DISPERSION CORRECTION IN HERA

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Abstract
The electron-proton collider HERA (Hadron Electron Ring Accelerator) at the DESY laboratory in Hamburg has been in operation since 1991. After the luminosity upgrade of HERA in 2001 the control of the horizontal and vertical dispersion function of the positron beam became more important than before. Deviations from the design dispersion in the horizontal plane can change the emittance of the electron beam significantly, leading to a reduction of the luminosity. For optimizing the polarization of the electron beam the reduction of vertical orbit and dispersion distortions is important. In this paper the combined dispersion and orbit correction in HERA is described and first results are reported.

INTRODUCTION

In HERA positrons (or electrons) with an energy of 27.5 GeV and protons with an energy of 920 GeV are brought into collision in a double ring machine. Beside the unique feature of colliding different particle species the machine makes use of the self-polarization mechanism in combination with spin-rotators to provide a longitudinal polarized positron beam for experiments in HERA-e.

Until 2000 an integrated luminosity of $L = 180$ pb$^{-1}$ has been delivered for the two experiments ZEUS and H1. For an additional increase of $L$ the interaction regions (IRs) of the machine were upgraded in 2000/2001 to decrease the spot size of both beams in the two interaction points (IPs) by a factor of three. Furthermore two more spin-rotator pairs were installed to provide longitudinal polarized positrons also for H1 and ZEUS [1]. Polarization has been used before the luminosity upgrade only by the internal target experiment HERMES.

The increase of the specific luminosity is achieved by stronger focusing at both IPs. The final magnets of the quadrupole triplets of HERA-e were installed partly inside the experimental detectors to reduce the beta function $\beta z^*$ at the IPs. To achieve a smaller emittance in HERA-e the focusing in the arcs is increased and the machine is operated with a RF-frequency slightly above the center-frequency.

ORBIT CONTROL AND CORRECTION

The correction and control of the closed orbit and the dispersion function became far more important for HERA-e after the luminosity upgrade:

- To optimize the luminosity the beam spot sizes of the colliding beams at the IP $\sqrt{\epsilon \beta y}$ have to be equal. As the emittance $\epsilon$ depends on the dispersion function $D_x$ and its derivative $D'_x$ in the bending magnets, deviations in $D_x$ can reduce the luminosity drastically.
- Synchro-betatron resonances near the working point of HERA-e can be driven by oscillatory closed orbit and dispersion distortions [2].
- Correction of the vertical closed orbit $y(s)$ and the vertical dispersion $D_y(s)$ is of particular importance to achieve high polarization in HERA-e, because unwanted horizontal magnetic fields can reduce severely the equilibrium polarization. Also the horizontal orbit correction is important due to the longitudinal spin orientation in the straight sections. A good correction is the base for further polarization optimization using harmonic bumps.

During normal operation only global orbit corrections with a small number of corrector magnets based on the MICA O [3] algorithm are used to reduce deviations between the measured orbit $\vec{u}_m$ and a goal orbit $\vec{u}_g$ at the BPMs, with $u=x$ or $y$. The goal orbit takes care of the non-centered closed orbits in the arcs due to the RF-shift, the steering of the orbit to reduce the particle background at the IPs, and design beam offsets of BPMs near the IRs.

METHOD

The problem to solve for the combined orbit and dispersion correction can be formulated as

$$\vec{u}_g = \vec{u}_m + S \Delta \vec{u}'$$ and

$$\vec{D}_{u,g} = \vec{D}_{u,m} + R \Delta \vec{u}'$$,

using the response matrix $S$ for the orbit and $R$ for the dispersion function, which are the change of the orbit $\Delta u_i$ and the dispersion $\Delta D_i$ at BPM $i$ due to the kick change $\Delta u'_j$ of a corrector magnet $j$. The dispersive contribution $\vec{D}_{u,g}$ to the orbit has to be removed from $\vec{u}_m$ before. In total $M = 281$ BPMs and $N = 279$ horizontal and 277 vertical corrector magnets can be used for the correction.

In linear approximation the orbit change at BPM $i$ can be written as

$$\Delta u_i = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi Q)} \Delta u'_j \cos(|\phi_i - \phi_j| - \pi Q)$$,

with the beta function $\beta$, the phase function $\phi$ and the tune $Q$ in plane $u$. From these elements the orbit response matrix $S$ with the elements $S_{ij} = \Delta u_i/\Delta u'_j$ can be constructed.

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For the dispersion response matrix \( R \) with \( R_{ij} = \frac{\Delta u_{i,j}}{\Delta u'_j} \) a similar but more complicated expression can be given (e.g. [4] for \( R_{u,y} \)), which depends on the orbit change in quadrupole and sextupole magnets.

All these analytical formulas have to use approximations and neglect higher order effects. We use therefore the program MAD [5] to determine the response matrices \( S \) and \( R \) by computing the orbit and dispersion changes for realistic changes of all corrector magnets.

