DESIGN CONSIDERATIONS FOR A HELICAL UNDULATOR FOR THE PRODUCTION OF POLARISED POSITRONS FOR TESLA

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Abstract
An efficient and simple method for the production of positrons, in the necessary quantities, is one of the problems facing proposals for any future e+ e- Linear Collider project. The possibility of colliding polarised beams would also be an advantage. One method to produce a polarised positron beam uses circularly polarised radiation generated by the main electron beam passing through a helical undulator. Design considerations and calculations for two undulators, based on super-conducting and pure permanent magnet technologies, for the TESLA machine, are presented.

INTRODUCTION
The TESLA Technical Design Report [1] has a positron production system involving a planar undulator and target. High-energy electrons (250 GeV) pass through the undulator, creating high-energy photons. The electrons then go to the interaction point (IP). The photons hit the target creating electrons and positrons by interacting with the electromagnetic field of the target nuclei. To create enough positrons the general parameters for a planar undulator are that it will be ~140m long, have a period of 14.2mm and an on axis field of 0.75T at a 5mm gap. To create the maximum number of positrons via the pair production process 20 MeV photons are required [2]. Due to the high quality electron beam before the IP (after the IP the energy spread is too great [3]) there is the possibility of having a polarised positron source. To create polarised positrons circularly polarised (CP) photons are needed. One method to produce CP photons involves using the main electron beam and a helical type undulator [4]. A polarised positron source would be a possible upgrade for TESLA [3].

INITIAL PARAMETERS
To create the highest positron flux a high photon flux is needed. This means that as many periods as possible need to be fitted into the available space. Consequently the smallest possible period should be used. Shortening the undulator period requires the on-axis field to increase (to create 20 MeV photons with a 250 GeV electron beam). For an undulator with period, \( \lambda_u \), the photon energy of the first harmonic, \( E_1 \), of a helical undulator is [5]:
\[
E_1 = \frac{\hbar \omega}{2\pi} = \frac{2\hbar \gamma^2 c}{(1 + K^2) \lambda_u},
\]
where K is the undulator deflection parameter and the other symbols have their usual meaning. From this equation the required on axis magnetic field can be calculated, for a given period, to produce 20 MeV photons with a 250 GeV electron beam. The final parameter that was set was that the beam aperture (BA) through the undulator should be a circle of ~4mm diameter. To create the magnetic field three different designs based on super-conducting (SC) and pure permanent magnet (PPM) technologies have been considered.

PERMANENT MAGNET DESIGNS
Several types of permanent magnet design were considered with two being studied in some detail; one based on an APPLE-II type undulator [6] and one made of permanent magnet multipole rings [7]. The permanent magnet modelling was done using Radia [8].

APPLE-II Design
The Apple-II type design has been utilised in many light sources as a variably polarising undulator. The design has four arrays of planar undulators, two above and two below the beam axis. By adjusting the relative longitudinal position (phase) of the arrays the relative strengths of the transverse components of the magnetic field are altered. For CP photons the beam must travel in a helix and so the two transverse components of the magnetic field must be equal. Undulators with different periods were modelled and the peak field in CP mode recorded.

Ring Design
In this design each period of the undulator is divided up into slices of PPM. Each slice comprises of a ring of trapezoid shaped PPM blocks with magnetisation vectors that rotate is rotated through 720° to produce a transverse dipole field. Each slice is then rotated slightly so that over one period the total rotation of the dipole field is 360° (Figure 1) thereby creating a helical field. By changing the number of blocks in a ring and the number of rings in a period the field strength and quality is altered.

![Ring Design](image.png)

Figure 1: A ring of PPM blocks create a dipole field and many rings are stacked together to create a helical field.
**PPM Modelling Results & Conclusions**

Figure 2 shows a graph of $\lambda_u$ vs $B$, the magnetic field on-axis, required to produce 20MeV photons with a 250 GeV electron beam (black), the maximum on-axis field vs period for the APPLE-II design (red) and the ring design (green). The PPM material used was NdFeB with a remnant field of 1.3 T for all models.

![Figure 2: Required and calculated on-axis magnet field.](image1)

Clearly the ring design can achieve more field on axis and so a shorter period device can be used; 14mm compared to 18mm. This is because the APPLE-II type device lacks magnet blocks on the sides of the vessel and so flux is lost and not driven into the gap.

**Detailed Ring Design**

To achieve the preferred vacuum in the vessel of $10^{-8}$ mbar CO equivalent the vacuum vessel will have to be NEG coated. If this can be done then access to the vessel will be required to activate the NEG and so the undulator must be able to be split in two. To keep the design regular (i.e. smooth along the faces of each half) the number of blocks per ring must be an even number and must be a multiple of the number of rings per period. The force between the two undulator halves can be considerable and depends on the detailed configuration of the magnet blocks. Figure 3 shows the magnet forces between the arrays for only ten periods of a 14mm period device, as they are brought together. The nomenclature 10x5 means that there are 10 blocks in a ring and 5 rings in a period etc.

![Figure 3: magnetic forces for different configurations.](image2)

It can be seen that the force between the two halves can either be repulsive or attractive, depending upon the configuration. The 8x8 design experiences comparatively little forces at all separations and so this design has been pursued further.

