TUNE AND STABILITY OF HIGH INTENSITY BUNCH TRAINS IN THE CERN SPS AND LHC

L. Vos, CERN, Geneva, Switzerland

Abstract

The complex resistive wall impedance generates a non-uniform transverse focusing along a bunch train. The effect was first computed by V. Balbekov [1] for the UNK machine. The origin of this effect is discussed and then quantified for bunch trains in the CERN SPS and future LHC. Then the transverse resistive wall instability of bunch trains is examined and compared with the well-known case of the uniformly filled machine. It is shown that the frequency of the most unstable mode is centred around the inverse of the length of the bunch train with a growth rate that is larger than the one expected from linear extrapolation of a full machine.

1 INTRODUCTION

V. I. Balbekov has computed the tune shift along a uniform train or batch of proton bunches for the high intensity UNK machine [1]. The origin of this effect is the complex transverse resistive wall impedance. It is computed here both for the SPS (fixed target and LHC beam operation) and for the LHC. It modifies the stability situation of the bunch train when compared to a machine which is uniformly filled. In the latter part of the paper it is shown that the driving force of the resistive wall instability depends critically on the batch length for constant local density.

2 THE ORIGIN OF THE TUNE GRADIENT ALONG A BUNCH TRAIN

Consider a train of identical bunches. The tune of the train and of the individual bunches will be modified by the so-called Laslett tune shifts [2]. Let us examine the various types of these shifts and their ability to impose a tune gradient along a batch or a train of bunches.

The direct tune shift acts on each bunch. The forces on each charged particle emanate from neighbouring charges and currents in the bunch. The force and the source are in perfect phase since there is no time delay between them. The effect will be the same on all bunches since they are assumed to be identical, hence it does not contribute to a tune gradient.

The electrostatic tune shifts are driven by the images created by the wall conductor and the electric field of the beam. As in the previous case there is no delay between the force and its source. The argument is valid both for the incoherent and the coherent electrostatic effect. Hence the electrostatic tune shift does not contribute to a tune gradient across a uniform bunch train.

Two cases have to be considered regarding the magnetic tune shifts. The first one is produced by the image created by a magnetic surface. Only the DC magnetic field is involved, hence it acts on the bunch train in a uniform way and does not contribute to a tune gradient.

The second case concerns the image currents that drive the incoherent and coherent tune shifts. Again there is no delay between the source (beam and image current) and the force for the incoherent effect. Hence all bunches are perturbed in the same way such that the effect is uniform along the bunch. The same is no longer true for the coherent effect. A differential image current is set up that drives a wall voltage via the complex wall impedance (skin effect). The complex impedance causes a time delay between the source (the image current which is in phase with the beam current) and the electric field in the wall impedance that drives the tune shift. Consequently the focusing force will vary with time along the bunch train. The effect that was just described is nothing else but the resistive wall effect.

3 COMPUTATION OF THE TUNE GRADIENT ALONG A BUNCH TRAIN

The resistive wall impedance is proportional to $\sqrt{j/\omega}$. For convenience it is assumed that the thickness of the wall conductor is larger than the skin depth. The bunch train will be simply represented by a uniform current impulse of length $\tau$. The bunch structure can be neglected since the resistive wall impedance falls off quickly with frequency from the lowest frequencies onwards. The revolution time of the machine is $T = 2\pi/Q$ and $Q$ is the angular revolution frequency. The spectrum of a uniform bunch train can be written as:

$$B(\omega) = \frac{\sin(\omega\tau/2)}{\omega/2},$$

which assumes a charge in the bunch train proportional to its length $\tau$. The (de)focusing force will be proportional to the imaginary part of (only imaginary impedance causes a real tune shift):

$$q(t) = \sum_{\omega=\omega_{0}} e^{j\omega t} \frac{\sin(\omega\tau/2)}{\omega/2} \sqrt{j/\omega}.$$

The summation is taken over the fundamental betatron frequency and its harmonics $\Omega = (n-\Omega)\Omega$ where $Q$ is the transverse tune.

Fig 1 shows two examples for different bunch train length. The time scale $t=0$ starts in the middle of the bunch train for convenience. The vertical scale is arbitrary but identical for the two plots. The time scale is for the LHC ($T = 2\pi/Q = 89$ $\mu$s). The tune spread in the bunch train increases to a maximum for a length equal to half the circumference of the machine and decreases again when the whole machine gets filled progressively. The
evolution of the spread $\Delta q$ as a function of the fraction of the machine circumference $\tau/T$ occupied by the trains is shown in Fig 2.

Figure 1: Tune evolution across a bunch train, top: train (heavy line) occupies $\tau/T=1/12$ of circumference, bottom: train occupies $\tau/T=1/2$ of circumference

Figure 2: Tune spread in bunch train versus fraction of machine circumference $\tau/T$ occupied by the beam.

The normalising factor $q_{\text{max}}$ is the tune shift induced by the imaginary part of the resistive wall for a full machine but with the same local density as the bunch train.

