AIR CORE SUPERCONDUCTING CYCLOTRONS

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Summary

This paper summarizes the major conclusions of the design study of superconducting booster cyclotrons. The goal of the study was to provide preliminary design of air core superconducting cyclotron,which should be able to accelerate not only heavy ions, but also p,d and He particles as close as possible to the energies of bending limit K Z^2/A.The considerations demonstrate the encouraging possibilities of air core superconducting cyclotrons in matching the requirements for multipurpose cyclotron even at large K number.The guidelines in designing the main coil and active flutter configurations are given together with solutions envisaged for the problems arise from the presence of limiting phenomena.The great emphasis in design considerations was placed on the cost savings,that are,in this cyclotron concept possible.

Iron Core Superconducting Cyclotrons

The iron core superconducting cyclotrons produce the isohronous average magnetic field by combining contributions from superconducting main coil and concerned feromagnetic structure.In addition to an important role in the production of a significant fraction(about 40 % of maximum field) of average magnetic field over the operating range of cyclotron magnet from 2.5 to 5 T,the suitable designed iron sectors provide a necessary field modulation for the axial focusing.At these field values the iron of pole tips is completly saturated and the amplitudes of azimuthaly varying field remain constant over the entire range of the operating field values.This feature strongly influences the focusing limit:[1]

$$T/A=K_f(Z/A)$$

The additional type of the operating limit at the iron core superconducting cyclotrons is set by the presence of the resonances:[2]

$$\nu_r,\nu_z=N/2 \qquad \text{stopband resonance}$$

$$\nu_r+(N-1)\,\nu_z=N \qquad \text{coupling resonance}$$

The first type of the operating limit corresponds to the existence of stopbands in the radial and axial motion,setting the limit on the maximum energy achievable at a given field symmetry number N of the machine.Due to the effect of the higher order terms in the expressions for ν_r and ν_z[3],it appears that stop band limit is considerably less than the "smooth approximation" estimation:

$$T/A=(N/2-1)\,w_0 \qquad (w_0=931.5\ \text{MeV})$$

Therefore,in order to achieve the highest energies/ nucleon,one must use a highest possible symmetry number N.As was pointed out,in one of the early paper on this topic[2],the practicality of this scheme,seems highly questionable.An additional consequence of this approach is,that a number of the open possibilities for designing the accelerating system,at increased field symmetry number N,is highly reduced,thus making discussible the possibility of achieving the desired quality of the ion beam.

The second type of operating limit(coupling resonance),which is not dependent upon imperfections present in magnetic field,cannot be crossed unless the beam is perfectly centred,or the beam has the possibility to jump over the resonance.To make this possible a high degree of freedom in the choice of the suitable accelerating system design is required. Therefore,the nature of design compromise,that is to be made to meet the conditions under which above mantioned types of operating limit can be considerably improved is an highly intriguing question not easy to solve,unless the design approach,which allows practically mutually independent design procedure for:
- isohronous magnetic field configuration
- magnetic field modulation configuration
- and suitable accelerating system

is established.

It seems that this design approach at iron core superconducting cyclotrons is not practically possible.Thus,the air core superconducting cyclotrons are to be carefully examined,as a natural further step in the developement of AVF cyclotrons.

Air Core Superconducting Cyclotrons

Main coil structure considerations.The air core superconducting cyclotrons are characterized by the feature that the magnet structure is formed entirely from current-carrying superconductors(feromagnetic structure is completly excluded).The magnet structure is composed of two sets of superconducting current-carying configurations,one set of which(characterized by $\overline{B}=0$),has to be used to fulfil the axial focusing, and the other(characterized by $\tilde{F}=0$)to fulfil the field isohronism requirements.Obviously the requirement $\tilde{F}=0$ implies a main coil structure composed of sets of windings with cylindrical symmetry,whereas the requirement $\overline{B}=0$ implies a set of azimuthaly dependent current-carying superconducting configurations.

Among several types of possible current-carying configurations providing necessary range and shape of average field,the spherical-like coil structure seems to be the most suitable configuration choice. This design approach at which the radius of the circular windings is decreased with increasing the distance of the winding plane from median plane, obviously gives the considerable cost savings of superconducting material in comparison with cylindrically wound main coil configuration.These cost savings should be able to compensate the expenses of the additional superconducting material,which is to be used for producing an additional number of Amperturns,needed to replace the contributions of the "excluded"iron core.

