OPERATIONAL MEASUREMENT OF COUPLING BY SKEW QUADRUPOLE MODULATION[*]

Y. Luo, P. Cameron, R. Lee, A. Marusic, F. Pilat, D. Trbojevic, T. Roser, J. Wei

Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

The measurement and correction of the residual betatron coupling via skew quadrupole modulation is a new diagnostics technique that has been developed and tested at the Relativistic Heavy Ion Collider (RHIC) as a very promising method for the linear decoupling on the ramp. By modulating the strengths of the skew quadrupole families the two eigentune modulations are precisely measured with a high resolution phase lock loop system. The projections of the residual coupling coefficient onto the skew quadrupole coupling modulation direction are determined. The residual linear coupling could be corrected according the measurement. We report the results from the dedicated beam studies carried on at RHIC injection, store and on the ramp. A capability of measuring coupling on the ramp opens possibility of continuous coupling corrections during acceleration.

INTRODUCTION

An analytical solution [1] to the skew quadrupole modulation based on the Hamiltonian perturbation theory [2, 3] has be achieved, which gives the tune difference square as

$$
\begin{aligned}
(Q_1 - Q_2)^2 &= \Delta^2 + |C_{res}^2|^2 + \frac{1}{2}|C_{modu,amp}^2|^2 \\
&\quad + 2|C_{res}^-||C_{mod,amp}^-|\cos(\varphi)\sin(2\pi f t) \\
&\quad - \frac{1}{2}|C_{mod,amp}^-|^2\cos(4\pi f t),
\end{aligned}
\tag{1}
$$

where $Q_{1,2}$ are the eigentunes, C_{res}^-, $C_{mod,amp}^-$ are residual coupling and the induced coupling modulation amplitude, φ the angle difference between them, f the modulation frequency. C^- is coupling coefficient,

$$
C^- = \frac{1}{2\pi}\oint \sqrt{\beta_x\beta_y}k_s e^{i(\Psi_x - \Psi_y - \Delta\frac{2\pi s}{C})}dl.
\tag{2}
$$

The projection ratio κ of the residual coupling onto the modulation coupling direction is defined as:

$$
\kappa = \frac{|C_{res}^-|\cos(\varphi)}{|C_{mod,amp}^-|}.
\tag{3}
$$

A straight forward way to obtain the projection ratio is to perform the FFT of $(Q_1 - Q_2)^2$, which can be obtained from

$$
|\kappa| = (\frac{A_{1f}}{A_{2f}})/4,
\tag{4}
$$

where $A_{1f,2f}$ are the amplitudes of the $1f$ and $2f$ peaks. κ's sign is same to that of $\sum_{i=1}^{N}(Q_1 - Q_2)_i^2 \times i_{mod,courrent}$. Knowing the residual coupling projections onto the coupling modulation directions, correction could be carried out with known skew quadrupole families.

DATA ACQUISITION AND ANALYSIS

PLL Tune measurement

The RHIC phase lock loop (PLL) system has been used for fast and high resolution measurement of the eigentunes during the skew quadrupole modulation. Its data acquisition frequency is 177 Hz, which is much faster than the skew quadrupole modulation frequency, ranging from 0.2–1.0 Hz. The full frequency span of the $(Q_1 - Q_2)^2$ FFT spectrum is 88.5 Hz. Figure 1 shows the PLL readings from three different modulations of the RHIC blue ring three skew quadrupole families. Figure 2 is the zoom-in plot of Fig. 1.
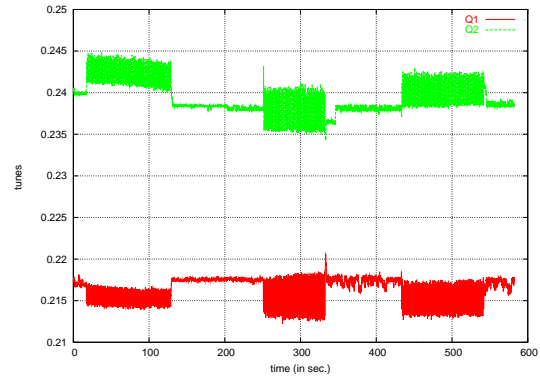

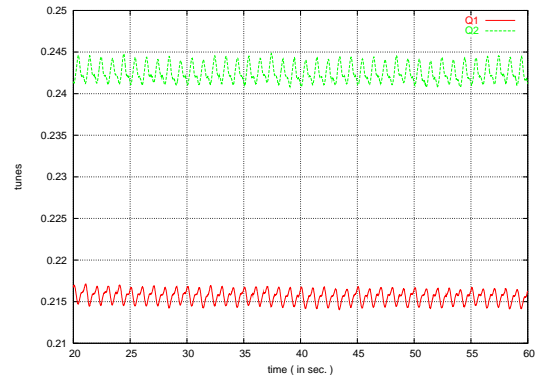Figure 1: PLL tune readings for 3 modulations at injection.

