POSSIBLE POLE GEOMETRY DETERMINATION METHOD FOR AN ISOCHRONOUS SUPERCONDUCTING CYCLOTRON

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Abstract

If the determination of the hill gap is simply dictated by the extraction elements, two important questions arise as soon as the design of the pole tips is considered : How to choose the valley depth and the spiralling law in order to locate in valleys accelerating electrodes of simplest shape.

The answers to these questions, fed in the scope of the IPN Orsay project, led the authors to define some helping criteria, and to elaborate a determination method for the pole tips geometry.

The method makes use of 4 different simple "low-consuming" codes. These are SPIRSI, SPIRAL, FI3DR2, ALADIN.

The latest has the specific interest to calculate directly the first Fourier Components of the magnetostatic field created by the saturated pole tips in a time an order of magnitude lower than ALACER.

I. Basic Data, Remark on K_F

The proposed method can start only when a choice of some basic parameters has been made.

These are :

- N : periodicity of the machine
- KB : bending parameter
- KF : focusing parameter
- R_{ext} : extraction radius
- KB_{min} : or $B(R_{ext})_{min}$: the lowest working mean induction
- gh : the hill gap
- $B_{iron}(r)$: mean contribution of the pole saturated iron as a function of the radius.

From this list of "basic data" we can deduce and draw the expected operating diagrams as T(MeV/n) the kinetic energy per nucleon, B(Rext) the working induction at the extraction radius, B(o) the central induction; F_p the rotating frequency as functions of Zi/A. These quantities are calculated by our code ENERG2. Some papers have been written that can help for choosing the main [1] parameters as N, KB, KF, R_{ext}, gh, so we will just discuss here the choice of $B_{iron}(r)$, KB (or more precisely the lowest working induction $B_{min}(R_{ext})$ which is directly related to), and make a short remark on KF.

The numerical value of the focusing parameter KF may delude because it is not directly related to the consequent difficulty of the design. More precisely the real difficulty of the design is due to the extension of the $\gamma(r)$ values range, because it reflects exactly the range of magnetic field radial gradient to create.

For example the IPN Orsay KF = 220 MeV value is twice as constraining as the Milano KF = 200 MeV and equivalent to the K800 MSU KF = 400 MeV value.

II. The Choice of the $B_{iron}(r)$ Function

G. Bellomo and R. Resmini have developped a powerful method [2] based on the minimization of the Trim coil power requirements which delivers the $B_{iron}(r)$ function as soon as the overall dimensions of the main coil are chosen, and with only the height of the partition as a parameter.

But an unrevealed constraint has been introduced which affects the results, not due to the method itself but in the way to use it. This constraint introduced at Milano and MSU has been the starting hypothesis of the solenoīdal simple form of the main coil.

The only article[3] that speaks about another way for choosing the law of $B_{iron}(r)$, starts from the fact that,

1. there exist an amazing similitude in shape between $B(o)\gamma(r)$ laws and fields created by very flat circular coils,
2. there exist solutions for coils able to create flat fields (Helmoltz position),
3. there exist also approximate solutions derived from the uniformly magnetized ellipsoīd that give $B_{iron}(r)$ = Cte.

The consequences of these starting choices not completely calculated at the time of writing the mentioned paper applied to the IPN Orsay project are now well established.

- The main coil has an uncommon form as it can be seen Fig.1.