The typical spherical-like coil-former structure is shown in fig.1.The coil-former steps are confined between two concentric spheres and are closely wound with superconducting cable of a constant cross-section in a necessary number of layers.This former approximates a constant current distribution per unit of axis of axial symmetry,whose value determines the necessary number of coil layers.

Spherical main coil structure. The spherical coil with a constant current per unit of z axis

$$di/dz = di/(r_0 \sin\theta_0 d\theta_0) = 3/2\ H_z = J_0$$

generates a homogenous fieldstrength H_z along z-axis inside the sphere of radius r_0. The magnetic field is given by:

$$B = \mu_0 H_z = 2/3\ \mu_0 J_0 \qquad (r < r_0)$$

$$B_0 = -1/2\ B\ (r/r_0)^3 \qquad (r > r_0)$$

The number of Amper-turns required to generate this field value

$$N\ di = 3 H_z r_0$$

is considerably smaller than the number of Amper-turns required to produce the same value of axial field at origin, by cylindrical coil of same radius r_0.

The actual median plane field, generated by spherical coil composed of m independently powered coil sections, with a constant current per unit of z-axis $J_0, J_1, J_2, .. J_m$ where each coil section is composed of two sections positioned symmetricaly to the median plane between $\theta_0^{j-1}, \theta_0^j$ and $(\pi-\theta_0^{j-1}, \pi-\theta_0^j)$ is given by:

$$B_{ac} = 2/3\ \mu_0 J_0 + \sum_{n=0} \binom{-1/2}{n} L_{2n+1}\ \rho^{2n} \qquad (1)$$

where

$$L_{2n+1} = \mu_0 \sum_{j=0}^{j=m} I_j^{2n+1}(\theta_0^j, \theta_0^{j+1})\ J_j$$

Putting

$$L_{2n+1} = b - 2/3\ \mu_0 J_0 \qquad (n=0)$$

$$L_{2n+1} = (-1)^n b (\rho\omega/c)^{2n} \qquad (n \neq 0)$$

into equation (1) the actual median plane field can be rewritten as:

$$B_{ac} = b \sum_{n=0} (-1)^n \binom{-1/2}{n} \rho^{2n} = b(1-\beta^2)^{-1/2}$$

which can be recognized as a "zero-flutter" isohronous field $B = b\gamma$.

Thus it is obvious that a spherical superconducting coil splitted into a suitable chosen number of symmetricaly positioned, independently powered, sections can be used for triming the radial field profile. This magnetic configuration should be able to shape the required field profile to within a small difference. If necessary the additional suitable positioned circular currents at spherical surface can be used to compensate the remaining difference field.

Active flutter azimuthaly dependent superconducting current-carying configurations. The azimuthaly dependent current distributions on spherical surface of radius r_f given by:

$$K_{si}^2 = (\sin\theta_0)^{2(n-1)}(\cos^2 n\Theta(1-(n^2+2n+2)\cos^2\theta_0 + (n+1)^2\cos^4\theta_0) + n^2\cos^2\theta_0)$$

$$K_{ti}^2 = (\sin\theta_0)^{2(n-1)}(\sin^2 n\Theta(1-(n^2+2n+2)\cos^2\theta_0 + (n+1)^2\cos^4\theta_0) + n^2\cos^2\theta_0) \qquad (2)$$

where n=N is the number which determines the magnetic field symmetry and

$$K_{si} = (n+2)/(n+3)\ \mu_0 J_{si}/B_{si}$$

$$K_{ti} = (n+2)/(n+3)\ \mu_0 J_{ti}/B_{ti}$$

generate an azimuthaly varying field:

$$B_f = (B_{si}\cos n\Theta + B_{ti}\sin n\Theta)(\rho/r_f)^n \qquad (r \le r_f)$$

$$B_{f0} = -(B_{si}\cos n\Theta + B_{ti}\sin n\Theta)(n+1)(n+2)^{-1}(r_f/r)^{n+3}(\rho/r)^n \qquad (r > r_f)$$

