation theory is given, and simulation code using smooth accelerator model is also developed. Some issues concerning the practical applications of this technique are discussed.

INTRODUCTION

For a hadron machine like RHIC, the working points are constrained in a very narrow space. It is demanded to keep the two tunes close to the linear difference resonance line in order to get better beam lifetime. RHIC presently has been equipped with applications for static linear coupling correction at injection and store. However, the coupling correction on the ramp is still an open problem, which become more and more important in the past few years, especially during the polarized proton operations. In order to avoid polarization loss, it is essential to keep the tunes almost constant on the ramp for RHIC polarized proton operation. The coupling changes on the ramp with variations of beam optics and closed orbit distortion. A fast, robust mechanism and a set of reliable diagnostics are needed to accomplish the task.

A skew quadrupole strength scan is the general way to decouple the machine globally. However, it is not suitable for coupling correction on the ramp because of the needs of moving tunes to get the minimum separation before correction, slow and large range scan to find the correction strength, bad resolution to determine the minimum tune separation, and possibility of causing beam abortion during scan. As a logical extension, T. Roser suggested skew quadrupole modulation to replace the skew quadrupole scan for the ramp global coupling correction [1]. The basic point is to reduce the amplitude ratio of $1f$ peak to $2f$ peak in the fast fourier transform (FFT) plot of $(Q_1-Q_2)^2$, where the f is the skew quadrupole strength modulation frequency, Q_1, Q_2 are two tunes' decimal parts. The idea is proved in the analytical solution shown below. To track the tune changes during modulation, phase lock loop (PLL) tune measurement data are used. The PLL system has been commissioned in RHIC for years and has become an indispensable tool in the routine performance of RHIC. This article focuses on the analytical solution, simulation study and discussions about the use of this technique. Results from beam experiments in RHIC are presented in these proceedings [2].

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ANALYTICAL SOLUTION

There are several approaches to tackle the linear coupling. Since we are interested in the two global tunes' response to the skew quadrupole modulation, perturbation theory [3, 4] based on Hamiltonian mechanism is more straight and simple and is therefore adopted here. The two eigentunes Q_1, Q_2 are given by

$$Q_1 = Q_{x,0} - \frac{\Delta}{2} + \frac{1}{2}\sqrt{\Delta^2 + (C^-)^2}, \qquad (1)$$

$$Q_2 = Q_{y,0} + \frac{\Delta}{2} - \frac{1}{2}\sqrt{\Delta^2 + (C^-)^2}. \qquad (2)$$

the tune separation's square is

$$(Q_1 - Q_2)^2 = \Delta^2 + (C^-)^2, \qquad (3)$$

where $Q_{x,0}$ and $Q_{y,0}$ are the uncoupled tunes without any couplers, Δ is the distance to the linear difference coupling resonance, C^- is the total coupling coefficient at one observation point, which normally is a complex number. Δ and C^- are defined as:

$$\Delta = Q_{x,0} - Q_{y,0} - p, \qquad (4)$$

$$C^- = \frac{1}{2\pi}\oint \sqrt{\beta_x\beta_y}k_s e^{i(\Psi_x-\Psi_y-\Delta\frac{2\pi s}{C})}dl. \qquad (5)$$

In order to distinguish the different sources of coupling coefficient, in the following we design C^-_{tot} as the total coupling coefficient, C^-_{res} the residual coupling coefficient, C^-_{mod} the induced coupling coefficient by the skew quadrupole modulation. To simplify narration, we will not distinguish the coupling coefficient and coupling, either.

If one skew quadrupole's current modulates, so does the induced coupling too. Assuming the modulation amplitude of the coupling is $C^-_{mod,amp}$, the induced and total coupling are given by

$$C^-_{mod} = C^-_{mod,amp}\sin(2\pi f t) \qquad (6)$$

$$C^-_{tot} = C^-_{res} + C^-_{mod} \qquad (7)$$

We introduce the slow varying approximation, that is, the skew quadrupole modulation frequency is slow enough so that the eigentunes are only determined by the modulation strength. Of course, the modulation amplitude should be small so that the above perturbation theory still holds. Substituting Eq. 6 into Eq. 3, we get

$$\begin{aligned}(Q_1 - Q_2)^2 \quad &= \quad \Delta^2 + |C^2_{res}|^2 + \frac{1}{2}|C^2_{mod,amp}|^2 \\ &\quad +2|C^-_{res}||C^-_{mod,amp}|\cos(\varphi)\sin(2\pi ft) \\ &\quad -\frac{1}{2}|C^-_{mod,amp}|^2\cos(4\pi ft) \qquad (8)\end{aligned}$$

where the φ is the angle difference between C^-_{res} and $C_{mod,amp}$. $|C^-_{res}|\cos(\varphi)$ is the projection of the residual coupling onto the modulation coupling.