To maintain a high level of polarization the beam energy \( E \) has to be kept constant; otherwise the spin-tune changes and the beam can be depolarized. As the kick of a corrector magnet would result in a change of the length of the closed orbit by \( \Delta L = D_{u,j} \Delta u'_j \), but the length \( L \) can’t be modified due to the fixed RF-frequency, the momentum of the beam is changed by

\[
\frac{\Delta p}{p} = \frac{\Delta u'_j D_{u,j}}{\alpha_c L}, \quad (4)
\]

with the dispersion \( D_{u,j} \) at the corrector magnet and the momentum correction factor \( \alpha_c \).

Putting all together the set of equations to be solved can be written as

\[
\begin{pmatrix}
(1 - w_R) \tilde{W}_S \Delta \tilde{u}' \\
 w_R \Delta \tilde{D}_u \\
0
\end{pmatrix}
\begin{pmatrix}
\Delta \tilde{u}'
\end{pmatrix}
= \begin{pmatrix}
(1 - w_R) \tilde{W}_S \cdot S \\
w_R \cdot R \\
w_D \cdot \tilde{D}_u
\end{pmatrix} \cdot \Delta \tilde{u}' \quad (5)
\]

with \( \Delta \tilde{u}' = \tilde{u}_g - \tilde{u}_m \) and \( \Delta \tilde{D}_u = \tilde{D}_{u,g} - \tilde{D}_{u,m} \) (comp. [6]).

Weighting matrices \( W_S \) and \( \tilde{W}_S \) have been introduced to take care of the different BPM weightings at different positions where the orbit should be kept constant. The scalar factor \( w_R \) is used to correct the different amplitude of the orbit response and dispersion response due to a corrector kick and allows the shifting between a pure orbit correction or a pure dispersion correction. In \( w_D \) the different factors of (4) have been collected.

**Solution of the Equations**

For HERA-e the number of corrector magnets \( N \ll 2M + 1 \), were \( M \) is the number of BPMs. Therefore (5) is over-determined and can only be solved approximately. Writing (5) in short form as

\[
\bar{b} = M \tilde{x}
\]

minimizing the square of the Euclidean norm of the residual vector

\[
\| \bar{b} - M \tilde{x} \|^2 \rightarrow \min \quad (6)
\]

leads to the normal equations of the least-squares problem

\[
M^T \bar{b} = M^T M \tilde{x} \quad . \quad (7)
\]

For solving the nearly degenerated equations (7) we use the singular value decomposition [7] of the matrix

\[
M^T M = USV^T, \quad (8)
\]

with the diagonal matrix \( D \), with \( D_{ii} = 1/S_{ii} \) and \( D_{jj} = 0 \) for all singular values \( S_{jj} < c S_{11} \) below a threshold \( c < 1 \), where \( S_{11} \) is the biggest singular value. The solution can then be written as

\[
\tilde{x} = (VDU^T) M^T \bar{b} \quad (9)
\]

Variation of the threshold \( c \) allows to increase the quality of the correction by using stronger corrector strength. The usage of a small number of singular values removes effectively the dominating harmonics from the orbit and dispersion function.

**IMPLEMENTATION**

The algorithm of the combined orbit and dispersion correction – which has been already tried out before the luminosity upgrade [8] – has been implemented as a MATLAB [9] program.

After the selection of machine parameters the program loads the optics information and the pre-computed orbit and dispersion response matrices. Within a loop the program measures the closed orbit for different RF-shifts, computes the dispersion function and removes the dispersive component from the horizontal closed orbit.

After that the user can choose the plane for the correction and the SVD-threshold \( c \). In addition the weight parameters and a global scale factor of the correction can be changed. If the correction is accepted by the user the new currents of the corrector magnets are computed, send to the magnet power supply controllers and set for all magnets simultaneously. The combined orbit and dispersion correction has to be iterated due to sextupole and skew quadrupole magnets.

To establish an orbit with good background conditions in the IRs was a long and tedious work. Wrong steering in the IRs can result in heating up the vacuum chamber by synchrotron light. Bad vacuum conditions or even damages can occur. Therefore important constraints are fixed BPM positions near the IRs. These BPMs have higher weight factors compared to BPMs in the arcs.

The program can also work with a machine model of HERA-e by communicating with MAD [5]. The program was tested in advance by using a machine model with realistic field and alignment errors of the magnets.

**RESULTS**

An example of the combined correction of orbit and dispersion is shown for the luminosity optics in fig. 1 and fig 2. BPMs with fixed positions are marked with crosses in fig. 1.

The root-mean-square (RMS) value of the horizontal closed orbit was reduced from \( x_{rms} = 1.3 \text{ mm} \) to \( 0.86 \text{ mm} \) and the dispersion from \( D_{x,rms} = 34 \text{ mm} \) to \( 19 \text{ mm} \). The vertical closed orbit was reduced from \( y_{rms} = 1.1 \text{ mm} \) to \( 0.81 \text{ mm} \) and the dispersion function \( D_{y,rms} = 58 \text{ mm} \) to \( 12 \text{ mm} \). It should be remarked that the horizontal disper-
Figure 1: Orbit before (above) and after correction (below). Crosses show fixed BPM positions in the IRs.

Figure 2: Dispersion before (above) and after correction (below).

**REFERENCES**