OPTIMIZING NON-SCALING FFAG LATTICES FOR RAPID ACCELERATION

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Abstract

An approach to fixed field acceleration using exclusively linear optical elements was first proposed[1, 2] and successfully developed[3, 4] to support rapid, large-emittance muon acceleration required by a Neutrino Factory or Muon Collider. This approach was termed, simply, a non-scaling, fixed field alternating gradient (FFAG) accelerator. Lattices have evolved from the simple F0D0-cell baseline first proposed to a slightly more complex layout that has usually been referred to as a triplet configuration. In this work a methodology is developed for optimizing non-scaling lattices which demonstrates that the appropriate description is minimum momentum compaction; that is least change in path length with momentum. This framework is then used to generate and compare lattices for rapid acceleration as for muon applications.

INTRODUCTION

The production, acceleration, and storage of a muon beam sufficiently intense to drive a Neutrino Factory or Muon Collider[5] requires multi-stage preparation. In a neutrino factory, the ability of, or limits to, accelerating large-emittance beams determines the specifications which upstream systems must meet, particularly the cooling. The downstream storage rings and experiments are presently not the limiting constraint. Acceleration proves, then, not only a difficult stage to develop, it becomes a pivotal one, particularly in the path to this facility. To further complicate issues, acceleration must occur rapidly because of potentially heavy losses from decay. Linear accelerators are the optimal choice in this respect, but, above a few GeV, they become prohibitively expensive. Conventional synchrotrons cannot be used because normal conducting magnets cannot readily cycle in the ramping times required by muon decay, nor do they support ultra-large beam emittances. In the past, the U.S. baseline relied on recirculating linear accelerators (RLAs) with separate, fixed-field arcs for each acceleration turn. Separate arcs permit control over the path length as a function of energy, allowing traversal times to be matched to the rf phase requirements for stable acceleration. Alternative approaches have focused on adapting the Fixed Field Alternating Gradient (FFAG) accelerator first developed and tested at MURA, primarily because of its inherently large longitudinal acceptance. The Japanese approach (KEK), for example, supports a radial-sector FFAG accelerator, but primarily in the context of a single-muon bunch and low frequency, broadband rf. A linear optics approach to fixed field acceleration was also proposed[3] and recent breakthroughs have resulted in a new design for a FFAG accelerator that can support a high-frequency bunch train, or the U.S. scenario.

The first successful lattices were based on a short, $\simeq$ 5 m long, F0D0 cell. The cell structure was simple: a horizontally-focusing quadrupole followed by a combined-function, vertically focusing magnet with the elements separated by 2-3 m for the rf cavity. Alternative base structures have since been explored[6] and a triplet quadrupole structure has been shown to improve performance relative to the F0D0 optics, but only in a particular configuration and strength. This paper discusses the performance issues associated with rapid acceleration and the parameters which underlie critical behavior and differences between lattice schemes. A thin-lens approach is developed to understand and optimize lattices and explains the improved performance of the so-called triplet versus F0D0. Important results include: (1) the triplet configuration is described by a parameter set that is F0D0-like, rather than the conventional focusing telescope; and (2) although the lattices resemble the minimum emittance lattices developed for electron rings, in this application, they represent rather minimum momentum-compaction lattices.

NON-SCALING FFAGS

When acceleration is sufficiently rapid, as is the case in muon acceleration, the beam experiences only a few turns in the accelerator and betatron resonances do not have to be avoided, allowing "instantaneous" crossing of resonance tunes. Optical parameters can then vary with momentum and the lattice constructed solely from linear elements, quadrupoles and dipoles. Linear elements in turn imply a large transverse dynamic aperture in addition to "unlimited" momentum acceptance. Stability over a large range in momentum, however, requires these lattices to be constructed from a single, simple optical structure and are, therefore, completely periodic with no insertions or long straight sections. The signature of fixed-field acceleration is that the particle beam moves across the radial aperture. The changes in orbit and traversal time are significant, leading to a phase-slippage between the beam and the rf waveform which eventually prevents further acceleration. Moreover, because the acceleration occurs over a submillisecond cycle, the magnetic field strength and the parameters of the radio-frequency system cannot be adjusted on a corresponding timescale.

The phase-slip profile of the lattice determines the characteristics of the extracted phase space and its evolution with the number of acceleration passes. The goal of lattice design, then, is to minimize phase slippage within the constraint of fixed, high radio-frequency (200 MHz for the U.S. Neutrino Factory scenario). Minimizing the overall phase slip, or the overall path-length differences for different momenta, becomes the fundamental problem of the lattice design. Since the lattice is completely periodic, it is therefore instructive to examine the momentum compaction dependence of the base cells.