Now it is clear from Eq. 8 that the $2f$ item is only related to skew quadruple modulation amplitude, the $1f$ item is related to the dot multiplication of the modulation coupling and the residual coupling. In the frequency domain of $(Q_1 - Q_2)^2$ during skew quadruploe modulation, we will see two peaks located at $1f$ and $2f$ if the machine is originally coupled, or only the $2f$ peak if the machine originally well decoupled. So the $1f$ peak is one reflection, or observable of the global residual coupling in one accelerator.

In a real machine it is therefore useful to use this technique to do fast measurements of residual coupling with two or more different skew quadruple modulations at injection, store and hopefully on the ramp. Knowing the residual coupling's amplitude and angle, it is possible to carry out global coupling correction with known skew quadrupole correctors. This correction technique could be used in feed forward or feedback scenarios.

SIMULATION

With the above analytical solution, a simulation program has been developed based on smooth accelerator model. The uncoupled motion is simply represented by

$$\begin{aligned} x^{''} + (\frac{Q_{x,0}}{R})^2 x \quad &= \quad 0, \\ y^{''} + (\frac{Q_{y,0}}{R})^2 y \quad &= \quad 0. \qquad (9)\end{aligned}$$

where R is the average radius of the circular accelerator. The artificial skew quadrupoles with zero length are distributed along the ring, the 4×4 transfer matrix between skew quadrupoles are diagonalized.

For each RHIC ring, there are three families of correction skew quadrupoles due to the six-fold structure of the ring. If appropriately powered, the coupling coefficients from the three families are $120°$ apart. So in the simulation, we insert three skew quadrupoles equidistantly in the ring model. Each quadrupole could have constant, modulated strengths, or both. The constant strengths of three skew quadrupoles present the residual coupling. The uncoupled tunes are $(Q_{x,0}, Q_{y,0}) = (28.22, 29.23)$, the average radius $R = C/2\pi$, C is the RHIC ring circumference 3833.84 m. For every tracking, one particle with initial coordinates is launched at the location of first skew quadrupole and successive particle x and y coordinates are recorded at the third quadrupole location.

Fig. 1 shows the two tunes and Fig. 2 is the FFT of the $(Q_1 - Q_2)^2$ from simulation when the third skew quadrupole strength modulates with integrated strength amplitude $((k_s dl)_3)_{amp,mod} = 0.0005$ m^{-1}, modulation frequency 0.5 Hz, under condition that the first and the second skew quadruple strengths are constant at $(k_s dl)_{1,2} = 0.0005$ m^{-1}.

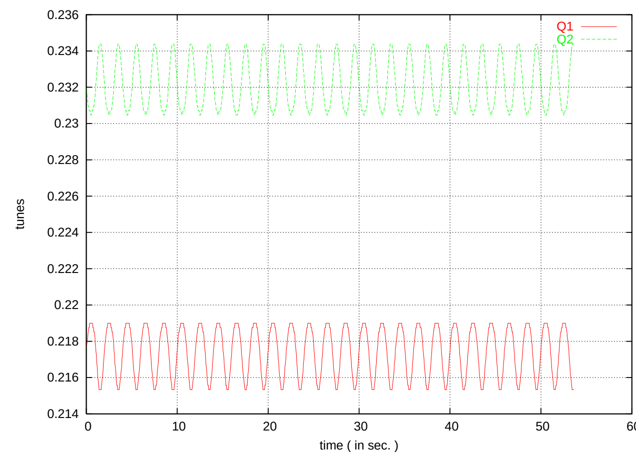

Figure 1: An example of tunes from simulation during skew quadruploe modulation. Every set of tunes (Q_1,Q_2) are achieved from FFT of 4096 turns' $(x + y)$ data.

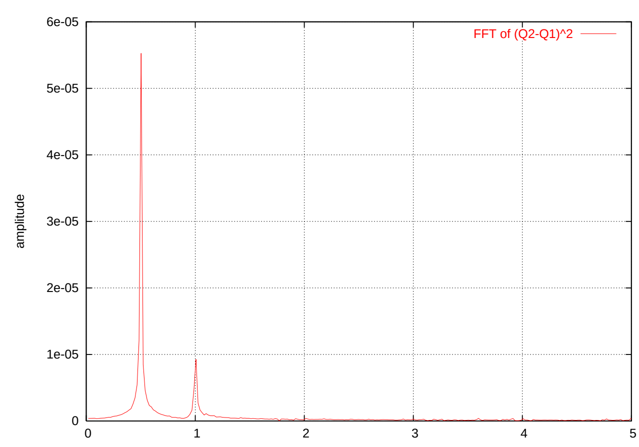

Figure 2: An example of $(Q_2 - Q_1)^2$ FFT spectrum from simulation. This plot is obtained from 1024 sets of tunes from Fig. 1.

