A MODEL FOR CALCULATING IN TWO DIMENSIONS A MAGNET WITH REPETITIVE STRUCTURE IN THE THIRD DIMENSION*

L.H. Harwood and D. Johnson
National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

Abstract

We have derived, implemented, tested, and used a procedure for calculating the one-dimensionally averaged field of a three-dimensional magnet with a two-dimensional code with the averaging occurring in the dimension perpendicular to the plane of the calculation. The magnet may have repetitive structure in this third dimension. The model involves a simple modification to the permeability curve of the iron.

Introduction

Calculations of magnets was greatly simplified by the introduction of the computer code TRIM and subsequently POISSON. These programs solve the nonlinear Poisson's equation by over-relaxation. Accurate, three-dimensional calculations of the fields of magnets of the complexity of the NSCL cyclotrons surpass the capabilities of existing computer codes. Two-dimensional calculations are poor representations because of the distortions of the flux paths due to the holes in the yokes. We have derived a model (a similar derivation was done[1] independently at Lawrence Berkely Laboratory) which accurately represents the average effect of the holes on the field (with the averaging being done in the coordinate perpendicular to the plane of the calculation) by modifying the permeability curve of the iron in the region where there is structure in the third dimension to reflect the missing iron. This model has been incorporated into the code POISSON in place of the existing "stacking factor" option. Experimental and computational tests of the model have been done. With an efficient code for doing the average field calculations, it is possible to use features of the POISSON family of programs to study field shapes, average forces on parts of the magnet, etc. This paper will describe the physics of the model and describe the tests of the model.

Description of the Model

The premises of our model are: 1) that the problem being calculated has infinitely repetitive structure in one dimension (we will use "z" for this dimension although in a cylindrical case the it would actually be "θ") and 2) that all conductors lie in this dimension. A two dimensional calculation is then possible since the third dimension of the problem can then be characterized by the fraction of air/iron at every point in the plane of the calculation. The second condition exists because the model makes no attempt to include effects of conductors which are not perpendicular to the plane of the calculation. With these two conditions, we can interpret the B and H which come from the two-dimensional calculation not as "absolute" fields but as "z"-averaged values. Codes such as TRIM and POISSON were designed to calculate two-dimensional cases when there is no structure in the third dimension. The technique for including the regions that are fractionally iron is at the heart of the problem.

Consider the relation

$$\vec{B} = \vec{H} + \vec{M}$$

where $\vec{M}$ is the magnetization (and includes the 4π usually seen in this equation); its validity is unchanged by the addition of a term Y (where Y=1 if there is iron at that point and Y=0 if there is air) with the resulting form

$$\vec{B} = \vec{H} + \vec{M}\cdot Y.$$

This relation holds at all points in space. If $\vec{B}_{av}$, $\vec{H}_{av}$, and $\vec{MY}_{av}$ are the "z"-averaged values of $\vec{B}$, $\vec{H}$, and $\vec{M}\cdot Y$, then

$$\vec{B}_{av} = \vec{H}_{av} + \vec{MY}_{av}.$$

If we assume that the iron is nearly saturated (valid for the NSCL cyclotron magnets) and that there are not too many "corners" in the "Z" dimension of the calculation, then $\vec{H}$ has only a two-dimensional dependence. Likewise, the three-dimensional character of the magnetization, $\vec{M}$, effectively reduces to a two-dimensional dependence with Y reflecting the presence or absence or iron at a given "z" value. Thus,

$$\vec{B}_{av} = \vec{H} + \vec{M}\cdot F$$

where F is the "z"-averaged value of Y (note: F is a function of both position coordinates in the plane of the calculation.). The resulting equation is quite similar to the original with $\vec{B}_{av}$ and F replacing $\vec{B}$ and Y, resp. Since the relation between H and M is represented by the permeability curve of the iron, we have effectively only changed the permeability of the iron. Thus, the equations used in POISSON need not be changed if we can modify the permeability curves to reflect the new relation above. Obviously, the only new parameter is F. POISSON does not use the relation in the form shown above to get H from B and the permeability curve; it uses

$$\vec{H} = \gamma\cdot\vec{B} = \vec{B}/\mu$$

with

$$\mu = 1 + (M/H).$$

We need only insert F into the last relation yielding $\mu_{av} = 1 + (M/H)\cdot F = (1-F) + \mu\cdot F$ which gives

$$\gamma_{av} = 1/[(1-F)+F/\gamma].$$

If F is near 1 and μ is large, as it is for low-field laminated magnets such as might be used in synchrotrons, this relation holds as well and simplifies to

$$\mu_{av} \approx \mu \cdot F .$$

The standard version of POISSON assumes this last form of the equation for all conditions. For small F and

CH1996-3/84/0000-099 $1.00

small μ, this leads to rather unphysical results, ie. that the permeability of the region would drop below that of free space. The NSCL version of POISSON has been modified to reflect the more general form.