The field flutter is then given by

$$F = F_0(\rho/r_f)^{2n}$$

where

$$F_0 = 1/2 \sum_i (B_{si}^2 + B_{ti}^2)/B^2$$

Limiting phenomena

Isohronous magnetic field. An expansion of a "zero flutter" isohronous magnetic field $B = b\gamma$ in terms of

$$\beta = ((-1)^n L_{2n+1}/b)^{1/(2n)} \qquad (n \neq 0)$$

$$L_1 = b - 2/3\ \mu_0 J_0$$

makes it possible to estimate the number of terms in power series expansion, needed to achieve the sufficient accuracy in matching the field isohronism requirements. This estimation gives the possibility to determine the number of coil sections needed to shape the desired field profile. It can be shown that the number of the coil sections m is equal to the number of terms in power series expansion of relativistic factor required to reproduce the maximum achievable $\gamma = \gamma_0$ at a given accuracy. From these considerations it arises, when allowing a resonable value of the fractional error:

$$p = \sum_{n=0}^{m} (-1)^n \binom{-1/2}{n} (1-\gamma_0^{-2})^n / \gamma_0 - 1$$

in producing the "zero flutter" isohronous field, that the number of the coil sections m, needed for matching the isohronism requirements[4], has an energy dependence shown in fig. 1.

Axial focusing and resonance operating limits. The azimuthaly dependent current distributions on spherical surface of radius r_f, given by equation(2), are the unique distributions for generating an azimuthaly varying field of a given field symmetry number N. These distributions at a "zero-flutter" isohronous field produce a flutter form factor:

$$F = F_0'\ (\gamma^2-1)^n/\gamma^{2(n+1)}$$

where

$$F_0' = F_0\ \gamma_0^{2(n+1)}/(\gamma_0^2-1)^n$$

Using well-known analytical formulae[3] for ν_r and ν_z it is possible to estimate the focusing power and operating limits set by stopband and coupling resonances. These considerations make it possible to obtain the set of values of parametar F_0', concerning the most important information about maximum bending and focusing power of air core superconducting cyclotron magnet at agiven magnetic field symmetry number N. The permissible range of the values F_0' is determined, in most cases, by the conditions:

$$\nu_z^2 = 0 \quad \text{and} \quad \nu_r = N/2$$

while the two additional important requirements:

$$\nu_z = N/2 \quad \text{and} \quad \nu_r + (N-1)\nu_z = N$$

appear to be of importance only at more detailed design considerations. It can be shown that the upper limiting value decreases faster than the lower limiting value, when the energy of the particle increases. The minimum value of F_0' which can be used at a given symmetry number N is determined by the point of the intersection between lower and higher operating limit curves.The position of the intersection point is shifted to the higher energies as the symmetry number N is increased,having an energy dependence behaviour given approximatively by:

$$T/A=((N-1.5)^{1/2} - 1)w_0$$

Once having determined the minimum permissible value for F_0' it is possible to find,at a given symmetry number N,the maximum bending power of cyclotron magnet,which equales the focusing power determined by the performance of the applied superconducting current-carying configuration in the active flutter structure at a given magnetic field intensity.

These performances can be characterized by maximum value of F_0 given by:

$$F_0=(s-1)^2$$

where s is the ratio of the maximum permitted value of magnetic field at the active flutter structure to the maximum value of median plane magnetic field.

In order to illustrate some of the points emerging from the previous considerations the graphical representation of trends in limiting phenomena together with the tentative data about the construction parameters for the air core superconducting cyclotrons are shown in figs. 1-4.

- -Fig.1a shows a typical spherical magnetic configuration which should be able to shape the "zero flutter radial field profile up to $T/(Aw_0)=0.21$,with m=3 and $\theta_0^1=63.4°$, $\theta_0^2=73.4°$
- -Fig.1b shows the maximum achievable T/A as a function of field symmetry number N and the number of the coil sections necessary for shaping the "zero flutter" isohronous field profile.
- -Fig.2 shows the typical current distribution on spherical surface which should be able to generate the necessary modulation for axial focusing at $K_{si}=1$ and $K_{si}=0.1$.
- -Fig.3 shows the necessary current settings for the production of isohronous "zero flutter" field at maximum bending power of the cyclotron magnet for the case given in fig.1,together with the values of factor s needed to make achievable maximum possible energy/nucleon at a given symmetry number N,with minimum value of F_0.
- -Fig.4 shows the shape of actual median plane magnetic field at current settings $J_0=J_1=J_2=6\cdot10^6$A/m together with the modulation field amplitude form factor B_f at N=3.

