A METHOD FOR REDUCTION OF THE REFLECTED POWER AND OVERVOLTAGE IN RF SYSTEMS

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Abstract
The basic elements of the TESLA linear collider with integrated X-ray FEL are superconducting accelerating cavities. The standard TESLA rf system is designed for a long (1300\(\mu\)s) rf pulse with a flat-top of 800\(\mu\)s and has several elements for the rf power distribution and protection against reflected rf power. Together with the superconducting cavities, normal conducting (NC) cavities are used in TESLA for different purposes, for example as a rf gun. As a rule, one NC cavity uses the total rf power from one klystron (5 MW or 10 MW). During the short transient period (the filling time for the L-band NC cavity is near 3\(\mu\)s) the reflected rf power and overvoltage in the waveguide (due to the interference of the forward and backward waves) exceeds the safety limit for the L-band waveguide operation (without SF6 filling). With a special shape of the beginning of the rf pulse it is possible to reduce the reflected power several times and limit the overvoltage to a safe level. The increasing of the total pulse length is in comparison with the total pulse length of 800\(\mu\)s. Different shapes of the beginning of the rf pulse are both considered to reduce the reflected power and to restrict the overvoltage. Results of experiments are presented.

1 INTRODUCTION
The power limit of the WR650 waveguide with air filling for stable operation is \(P_{\text{crit}} \approx 14\,\text{MW}\) [1]. But in the rf pulse beginning a reflection from the cavity occurs. If the driving rf pulse has a simple rectangular shape, at the moment \(t \approx 0\) the reflected wave amplitude \(E_{\text{ref}}^{\text{dr}}(t)\) is comparable with the forward wave amplitude \(E_{\text{dr}}^{\text{nom}}\). In the waveguide there will be regions where two waves add. If \(E_{\text{dr}}^{\text{nom}}\) corresponds to the nominal rf power \(P_{\text{nom}} = 10\,\text{MW}\), the maximal amplitude of the field possible in the waveguide is \(E_{\text{dr}}^{\text{nom}} + E_{\text{ref}} \approx 2E_{\text{dr}}^{\text{nom}}\), corresponding to rf power \(P_{\text{max}} \approx (2E_{\text{dr}}^{\text{nom}})^2 \sim 4P_{\text{nom}} = 40\,\text{MW}\). It exceeds the safety limit for a WR650 waveguide with air filling. Due to the low - as compared to a superconducting structure - \(Q_0\)-factor (\(\approx 23000\)), the field rise-time \(\tau_1\) for L-band NC cavities is quite short \(3\tau_1 \approx 7.5\,\mu\)s. The enlarged reflection and overvoltage takes place during the rather short transient period \(3\tau_1 \approx 7.5\,\mu\)s, which is small in comparison with the total TESLA pulse length \(\approx 820\,\mu\)s. If the overvoltage is a serious problem, it can be diminished by a special shape of the rf pulse beginning. The cavity becomes saturated with a forward wave from the rf source \(E_{\text{dr}}^{\text{dr}}(t) \exp(i\omega_0 t)\). Starting with the diminished driving amplitude in the beginning of the rf pulse, we reduce the reflected wave amplitude and significantly reduce the overvoltage. Then the amplitude of the forward wave is increased to its nominal value, while the reflected wave amplitude becomes smaller than for a simple step-like rf pulse beginning.

2 A SPECIAL RF PULSE SHAPE
The reflected wave amplitude \(E_{\text{ref}}^{\text{dr}}(t)\) can be easily found as:

\[
E_{\text{ref}}^{\text{dr}}(t) = L^{-1}(L(E_{\text{dr}}^{\text{dr}}(t))F_{\text{rc}}(p)),
\]

where \(L()\) and \(L^{-1}()\) are operators of direct and inverse Laplace transformation, \(F_{\text{rc}}(p)\) is the cavity reflection response.

Single cavity. In the general case of a slightly detuned single mode cavity \(F_{\text{rc}}(p)\) is [2]:

\[
F_{\text{rc}}(p) = \frac{\eta - 1 - \eta(1 + \eta)(i\Delta\omega + p)}{\eta + 1 + \eta(1 + \eta)(i\Delta\omega + p)},
\]

where \(\eta\) is the coupling coefficient between the cavity and the waveguide, \(\omega_0\) is the operating frequency, \(\Delta\omega = \omega_0 - \omega_1, |\Delta\omega| \ll (\omega_1,\omega_0),\omega_1\) is the cavity resonance frequency.

Let the driving rf pulse envelope \(E_{\text{dr}}^{\text{dr}}(t)\) be described as:

\[
E_{\text{dr}}^{\text{dr}}(t) = \begin{cases} 
E_{\text{dr}}^{\text{dr}}(t) & t \leq t_b \\
E_{\text{nom}}(t) & t_b \leq t \leq t_{dr}
\end{cases}
\]

where \(t_b\) is a time duration for the special pulse shape beginning, \(t_{dr}\) is the rf pulse duration. Different approaches are possible. Defining the reasonable overvoltage \(E_{\text{ov}}(t) = E_{\text{dr}}^{\text{dr}}(t) + E_{\text{ref}}^{\text{dr}}(t)\) needed for the safe operation, one can find:

\[
E_{\text{dr}}^{\text{dr}}(t) = L^{-1}\left(\frac{E_{\text{ov}}^{\text{dr}}(p)}{1 + F_{\text{rc}}(p)}\right)
\]

In this case only one limitation exists - not all shapes for \(E_{\text{ov}}(t)\) and \(E_{\text{dr}}^{\text{dr}}(t)\) can be realized.