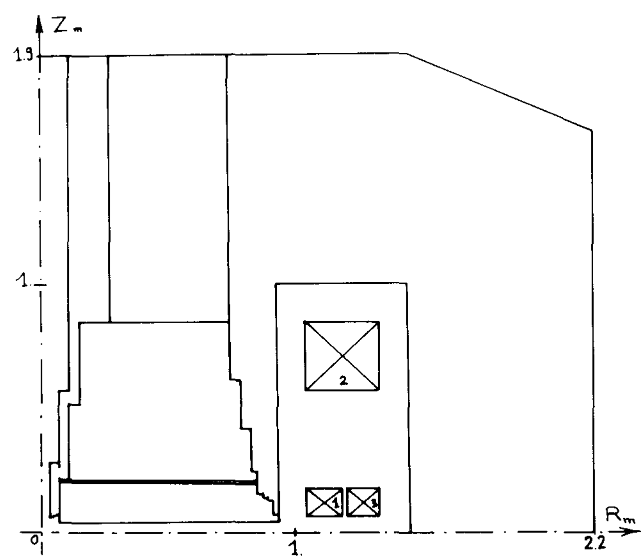

Fig. 1 - Sectional view of a quarter of the IPN Orsay project magnet showing the uncommon main coil cross section. Part.1, creates the radial gradient; parts 2 and 3 in series flat fields.

- The consumed power in Trim coils reaches its minimum minimorum of 20 kW for greater γ and B(o) range. (See Fig. 2)
- The signs of the two independant currents in the main coil partitions are the same. This has the great advantage of simplifying the technology of the winding by the much more lower precompression needed.
- The starting hypothesis of the uniform magnetization of the pole iron is in that case better respected than in the case of opposite currents (K 800 MSU-MILANO) and should preserve the possibility to work at as low an induction as 1.75 T.

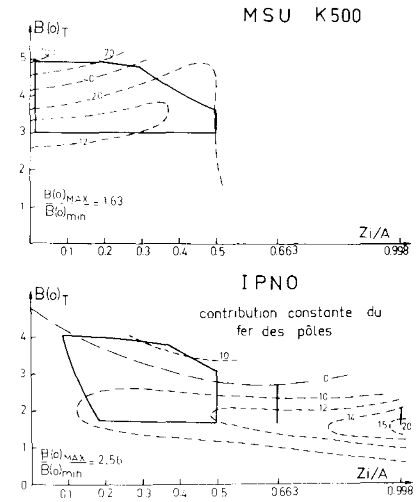

Fig.2 - Trim coil consumed power comparison between MSU-K 500 and IPN-Orsay project.

III. Choice of the minimum working induction

There is a great interest to be able to work at the lowest possible induction $B(r\,ext)_{min}$ for acceleration of the lightest particle. This is due to the flutter $F \sim \tilde{/B^2}_{min}(r)$ which reduces the spiralling and consequently the mean fiber radial length of the accelerating electrodes.

We present here an approximate reasoning for demonstrating that a working induction in the median plane of 1.7 T keeps saturated the iron of the hills.

Let us suppose an ellipsoïdal cavity digged out of a soft iron cylinder with a circular basis (Fig.3) flat enough for developping an inside 1.8 T = B_{in} homogeneous induction.

The iron is homogeneously saturated at M = 2.14 T
We will isolate by mind a little circular surface "S" at the top around the axis and will consider two points a and b on the axis close enough to the S surface to see it under 2π steradian.

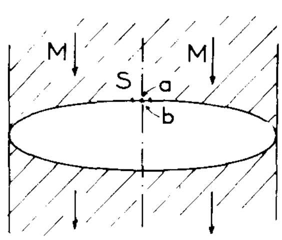

Fig. 3

Due to the uniformity of $B(b) = 1.8\ T = B(S)+B(E-S)$

B(S) being the contribution of S in "b"

$$B(S) = \frac{2.14}{2} = 1.07\ T$$

B(E-S) being the contribution of all the magnetized surface excepted S.

So $B(b) = 1.07 + B(E-S) = 1.8\ T$

$\rightarrow B(E-S) = 0.73\ T$

Now applied at "a"

$$B(a) = -B(S) + B(E-S) = -1.07 + 0.73\ T$$
$$B(a) = -0.34\ T.$$

this means that such a magnetized state can be realized only if the whole system is plunged in a uniform external magnetic field created by coils $\mu_0 H_c \geq +0.44\ T$ because the soft iron need an 0.1 T excitation to reach the saturation.

Finally the total induction inside the cavity may at least be

$$B_{inside} = 0.44 + 1.8 = 2.24\ T$$

Now, by mind, let cut the cylinder in 2N radial sectors of π/N in opening and put away half of them, keeping N alternating iron and air sectors. Such a system, when homogeneously magnetized, is a special cyclotron with infinite depth valleys.