Figure 2: Zoom-in of the above PLL tune readings.

FFT

FFT is a natural choice for the data processing according to Eq. 1. However, in order to get better resolution of the FFT spectrum in the low frequency range below 2Hz, at least 2048 continuous sets of $Q_{1,2}$, or about 12 second of skew quadrupole modulation time is needed. In the beam experiment, 4096 continuous $Q_{1,2}$ points are normally used for FFT. Figure 3 shows the FFT of the first modulation in Fig. 1, the projection ratio from FFT is 0.622.
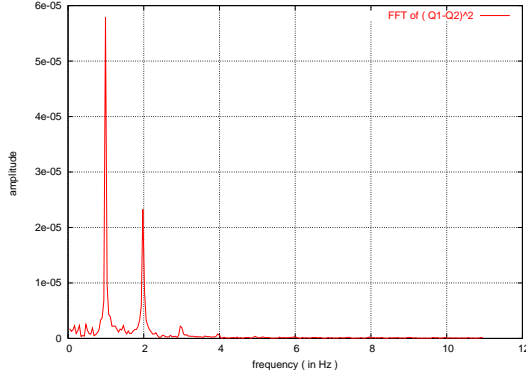

Figure 3: FFT spectrum of $(Q_1 - Q_2)^2$ for Fig. 2 data.

Linear Regression

In order to shorten the PLL data taking time on the ramp, linear regression on Eq. 1 is used. The fitting function is assumed as:

$$
\begin{aligned}
f(t_i) \quad = \quad & A + B_1 \sin(2\pi f t_i) + B_2 \cos(2\pi f t_i) \\
& + C_1 \sin(4\pi f t_i) + C_2 \cos(4\pi f t_i) \\
& + E t_i + F t_i^2.
\end{aligned} \tag{5}
$$

We use the linear regression technique to minimize the error function χ^2:

$$
\chi^2 = \sum_{i=1}^{N} [(Q_1 - Q_2)^2 - f(t_i)]^2. \tag{6}
$$

The projection ratio is then obtained:

$$
|\kappa| = \frac{\sqrt{B_1^2 + B_2^2}}{\sqrt{C_1^2 + C_2^2}}. \tag{7}
$$

In Eq. 5 the linear and quadratic terms of time t_i are introduced due to a shift of the uncoupled tunes during the measurements, as observed on ramp.

Figure 4 shows the fitting result for $(Q_1 - Q_2)^2$ from the same PLL tune measurement data as that for Fig. 3. Eight modulation PLL periods' of the raw data are used for fitting (normally 2 or 3 modulation period's data are enough). The projection ratio obtained from fitting is 0.574. Figure 5 represents the time dependence of the projection ratios. Here the linear regression is performed every 2 s, or 2 periods of skew quadrupole modulations. Using this

method the modulation measurement time can be reduced below 10 s and it opens the possibility to do continuous measurement of coupling on the whole ramp. However FFT data analysis is still needed sometimes, when the PLL data qaulity is low. FFT is more robust and reveals more physics than the linear regression.
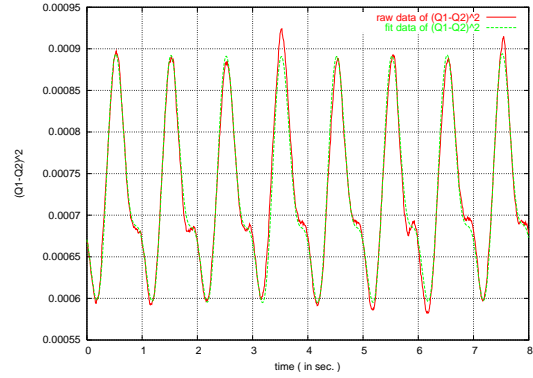

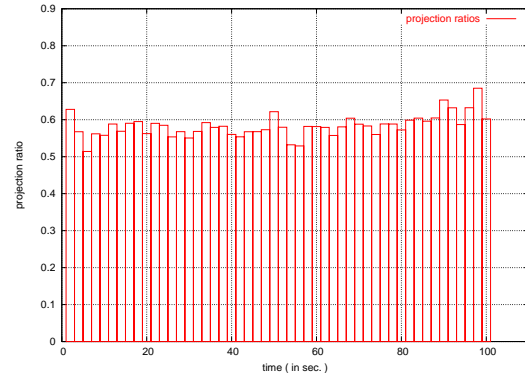Figure 4: Linear regression of $(Q_1 - Q_2)^2$ at injection.

Figure 5: Continuous measurements at injection.

MEASUREMENTS

During the experiment time the measurement data analysis is basically off-line. Results of the measuremnts from modulations are compared with the known coupling sources, which were artificially induced into a well decoupled machine. Much more experiment time was given to measurements on ramp since there good quality PLL data were not easy to get. An on-line application is being developed that will help operational development in Run 2005.