Dispersion in F0D0 optics

Momentum compaction α is defined as the relative fractional change in orbit length $\Delta L/L$ for a relative change in momentum offset $\Delta p/p \equiv \delta$, that is $\Delta L/L = \alpha \times \delta$. For a periodic structure with cell length L,

$$\alpha = \frac{1}{L}\int_0^L \frac{\eta}{\rho} ds \ , \tag{1}$$

where η is the periodic dispersion function and ρ is the magnetic bending radius. Using simple thin-lens matrices, calculation of the maximum and minimum locations of dispersion and their dependence on cell properties is straightforward and commonly derived[7]. Their dependence on cell properties provides insight into effective design principles for minimizing momentum compaction. The periodic dispersion η gives the closed orbit of an off-momentum particle and is the solution of

$$\begin{bmatrix} \eta \\ \eta' \\ 1 \end{bmatrix}_{s+L} = \mathbf{M}(s, s+L) \begin{bmatrix} \eta \\ \eta' \\ 1 \end{bmatrix}_{s} \tag{2}$$

where $\mathbf{M}$ is the usual 3×3 transfer matrix.

In a cell with reflective symmetry, one may consider a half cell of length l. At the symmetry points, the slopes of all optical parameters are zero in the closed solution, and they correspond to extrema of the dispersion. The center of the vertically-focusing element has a minimum ($\check{\eta}$) and the horizontally-focusing element has a maximum ($\hat{\eta}$) of the dispersion, as is clear after solving the thin-lens equations.

$$\begin{bmatrix} \check{\eta} \\ 0 \\ 1 \end{bmatrix} = \mathbf{M}^{1/2\mathrm{F0D0}} \begin{bmatrix} \hat{\eta} \\ 0 \\ 1 \end{bmatrix} \tag{3}$$

The results for the conventional F0D0 cell with the bend centered between focusing elements are:

$$\hat{\eta}_{\mathrm{F0D0}} = \frac{f^2}{l}\theta_b \left[1 + \frac{1}{2}\frac{l}{f}\right] \ , \ \check{\eta}_{\mathrm{F0D0}} = \frac{f^2}{l}\theta_b \left[1 - \frac{1}{2}\frac{l}{f}\right] \ . \tag{4}$$

The corresponding results for the combined function (CF) F0D0 with the bend centered on the vertically focusing element are:

$$\hat{\eta}_{\mathrm{CF}} = \frac{f^2}{l}\theta_b \ , \quad \check{\eta}_{\mathrm{CF}} = \frac{f^2}{l}\theta_b \left[1 - \frac{l}{f}\right] \ . \tag{5}$$

Here $\theta_b = l/\rho$ is the bend angle of the half cell.

Comparing the two equations (4, 5), it is clear that $\check{\eta}$ and $\hat{\eta}$ are smaller in the second or combined function case, noting that dispersion remains positive in these lattices. One immediately concludes from these simple equations that locating the dipole bend component in the vertically-focusing element minimizes both dispersion and momentum compaction in a F0D0-based lattice design and, therefore, also the excursion or phase slip of off-momentum orbits. Conversely, dispersion and momentum compaction are maximized accordingly if the dipole field is instead added to the horizontally-focusing quadrupole. Another very important observation is that value for dispersion/momentum compaction is strongly influenced by the focal length for a given bend angle which, in the thin-lens approximation, can be set at the limit of stability for the lowest momentum (which corresponds to 180° of phase advance across the full F0D0 cell). At the high momentum, $f \gg l$, and l/f terms approach zero. Setting the lowest momentum at or near the limit of stability represents the shortest focal length, the lowest dispersion and therefore the smallest values of momentum compaction achievable for a given cell design. For the realistic, thick-lenses design, one stays a conservative distance away from the limit of stability; typically one reduces the focusing strength of the quadrupoles to stay below 0.8π phase advance. The limit of stability, however, is readily calculated for the different cell configurations and remains a useful benchmark of their relative performance. These two limits determine the range in dispersion and values for momentum compaction between injection and extraction energies.