The mean contribution of the magnetization in the median plane is $\tilde{B}_{in} = 0.9\ T$.

Now we will consider a new point "a" in the iron and a surface S just sligthly displaced from the vertical axis and write

$$B'(a) = -B(S) + B'(\frac{E}{2} - S)$$

where $B'(\frac{E}{2} - S)$ is the new contribution of all the remaining charged surfaces excepted S.(The alternating part of which is vanishing with its distance from the axis)

so $B'(\frac{E}{2} - S) = 0.365\ T$

and $B'(a) = -1.07 + 0.365 = -0.705\ T$

This means that such a magnetized state can be reached only if the external coil excitation equals $\mu_0 H'_c = 0.805\ T$ and the mean total induction in the median plane will be $B'_{inside} = 0.9 + 0.805 = 1.705\ T$.

It can be checked that this value is always reached whatever the excentricity of the cavity generating ellipse has been chosen and for keeping in "hills" half the initial iron volume. Under these conditions

$$B'_{\substack{inside\\minimum}} = \frac{3}{4} \times 2.14 + 0.1 = 1.705\ T.$$

This means that for an actual design where the remaining fraction of the hills is lower than 1/2 and due to the non infinite valleys presence which enhances the magnetization in hill iron we certainly can work at levels as low as 1.75 T.

Conversely, the full saturated iron in hills reduces the magnetization level in the valley bottom and we always have interest to choose them as deep as possible which means a low valley contribution (in any case the RF holes will reduce it).

To our opinion a good choice for the $B_{iron}(r)$ function would be flat with the radius and in the range

$$1.1\ T < B_{iron}(r) = Cte < 1.3\ T.$$

IV. Valley profile Determination Method

Our valley profile determination method is based upon the mentionned basic data applied to a fictive right-sectored machine (Fig. 6).

The hill geometry takes into account the electric half gap width and the trim coil thickness (SPIRSI code)

The hills contribution to the mean field $B_H(r)$ is calculated by "ALADIN" and added to the contribution of a central plug, the height of which Z_P is chosen in order to fill in the central depression;so we get $B_{(H+P)}(r)$. (See Fig.4).

Introduced in "SPIRAL", with the total expected contribution of the pole iron $B_{iron}(r)$, we get the height of each valley zone by considering this zone as a part of an ellipsoïd able to deliver by itself

$$\overline{B}_V(r) = \overline{B}_{iron}(r) - \overline{B}_{H+P}(r)$$

The results of such a calculation applied to an example whose basic data are
$N = 3$, $K_B = 500$, $K_F = 130$, RP = 0.77 m, gh = 0.07 m , $B_{iron}(r) = 1.3\ T$ are shown Fig.4 and Fig. 5.

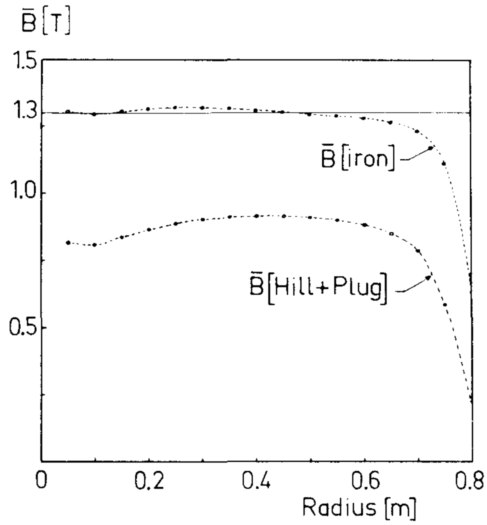

Fig.4 - Contributions to the mean induction, lower curve : hills and central plug ; upper curve in dotted line hills + plug + valleys continuous line : expected $B_{iron}(r)$.