Using some simple calculations,one can be shown that the ratio of the number of Amper-turns of the spherical coil to the number of Amper-turns of the cylindrical coil,producing the same value of center field b,at the same radius r_0 is approximatively $NI_{sph}/NI_{cyl}=.32$, while the ratio of length of these Amper-turns is approximatively 0.25,when the same value of surface current density is used.

Conclusion

The air core superconducting cyclotrons are conceptually characterized by folowing basic features:

1. The isohronous magnetic field and cyclotron bending power are completly produced by suitable superconducting current-caying configuration of main coil structure.
2. The modulation of magnetic field,producing the cyclotron focusing power is generated by active flutter superconducting current-carying configurations.
3. The spherical-like coil structure is used for producing the isohronous magnetic field,giving the considerable cost savings of superconducting material in comparison with cylindrically wound main coil structure.
4. The azimuthaly dependent active flutter configurations are placed on the spherical surface,out of the median plane,allowing sufficient empty space in the interior of the coil structure for placing the suitable number of conveniently shaped dees,close to the median plane.High energy gain per turn,needed to separate the orbits and make the jumps over critical points in ν_r,ν_z diagrams,thus can be obtained.
5. The settings of isohronous average magnetic field and of the field flutter are mutually independent.This feature gives the possibility to improve the focusing and resonance operating limits by suitable choice of the flutter form factor.
6. Suitably chosen superconducting current-carying configurations at coil structure,should be able to produce the constant field value along the line of the axial injection and the fast field fall-of in extraction region,shaping thus the field profile close to the optimal field configuration for axial injection and efficient beam extraction.

References

1. H.G.Blosser and D.A.Johnson,Nuc.Inst.and Meth. 121 (1974)301
2. F.G.Resmini,Report MSUCL-341(1980)
3. H.L.Hagedoorn and N.F.Verster,Nucl.Inst.and Meth.18,19(1962)201

   W.M.Schulte,Thesis,Eindhoven University 1978
4. M.M.Gordon,Particle Accelerators 13 (1983)67-84

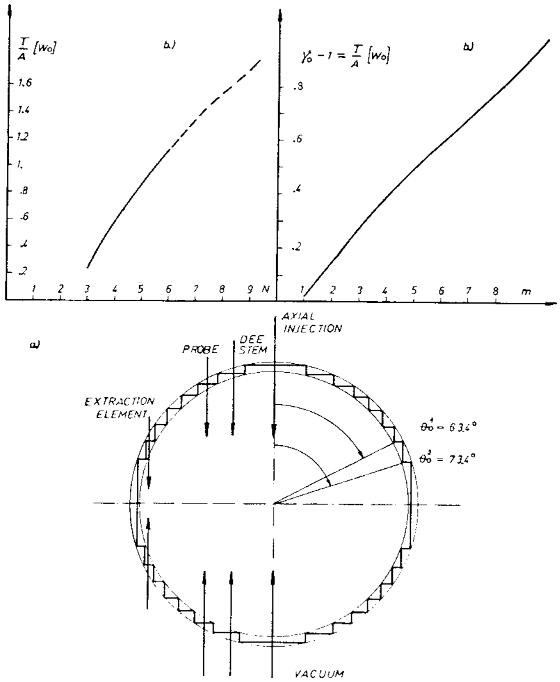

Fig. 1
a.)Coil-former for generating approximately a isohronous field profile up to $T/(Aw_0)$=.21. The steps are confined between two concetric spheres and are closely wound with superconducting cable of constant cross-section in a necessary number of layers.The position of the coil sections are indicated by their position angles.
b.)Maximum achievable T/A vs. field symmetry number N and number of the coil sections m.