Tests of the Model

We have tested the relation derived above numerically and experimentally. POISSON was used for the numerical tests; the idea was to compare a repetitive structure (seen from the "side") with "stacked" iron; the geometry of the model is shown in Fig. 1 along with the two "views" used for the POISSON calculations. The coordinate plane for the first calculation was the "z"-y plane and the x-y plane for the second. A "uniform" H "bath" was produced using a

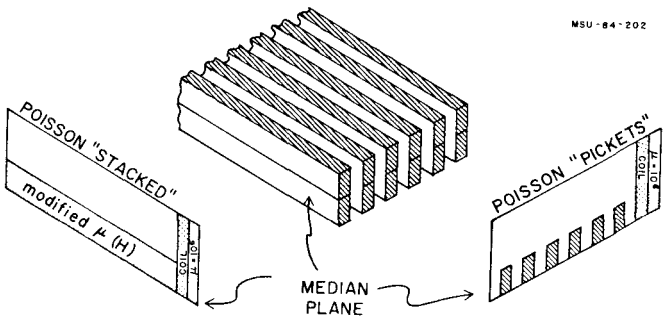

Fig. 1. Isometric view of the model used for the numerical test of the "stacking factor" prescription. The bars described in the text are shown in the center of the figure. The two projections (A and B) used in the POISSON calculations are also shown in their appropriate orientations relative to the bar configuration.

"window-frame" magnet. Six iron pickets were distributed in the median plane whose height to width aspect ratio was always at least 2:1. The "system" was then projected onto two planes; one projection ("z"-y plane) was parallel to the bars and gave view A; the other was perpendicular to the bars (x-y plane) and gave view B. The two views formed the basis of two POISSON calculations. In the first view, the bars can actually be seen; in the second, the projection of the bars is a region with a composite material, ie. one which we claim can be characterized with the "stacking factor.". For view A, $\vec{B}$ was then averaged over the three inner pickets and the intervening spaces. A corresponding calculation was made for view B, ie. the picket region was replaced by iron with a stacking factor; the median plane B values were extracted and compared to the results for the pickets. Pickets of various widths were used to change F. A number of field levels were tried to test the universality of the model. The results are summarized in Table 1. The agreement is quite good and is consistent with the conditions of the model. Our criteria are such that non-saturated iron or close-spacing of the iron pieces would distur the accuracy of the model. At low fields, eg. 1.5 kG, the iron is like an equipotential, and the field lines are perpendicular to the surfaces of the iron. Thus, there must be locations where the fields lines are nearly parallel to the "z" axis, and the "z" component of $\vec{H}$ must, therefore, be a function of "z", thus the requirement that $\vec{H} = \vec{H}_{av}$ is thus not always met. At higher fields, eg. 15 kG, the local effects of the neighboring pickets modify the field and produce "z" components in $\vec{H}$. Thus, the differences observed between the "picket" value and the "stacking factor" value are consistent with the conditions of the model, ie. that the model works when $\vec{H}$ is independent of "z".

TABLE I

COMPARISON OF AVERAGE FIELDS CALCULATED WITH "PICKETS" AND WITH A "STACKING FACTOR"

Iron Fraction	B(average) (Gauss) Pickets	"Stacking factor"
0.5	1529	1571
	14635	14713
0.33	1514	1571
	13983	13985
0.10	1488	1576
	13007	12995

The experimental test consisted of comparing the calculated fields for the K500 cyclotron to the measurements of the actual field. The K500 magnet designs were done with TRIM and, the modified permeability tables were entered manually. The comparison was quite involved since steps had to be taken to include the complete field of the pole tips before the comparison could be made. The details of the procedure used can be found in Ref. 2. Discrepancies of less than 1% were found.

Summary

We are confident of the formula we have derived, and it has replaced the one used in POISSON for the stacking factor option. If one is aware of the basic assumption of the model, ie. $\vec{H}$ is in the plane of the calculation, then this formula can be used over a large range of field levels to predict, in a two-dimensional calculation, the "z"-averaged field in a magnet which has some repetitive structure in the third, "z", dimension.

References

* Work supported by NSF under Grant No. PHY-83-12245.

1) K. Halbach, private communication.

2) Bellomo, et al., NIM 180 (1981) 285.