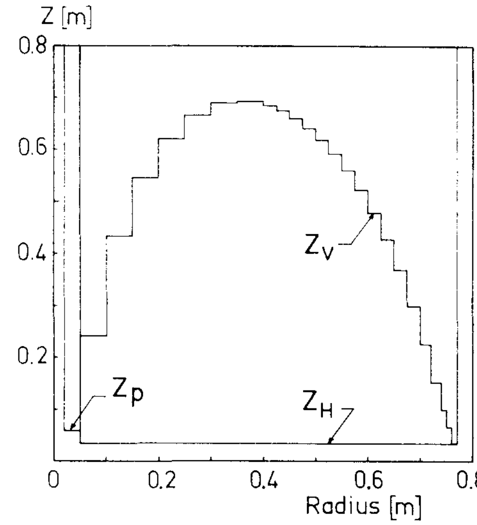

Fig.5 - Valley profile calculated by "SPIRAL" for the starting right-sectored machine, in the case mentionned in the text as an example.

V. Spiraling law Determination

At this stage the fictive right-sectored machine is only satisfactory for its $B_{iron}(r)$ law, using ALADIN we calculate the K^{th} first Fourier Harmonics of the field $C_{K.N}(r)$, $\phi_{K.N}(r)$ necessary for calculating the wave numbers by Hagedoorn Verster formulaes.

The spiralling law determination method consists in imposing to the more difficult particle to accelerate $\frac{Zi}{A}$ = KF/KB, $B(o)_{max}$ a vertical wave number

$\nu_z(r)$ = 0.1 in all the isochronous part of the field.

The code FI3DR2 numerically integrates the $\sum_K \frac{d\phi_{KN}(r)}{dr}$ functions and delivers a new field map having the same amplitudes but new phases $\phi_{K\ N}(r)$. This phase law is imposed to the axis of the hills by "SPIRSI". The implicit remaining constant amplitude hypothesis is approximative and can be corrected by a 2 steps iterative process Fig.7 and Fig. 8.

VI. Survey of the Computer Programs

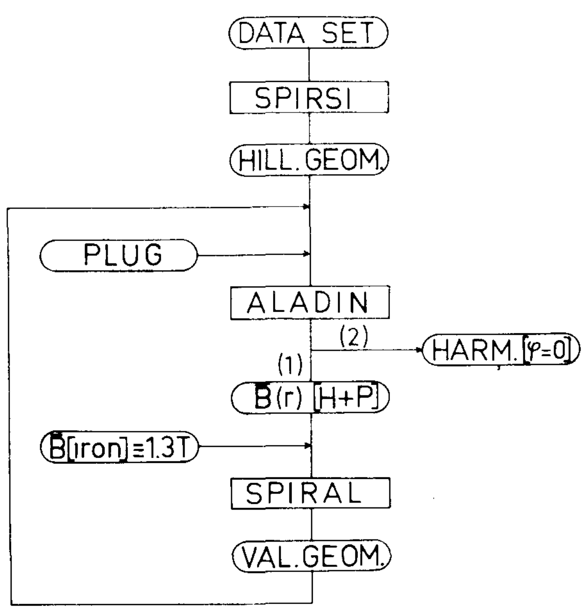

Fig.6 - Schematic organization of the computer codes for the treatment of the first part of the pole geometry determination method. The goal is the valley profile when g_h and $B_{iron}(r)$ are given. The method is applied to a fictive right-sectored machine.