Let us consider another algorithm, suitable for modern rf systems with digital control. It means that a rf control system has a small, but finite time step of discretization \(t_d\) to represent any signal. During this time step every signal is assumed as constant value. The control signal for \(E_{\text{dr}}^{\text{dr}}\) can be described as a sum of several steps:

\[
E_{\text{dr}}^{\text{dr}}(t) = E_{\text{nom}}^{\text{dr}} \sum_{n=1}^{N} a_n h(t - (n - 1)t_d),
\]
where \( h(t) = 0, t < 0, h(t) = 1.0, t \geq 0 \) is a usual step function. The amplitude of the first step \( a_1 \) is determined as:

\[
 a_1 = \left( \frac{P_{crit}}{4P_{nom}} \right)^{1/2} \quad (5)
\]

We limit the overvoltage in the waveguide by the equivalent power \( P_{crit} \) at the moment \( t = 0 \). Then the reflected wave from the first step will decrease and by the beginning of the second step \( t = t_d \) will be \( E_{nom}^0 \exp(\frac{-t_d}{\tau_l}) \). The second step amplitude \( a_2 \) can be found from the equation:

\[
 a_1(1+\exp(-\frac{t_d}{\tau_l}))+2a_2 = 2a_1, \quad a_2 = 0.5a_1(1-e^{-\frac{t_d}{\tau_l}}), \quad (6)
\]

In the beginning of the second step the overvoltage will be returned to the equivalent value \( P_{crit} \), decreasing during this step. In this equation, and further, we assume a tuned \((\Delta \omega = 0)\), perfectly matched \((\eta = 1.0)\) cavity and neglect the delay in the waveguide. To consider this delay time, one should artificially (in the consideration) increase \( t_d \).

Following the general idea - to keep in the beginning of each step the total overvoltage equivalent to \( P_{crit} \), for \( a_{n+1} \) step amplitudes one finds (defining \( x = \exp(-\frac{t_d}{\tau_l}) \)):

\[
 \sum_{k=1}^{n} a_k(1+x^{(n-k+1)}) + 2a_{n+1} = 2a_1. \quad (7)
\]

and

\[
 a_{n+1} = 0.5a_n(1+\exp(-\frac{t_d}{\tau_l})). \quad (8)
\]

The procedure should be finished with \( N \) if \( \sum_{n=1}^{N} a_n \geq 1.0 \) and the last step may be reduced to have \( E_{dr}^r(t) = N(t_d) = E_{nom}^0 \). It converges always if \( P_{crit} > P_{nom} \). But this ensures stable operation during the whole rf pulse.

For different \( P_{crit}/P_{nom} \) the calculated ratio for the forward and reflected waves, the overvoltage and the cavity voltage are plotted in (Fig 1) in terms of rf power (Fig.1a-c) and in terms of voltage (Fig. 1d) for \( t_d = 1 \mu s, Q_0 = 24000, \tau_l = 2.938 \mu s, P_{crit} = 14.0 MW \).

The total length of the pulse beginning \( t_b = t_d(N - 1) \) mainly depends on the \( P_{crit}/P_{nom} \) ratio. Assuming \( P_{crit} = 14 MW \), for \( P_{nom} = 5 MW \) only two additional steps are necessary, eight for \( P_{nom} = 10 MW \) and twenty one for \( P_{nom} = 13.5 MW \). But the total duration of the transient processes increases not so strongly. The total transient processes consist of a special pulse beginning and a natural tail. With low \( P_{nom} \) value one has a very short pulse beginning and a long natural tail. During the pulse beginning rf power comes into the cavity and to the end of the pulse beginning reaches \( P_{st} \) value (see Fig. 1b,c). With \( P_{nom} \) increasing or \( P_{crit}/P_{nom} \) decreasing more steps are necessary, but \( P_{st} \) becomes relatively higher, the natural tail begins from a higher value of the cavity voltage and takes a shorter time.