DISCUSSION

In the following we discuss how to use the results from the skew quadrupole modulation and some specific issues connecting its practical applications.

Projection Ratio

A straight use of Eq. 8 is the projection ratio of the residual coupling coefficient onto the modulation coupling direction. It is defined as:

$$\kappa = \frac{|C^{-}_{res}|\cos(\varphi)}{|C^{-}_{mod,amp}|}. \quad (10)$$

κ is a dimensionless quantity, however it has its sign which is decided by the difference angle φ between the residual and modulation couplings. From FFT of $(Q_1 - Q_2)^2$, we could get $1f$ and $2f$ peaks' amplitudes, say A_{1f}, A_{2f}, then

$$|\kappa| = (\frac{A_{1f}}{A_{2f}})/4. \quad (11)$$

The sign of κ is same as the constant part of FFT of the productions of $(Q_1-Q_2)^2 \times I_0 \sin(2\pi f t)$, where $I_0 \sin(2\pi f t)$ is skew quadrupole modulation power supply current, I_0 is normally positive.

Modulation Strength and Frequency

The skew quadrupole modulation have the advantage over skew quadrupole scans that the modulation strength could be small in operation. It is very important not to deteriorate the beam lifetime during measurement. Eq. 11 shows that the $1f$ peak amplitude is enlarged by 4 times comparing to the $2f$ peak in the $(Q_1-Q_2)^2$ FFT plot if the residual and induced couplings' amplitudes are the same. Skew quadrupole modulation is eligible to measure small residual coupling.

The skew quadrupole modulation frequency is chosen between 0.2 – 1.0 Hz for RHIC to fulfil the slow varying approximation and system limitation in the power supply system and PLL response. It has be shown from simulation and beam experiment that there will be spurious peaks in the $(Q_1 - Q_2)^2$ FFT plot, multipoles of modulation frequencies if the frequency is too large. On the other hand, if the modulation is too slow, the measurement can be too long to accommodate the ramp.

RHIC Coupling Measurement and Correction

Two projections onto two known directions are needed to determine the residual coupling. We use the same skew quadrupole families to modulation measurement and correction at RHIC. RHIC has three correction skew quadrupole families, F1,F2, F3. The contributions to the global coupling from each skew quadrupole in one family are same due to the six-fold lattice. The coupling from the three families with same strength and proper polarizations are only $120°$ phase different. Knowing the projections onto at least two families, the residual coupling will be obtained.

The above $120°$ difference is important to get specific residual coupling values from measurement. It holds for RHIC at injection, store and on the ramp, if the following conditions are met:

- the betatron phase advances in every sextant are same, or almost same.
- the Q_x, Q_y integer part difference equals 1.
- the Q_x, Q_y decimal part difference is small, say $\leq$ 0.015.

Orthogonal Modulations

It is wise to choose two skew quadrupole modulation families whose coupling contributions are orthogonal. Then the correction strength is straightly obtained from the projection ratio,

$$(k_s dl)_{corr} = -\kappa \times (k_s dl)_{amp,modu} \quad (12)$$

For RHIC the coupling contributions of three skew quadrupole are $120°$ apart, so it is easy to construct two orthogonal families. For example, we could modulate F1 and F3 simultaneously with same frequency and phase to produce a coupling normal to that only from F2 modulation.

Two Frequency Modulations

In order to shorten the time occupied by the measurement, we could modulate two skew quadrupole families with two different modulation frequencies at the same time. From analytical solution and simulation, it shows there are more peaks in $(Q_2 - Q_1)^2$ FFT plot. If the two modulation frequencies are f_1 and f_2, there will be peaks with following frequencies in the $(Q_2 - Q_1)^2$ FFT plot : f_1, $2f_1$, f_2, $2f_2$, and $|f_2 \pm f_1|$. The modulation frequency should be chosen so that these peaks do not overlap. The projection ratio defined in Eq. 11 is still valid for each modulation frequency. $|f_2 \pm f_1|$ peaks are of the same amplitude, which only decided by the two induced modulating couplings.

Simulation Studies

Simulation of coupling diagnostics and corrections are done based on the smooth accelerator model. It shows that the measurement and correction work well. In order to get better coupling correction, iteration of measurement and correction is necessary.

MADX simulations of the tune shifts with skew quadrupole modulation show that the tune change due to the residual orbit in the skew quadrupoles is small compared to that from the skew quadrupole strength modulation.

The element-by-element tracking based on the RHIC model with skew quadrupole modulation is in progress.

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