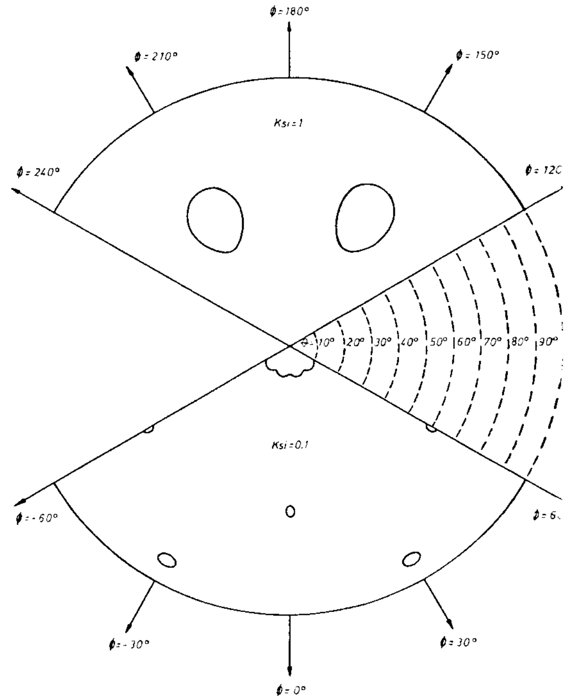

Fig. 2
Typical current distributions on spherical surface providing the field modulation of symmetry number N=3,for values of parametar K_{si}=1 and K_{si}=0.1.

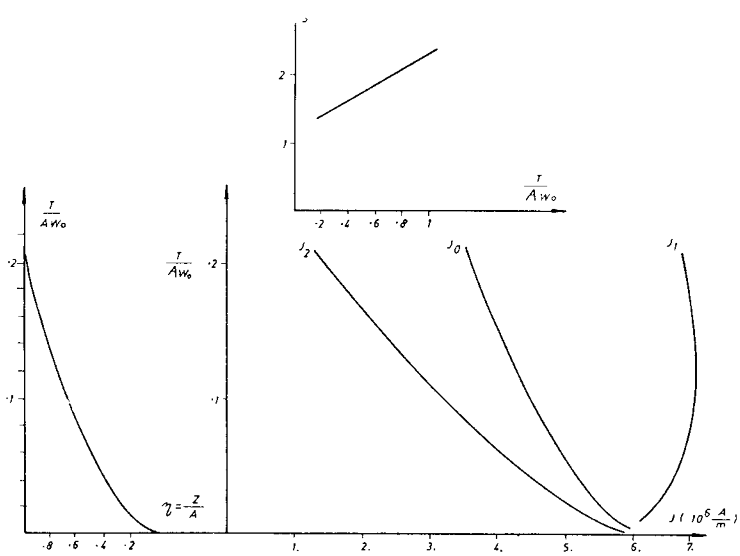

Fig. 3
Current settings in coil sections for the production of "zero flutter" isohronous field profile for the case shown in fig.1a,at maximum bending power,together with the values of factor s for the case given in fig. 1b.

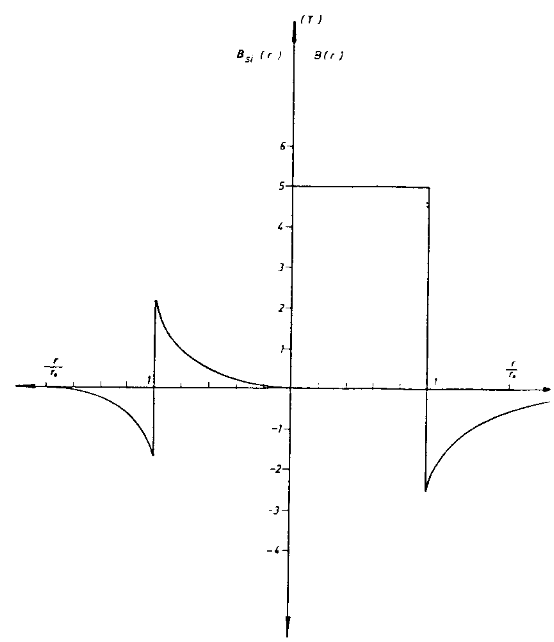

Fig. 4
Shape of actual median magnetic field at current settings $J_2=J_1=J_0=6\cdot10^6$ A/m,together with the amplitude form-factor of modulation field $B_f(r)$ at N=3.