Dispersion in FDF triplet optics

For the triplet case, which is seen to be essentially a modified-F0D0, the long drift space is simply placed at the center of the horizontally-focusing element thereby splitting it in two; minimal drifts are placed between the quadrupoles and combined function magnets (i.e. FDF-drift-FDF-drift ...). The extrema of the periodic dispersion are in this case:

$$\hat{\eta}_{\mathrm{FDF}} = f^*\theta_b \ , \quad \hat{\eta}_{\mathrm{FDF}} = f^*\theta_b \left[1 - \frac{D}{f_1}\right] \tag{6}$$

$$\text{where} \quad \frac{1}{f^*} = \frac{1}{f_1} - \frac{1}{f_2} + \frac{D}{f_1 f_2} \ . \tag{7}$$

Here D is simply the inter-magnet spacing of the FDF triplet, and f_1 and f_2 the focal lengths of the horizontally and vertically-focusing quadupoles, respectively. Understanding the improvement observed leads back to the solving for the focal lengths in the limit of stability. For the simple F0D0, the limit is $f = l$ for a thin lens. For the triplet with F–F spacing l, the limit depends on $l/2$ (for a half cell) relative to length D. A number of cases can be identified as follows:

$$f_1 = D \ , \quad f^* = D \tag{8}$$

$$f_2 = 2D \quad \text{for} \quad l \gg D\ . \tag{9}$$

$$f_2 = 1.4D \quad \text{for} \quad D = l/2 \tag{10}$$

$$f_2 = D \quad \text{for} \quad l = 0\ . \tag{11}$$

In the lattice models we have explored, $l \approx 1.5D$ ($f_2 \approx 1.3D$). Assuming the same bend per cell and substituting these values implies that the CF F0D0 cell has about a 50% larger momentum compaction than the triplet cell for rings with identical design constraints: poletip fields, magnet spacing and long drift. (These results scale as momentum increases so the momentum compaction is always less for the FDF-triplet versus the CF F0D0 for the designs here).

The differences are realized and accurately predicted for optimized lattices based on these different cell configurations. Examples of two 10-20 GeV lattices based on the CF F0D0 and the FDF triplet are given in Table1.

Table 1: Parameters for 10-20 GeV non-scaling FFAGs

Parameter	**Triplet**	**CF F0D0**
Circumference	607 m	616 m
#cells	110	108
cell length	5.52 m	5.70 m
CF length	1.89 m	1.31 m
F length	0.32 m (×2)	0.39 m
magnet spacing	0.5 m	0.5 m
long drift	2 m	2 m
central energy	20 GeV	18 GeV
F grad	60 T/m	60 T/m
D grad	20 T/m	20 T/m
F strength	0.99 m^{-2}	0.94 m^{-2}
D strength	0.30 m^{-2}	0.30 m^{-2}
Bend field	2 T	2.7 T
Orbit swing		
low momentum	-7.7 cm	-9.8 cm
high momentum	0 cm	3.8 cm
ΔC	16.6 cm	26 cm
$\beta_x$max/$\beta_y$max/	6.5/13.8 m	14.4/11.4 m
β(injection)	6.5 m	5.8 m

The central energy, to which the orbit offsets are referenced, corresponds to the orbit which traverses the horizontally-focusing quadrupole at the zero field point. ΔC is the total swing in path length from top to bottom of the parabolic curve which describes the circumference change dependence on momentum.

Electron Demonstration Model

Because the non-scaling FFAGs are newly-devised accelerators, a "proof of principle" electron model is being considered. For completeness, preliminary design specifications for comparable, optimized electron lattice are presented in Table 2. For this case, the small number of cells and the large bend angles of the sector magnets implies that care is demanded and higher order optics is required. Consequently, the initial parameter sets derived from MAD were fine tuned using the COSY optical computer program. Once more, the triplet configuration proves superior to the CF F0D0 as anticipated.

Table 2: Parameters for a 10-20 MeV electron machine

Parameter	**Triplet**	**CF F0D0**
Circumference	12.3 m	12.3 m
# cells	28	28
cell length	0.44 m	0.44 m
CF length	12 cm	9.2 cm
F length	3.5 cm (×2)	5.2 cm
magnet spacing	5 cm	5 cm
rf drift	15 cm	15 cm
central energy	19 MeV	18 MeV
F grad	7.0 T/m	6.1 T/m
D grad	3.7 T/m	3.5 T/m
Bend field	0.2 T	0.2 T
Orbit swing		
low momentum	-2.2 cm	-2.3 cm
high momentum	3.5 cm	1.2 cm
ΔC	5.3 cm	6.5 cm
$\beta_x$max/$\beta_y$max	1.0/0.6 m	1.0/0.8 m

SUMMARY

The objective of minimizing momentum compaction, leads to reducing the range of the periodic dispersion by manipulating the location of the bending magnets. This strategy leads first to a F0D0 cell with the bending centred on the D-element. Further refinement leads to the splitting of the F-element and moving the magnets into closer proximity, and at the same time introducing a drift for the accelerating cavity. This strategy is born out in detailed calculations of lattices intended for muon acceleration and for an electron model of this novel type of FFAG accelerator. The methodology developed thus far does not admit doublet lattices, since these do not have reflective symmetry. However, the doublet lattice enables shortening of one of the drift spaces, compared with the F0D0, leading to a reduced path length; and will be the subject of further study.

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