With \( t_d \to 0 \) we will have a smooth function \( E_{dr}^r(t) \). One can check directly, by using (3), that the function

\[
 E_{dr}^r(t) = E_{nom}^0 \left( \frac{P_{crit}}{P_{nom}} \right)^{1/2} (1-0.5 \exp(-\frac{t}{2\tau_l})), 0 \leq t \leq t_b, \quad (9)
\]

and

\[
 E_{dr}^r(t) = E_{nom}^0 \frac{P_{crit}}{P_{nom}} \leq t \leq t_{rf}, \quad \text{where}
\]

\[
 t_b = 2\tau_l \ln\left[2 - 2 \left( \frac{P_{nom}}{P_{crit}} \right)^{1/2} \right], \quad (10)
\]

is a continuous function for a \( E_{dr}^r(t) \) approximation, leading to the optimal overvoltage time dependence\( E_{ov}^r(t) = E_{nom}^0 \left( \frac{P_{crit}}{P_{nom}} \right)^{1/2} 0 \leq t \leq t_b, \) and for \( t_b \leq t \leq t_{rf} \):

\[
 E_{ov}^r(t) = P_{nom}^0 \left( \frac{P_{crit}}{P_{nom}} \right)^{1/2} - 1) \exp(-\frac{t-t_b}{\tau_l}) + E_{dr}^r. \quad (11)
\]

With such approximation for the rf pulse beginning we keep the overvoltage constant during \( 0 \leq t \leq t_b \) and as matching as possible, providing the most effective cavity excitation, then release the transient process and the overvoltage will decreases by the natural exponential law to the
normal operating value $E_{nom}^{dr}$. If we just want to restrict the maximal reflected power to the $P_{crit}^{ref}$ value, similar to (5) - (8), we get:

$$a_1' = \left(\frac{P_{crit}^{ref}}{P_{nom}}\right)^{1/2}, a_n = a_1'(1 - \exp\left(-\frac{t_d}{\tau_l}\right)).$$

(12)

If for a simple step-like driving pulse the maximal reflected power (at the moment $t = 0$) is $P_{ref}^{max} \approx P_{nom}$, with $N$ steps, following (12), the maximal reflected power can be reduced to:

$$P_{ref} = \frac{P_{nom}}{(1 + N(1 - \exp(-\frac{t_d}{\tau_l})))^2}.$$  

(13)

With $N = 8$ steps, $t_d = 1\mu s$, $\tau_l = 2.938\mu s$, the maximal reflected power can be diminished ten times. Multi-cell cavity. The transient effect in the multi-cell cavity can be considered as a summation of travelling waves propagation. A general theory of transients in multi-cells Accelerating Cavities (AC) has been developed in [2]. Two structure parameters are important - the time of travelling wave propagation along the cavity from the rf input point to the cavity end-wall and back $\tau_{tr}$ and the attenuation constant $\alpha$:

$$\tau_{tr} = \frac{2L}{c\beta g}, 2\alpha L = \frac{\tau_{tr}}{4\tau_l} = \frac{2N_c}{k_cQ_0},$$

(14)

where $k_c$ is the structure coupling coefficient, $c$ is the velocity of light, $L = \frac{N_c\beta g}{2}$, $N_c$ is the number of periods from the rf input point to the structure end. For long ACs of the Positron Pre-Accelerator (PPA) [3] $\tau_{tr} \approx 0.123N\tau$. For a step-like driving rf signal with $E_{nom}^{dr}$ amplitude, tuned ($\Delta\omega = 0$) and perfectly matched cavity (for multi-cell cavities a condition of critical coupling is $\eta = \exp(-2\alpha L)$) the reflected wave amplitude $E^{ref}(t)$ is [2]:

$$E^{ref}(t) = E_{nom}^{dr}\eta^{\left\lfloor \frac{t}{\tau_{tr}} \right\rfloor}.$$

(15)

where $\left\lfloor \frac{t}{\tau_{tr}} \right\rfloor$ is an integer part of the $\frac{t}{\tau_{tr}}$ ratio.

In spite of the fact that for transient description in multi-cell cavity we don’t use the quality factor $Q_0$ or rise-time $\tau_l$, the envelopes for reflected wave and cavity voltage exhibit the same behavior as for a single cavity, with the exponential law $\exp\left(-\frac{t}{\tau_l}\right)$. But these envelopes are modulated by steps with a time duration $\tau_{tr}$. Since the general behavior of the transient effects for a multi-cell cavity is the same as for a single one, we can apply the procedure of the overvoltage decreasing with the same result.

### 3 EXPERIMENTS

The method to reduce the reflected rf power has been proved at the Tesla Test Facility (TTF) rf-gun cavity. Because this procedure is based on general properties of linear circuits, the difference between expected and experimental values is in the frame of the measurement precision. With three steps in the rf pulse beginning (12), the maximal reflected rf power has been decreased $\approx 4.5$ times (Fig. 2b) (the calculated value is 5.05) in comparison with a simple step-like driving pulse (Fig.2a).

### 4 SUMMARY

A special shape for the rf pulse beginning is proposed to reduce the overvoltage in a transmission line. The procedure for the overvoltage limitation can be both applied for single and multi-cell cavities. To use the proposed procedure one should specify the tolerable overvoltage $E_{ov}$. For the end of the pulse such a procedure is not necessary, because there is no forward wave and the reflected wave can’t be higher than the forward one.

The proposed procedure does not protect against overvoltage if there is sparking in the AC during the rf pulse. But the latter is not an ordinary situation and should be removed during cavity rf conditioning. For rf systems, combining both high pulse and high average rf-power level, this procedure opens the possibility to restrict the number of such expensive rf elements as powerful rf circulator.

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### REFERENCES