**SC DESIGN**

This design is based on two helical super-conducting wires wound around the vacuum vessel with current flowing in opposite directions [9] (Figure 4). On axis the central axial magnetic field is cancelled and there is only a helical transverse field.

![Figure 4: Schematic of SC design.](image3)

**Magnet Modelling**

3D modelling using OPERA of the helix geometry, changing the period and bore was carried out. For a particular period the peak magnetic field at the wires was calculated for the current density, $J$, required to create the necessary on-axis field (with some margin for error). This can then be expressed in terms of a conductor margin or short sample (SS) %. At 100% the conductor becomes normal conducting. Table 1 gives the SS% for various different geometries with 4mm x 4mm wires of NbTi with a 1:1 SC normal conductor ratio and a packing ratio of 0.74.

<table>
<thead>
<tr>
<th>Period (mm)</th>
<th>Bore (mm)</th>
<th>$J$ (A mm$^{-2}$)</th>
<th>Baxis (T)</th>
<th>Bwire (T)</th>
<th>SS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>2105</td>
<td>1.6</td>
<td>3.8</td>
<td>137</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>1200</td>
<td>1.2</td>
<td>2.1</td>
<td>78.3</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>908</td>
<td>1.0</td>
<td>1.6</td>
<td>59.5</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>720</td>
<td>0.85</td>
<td>1.3</td>
<td>46</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>2027</td>
<td>1.2</td>
<td>3.5</td>
<td>131</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td>1302</td>
<td>0.9</td>
<td>2.2</td>
<td>84</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>1000</td>
<td>0.85</td>
<td>1.7</td>
<td>74</td>
</tr>
</tbody>
</table>

The wires must be wrapped around some former and so the minimum helix bore that can safely be used is 6mm. This should maintain a 4mm BA and allow the former to be a vacuum vessel with 1mm thick walls. If the walls were much thinner then winding the wires could deform the tube. Due to the brittleness of SC wires winding them round a smaller bore would also be more difficult. For these reasons a magnet with a period of 14mm with a helix bore of 6mm has been selected.
MODEL PRODUCTION

The studies have shown that there are two potential designs for this magnet. A PPM and a SC device each of 14mm period could provide enough on-axis field to create 20 MeV photons. To help in deciding which design is better short 10 to 20 period models are being designed and constructed. These will enable:

- The field quality and strength of each design to be checked
- Assessment of the size and probability of magnet errors
- The ease of constructing full size modules
- Improvement of cost estimates

Figure 6 shows the engineering drawing for the PPM design and Figure 7 a SC model.

Figure 6: PPM model engineering drawing.
Figure 7: SC model.

POWER DEPOSITED IN VESSEL WALLS

Heating of the vacuum vessel inner walls is an issue for both designs. In the SC device it could quench the magnet and in the PPM device it could cause demagnetisation of the blocks. The main source of power is expected to be synchrotron radiation although resistive wall wakefields will also contribute. The angular distribution of power from synchrotron radiation created in a helical undulator is [5]:

\[
\frac{dI}{d\Omega}(\theta) = \frac{8Ne^3\gamma^2\omega}{4\pi\varepsilon_0c} \frac{K^2}{(1+K^2+\gamma^2\theta^2)^{3/2}} \times
\sum_{n=1}^{N} \left[ J_n^2(x_n) + \left( \frac{n\theta}{K} - \frac{x_n}{\lambda_n} \right) J_n(x_n) \right],
\]

where \( x_n = \frac{2K\pi n\theta}{1+K^2+\gamma^2} \), \( N \) is the number of periods in the device, \( k_n \) is the wave-number of the undulator period, \( k_n = \frac{2\pi}{\lambda_n} \), \( \omega_n = k_n c \sqrt{1 - \frac{K^2}{\gamma^2}} \), \( m \) is the number of harmonics to be included and the other symbols have their usual meaning.

To calculate the power deposited in the undulator vessel wall the undulator was pessimistically modelled as 140 separate 1m long devices. It is likely that there will be diagnostic sections and focussing magnets distributed along the undulator length. Therefore after every two sections of undulator a 1m long drift space was assumed. The radiation from each undulator section into all the following sections was then calculated. A 14mm period device creating 0.85 T on-axis was considered. Only the first 20 harmonics were found to be significant. Figure 8 shows the power per metre along the device, for energies of 50 (blue), 150 (green) and 250 (red) GeV. Lower energies could be used for certain experiments and commissioning. The beam current is 45\( \mu \)A and the aperture 4mm.

Figure 8: Power deposited along undulator length.

As can be seen the power levels rise dramatically along the undulator length. If the power levels are too high for operation of the magnet then photon beam absorbers will need to be strategically placed. These will necessarily be smaller than the 4mm magnet aperture.

FURTHER WORK

After the models have been built and tested a recommendation of the preferred technology choice will be made. A single full size proto-type will then be constructed and tested, possibly with an electron test beam.

REFERENCES

[8] [http://www.esrf.fr/machine/groups/insertion_devices/Codes/Radia/Radia.html](http://www.esrf.fr/machine/groups/insertion_devices/Codes/Radia/Radia.html)