4 APPLICATION TO SPS AND LHC

The tune shift for a total beam current $i_B$, equal to the local beam current of the bunch train, is given by:

$$q_{\text{max}} = \frac{1}{4\pi} \frac{R/Q}{E/e} Z_{\perp} i_B,$$

(3)

where $Z_{\perp}$ is the (imaginary part) transverse impedance, $R$ the machine radius and $E$ the beam energy. Table 1 summarises the effect of the tune gradient caused by the resistive wall in the SPS and the LHC for various bunch train situations.

Table 1: Vertical tune spread in beam batches for various situations in the SPS and the LHC.

<table>
<thead>
<tr>
<th>$i_B$</th>
<th>$\tau/T$</th>
<th>$\tau_{\perp}/j_E$</th>
<th>$E/e$</th>
<th>$q_{\text{max}}$</th>
<th>$\Delta q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS A</td>
<td>0.27 1/2</td>
<td>200 14 13 11.7</td>
<td>10^{-3}</td>
<td>10^{-3}</td>
<td></td>
</tr>
<tr>
<td>batch</td>
<td>0.67 1/12</td>
<td>&quot; 26 17 7.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>batch</td>
<td>&quot; 1/6</td>
<td>&quot; &quot; &quot; 11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>batch</td>
<td>&quot; 1/4</td>
<td>&quot; &quot; &quot; 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC</td>
<td>0.46 1/12</td>
<td>52 450 0.46 0.2</td>
<td>10^{-3}</td>
<td>10^{-3}</td>
<td></td>
</tr>
<tr>
<td>1.7 batch</td>
<td>0.46 1/2</td>
<td>&quot; &quot; &quot; 0.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.7 batch</td>
<td>0.46 1/12</td>
<td>&quot; 117 116 0.114</td>
<td>7000 0.067 0.016</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It may be useful to compare the magnitude of this effect with the direct space charge tune shift which plays an important role both in the SPS and the LHC. The direct space charge tune shift can be written as [2]:

$$q_{\text{sc}} = \frac{1}{4\pi} \frac{R/Q}{E/e} \frac{1}{\gamma \varepsilon_{\gamma}} \{i_B\}.$$

(4)

The ratio of the tune shifts given by Eq. 3 and 4 is:

$$\frac{q_{\text{sc}}}{q_{\text{max}}} = \frac{Z_{\perp} \varepsilon_{\gamma}}{\gamma \varepsilon_{\gamma}} \frac{i_B}{i_B} = \frac{\varepsilon_0 Q Z_{\perp}}{\gamma \varepsilon_{\gamma} \sqrt{2\pi} \sigma_t} \{i_B\},$$

(5)

where $Z_0$ is the impedance of free space, $\varepsilon_{\gamma}$ the normalised transverse emittance, $\gamma$ the relativistic factor, $\varepsilon_0$ the Laslett coefficient, $i_B$ the bunch current. The tune shift ratio in the SPS for nominal LHC type beam parameters is 0.45 while it is 0.3 in the LHC, both at injection. The tune spread of a bunch train as a whole is equal to the tune spread of its constituent bunches augmented by tune differences between them. Clearly, the tune gradient along the LHC batches is a non-trivial component of the tune spread of a batch, especially for the leading one, which helps slightly for stability but it reduces the decoherence time which makes the transverse emittance conservation a more difficult task.

5 TRANSVERSE STABILITY OF BUNCH TRAINS

The beam envelope spectrum of a bunch train that occupies the full circumference consists of a single DC component. It couples with the lowest slow wave mode
and drives the transverse resistive wall instability. The situation for a partially filled machine is very different. The resistive wall impedance is proportional to a factor (see also above) that is called \( z(\omega) \):

\[
z_j(\omega) = \frac{\sin((\omega - \omega_i)\tau/2)}{\omega - \omega_i}/2.
\]  

(7)

Another way of describing the phenomenon is to say that the instability drives a higher mode of the bunch train spectrum. The driving force of the instability can be derived from the convolution of \( z(\omega) \) and \( B_j(\omega) \). The frequency \( \omega_i \) is such that the convolution maximises the level of anti-damping, considering the fact that positive frequencies (fast waves) are damped and negative frequencies (slow waves) are anti-damped. Hence, the maximum of the real part of following function yields \( \omega_i \):

\[
\sum_{\omega=\Omega} B_j(\omega)z(\omega) = \sum_{\omega=\Omega} \frac{\sin((\omega - \omega_i)\tau/2)}{\omega - \omega_i}/2 \sqrt{\omega}.
\]  

(8)

A plot of Eq. 8 (real part) is shown in Fig 3 for a full machine (lower curve) and a partially filled machine (upper curve).

6 CONCLUSIONS

The most unstable mode of the resistive wall instability of a beam in a machine that is being filled with a sequence of batches is centred around a frequency approximately given by \( f = 1/\tau \), i.e. the inverse of the length of the bunch train. The driving force increases more than linearly with the total charge in a train. Further more it is accompanied by an increase in tune spread due to the complex nature of the skin effect impedance. This effect is not negligible since it culminates at 45% of the space charge tune shift (the main contributor to the tune spread) in the SPS and at 30% in the LHC for bunch trains of nominal bunch intensity at injection energy. Landau damping is slightly better but its benefit is lost by the reduction of the decoherence time. Consequently the requirements on the transverse feedback are enhanced in view of the conservation of the transverse emittance especially for the leading batches of an injection sequence.

REFERENCES