Measurements at injection

Every RHIC ring has 3 skew quadrupole correction families. Table 1 shows one example of coupling measurement with the three families. In the table the coupling strengths are given in current unit A. The top block of Table 1 gives residual coupling projection amplitudes and directions on the three families. The bottom block of Table 1 gives the residual couplings calculated from the two above measured

projections in the three measurements. The residual couplings from different two projection combinations show good agreement in strength and direction. This measurements were taken in the RHIC Blue ring after changing F3's integrated strength by -0.0004 m^{-1}. Family F3's coupling contribution direction is 169.7° from model when positively powered. The average of the measured residual coupling direction is about 338.07°, about 10° different from the direction of -F3 from the model.

Table 1: Measurement results at injection

Family	Amplitude(A)	Direction
F1	2.04	289.5°
F2	0.92	49.0°
F3	3.45	169.7°
(F1,F2)	3.00	336.8°
(F2,F3)	3.59	333.8°
(F3,F1)	3.47	343.6°

Measurements at Store

Table 2 gives a measurement example at store, where we modulated skew quadrupole family F1 and F3 simultaneously with same strength and phase, which produced a modulation (F1F3) with direction orthogonal to that of family F2. The top and bottom block of Table 2 give the measure med projections before and after Family F3 integrated strength change from -0.0004 m^{-1} to -0.0002 m^{-1}.

Table 2: Measurement results at store

Family	Projection(A) from FFT	Projection(A) from FIT
F2	0.32	0.32
F1F3	0.22	0.19
F2	1.37	1.48
F1F3	2.53	2.45

The change in strength of F3 of 0.0002 m^{-1} requires a 2.2 A change of its power supply current. From the measurement the projection changes onto F2 and F1F3 coupling direction are ~ 1.0 A, ~ 2.2 A, respectively. The predications from model for them are 1.1 A and 1.9 A, respectively. The coupling amplitude measurement error is within 15%. To reduce the measurement error, more measuremnts are necessary.

Measurements on the Ramp

Coupling measurement on the ramp with the skew quadrupole modulation technique posed some challenges to the RHIC PLL system during the beam experiments last run. The observables for the skew quadrupole modulation are the eigentunes, and it is important for the PLL system to give stable and reliable tunes during the modulation. However, because of energy ramp, $\beta^\star$ squeezing, closed orbit changes and the induced modulating coupling, sometime PLL system couldn't keep track of the two tunes during beam experiments. Figure 6 shows an example of the PLL losing lock on the ramp, where the modulation frequency is 0.2 Hz. Figure 7 shows the projection ratios in the measurement correspnding to the valid data in Figure 6, where the projection ratios increase with time is due to the constant skew quadrupole modulation amplitude on energy ramp.
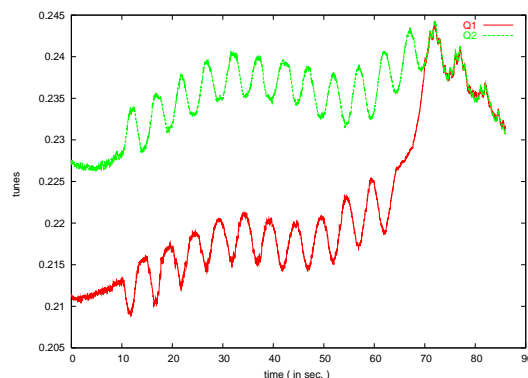

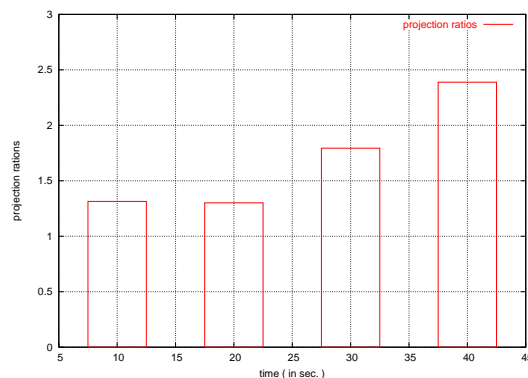Figure 6: An example of PLL losing lock on ramp modu.

Figure 7: Projection ratios on ramp with constant skew quadrupole current modulation amplitude.

In order to reduce the demands on the RHIC PLL system, we shortened the modulation time, or lowered the skew quadrupole modulation frequencies in the beam experiments. Besides the PLL development, several attempts are under investigation. A very promising scheme coupling phase modulation, which doesn't care too much about the detailed tune data during the modulation, was put forth by Y. Luo and is waiting to be tested in the next RHIC run.

REFERENCES

[1] Y. Luo, et al., these proceedings.

[2] G. Guignard, CERN Report No. 76-06, 1976 (unpublished).

[3] G. Guignard, Phys. Rev. E **51**, p6104, 1995.