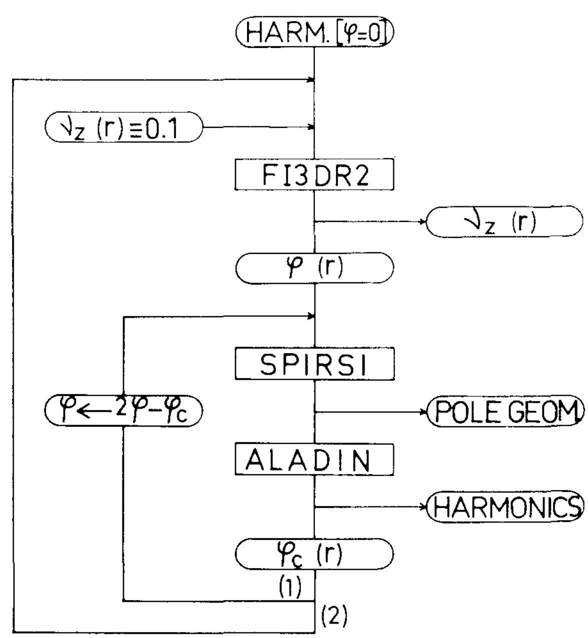

Fig.7 - Schematic organization of the computer codes for the treatment of the second part of the pole geometry determination method which delivers the spiraling law. A two steps iteration is necessary in which an enhanced convergence process is introduced.

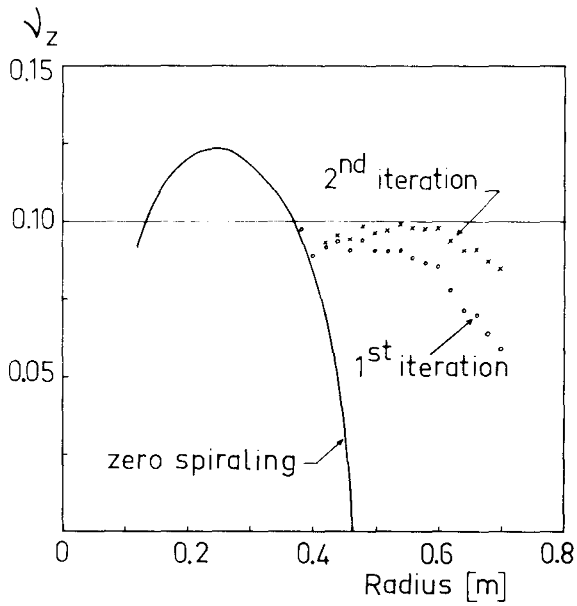

Fig.8 – ν_z as a function of the radius in the case of the machine treated as an example (Hagedoorn-Verster formula).

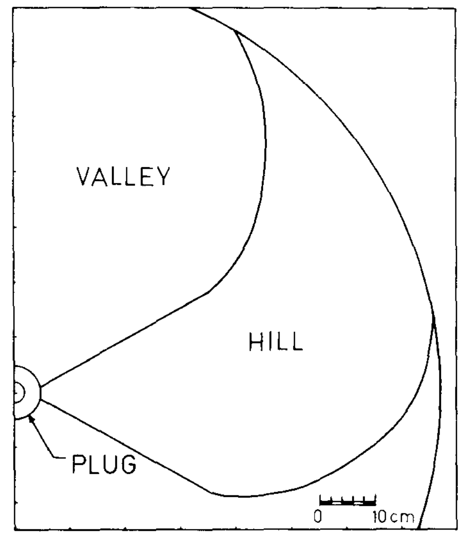

Fig.9 – Plan view of the pole tip.

VII. Conclusion

The results concerning the example defined above shows that with our starting hypothesis of $\overline{B}_{min}(r\,ext)$ = 1.75 T which is now included in the basic datas (Rext choice) the spiral starts only above R = 0.35 m as it is shown on Fig.9. It is also clear on Fig.8 that the ν_z value can't be reduced to 0.1 under this late radius. The consequence of these choices added to those of the method is a reduced to the minimum length for the radial mean fiber of the accelerating electrode.

This pole geometry determination method gives a very good first approach of the pole design. It presents the net advantage to be carried out in a very short time. The best improvement as far as the saving of computer time is concerned comes from the "ALADIN" code which is very low-time-consuming compared with ALACER. The mathematical basis of this code is presented in appendix.

Appendix

Mathematical basis of the program "ALADIN"

Let us consider a pair of plane pole-tips elements (dr', $r'd\theta'$) symmetrically disposed at coordinates (r', θ', $\pm Z$). For a given magnetization M, the z-component of the elemental induction at the observation point $(r,\theta,0)$ is given by :

$$d^2B_z(r,\theta) = \frac{MZ}{2\pi}\,\frac{r'dr'd\theta'}{D^3(r,r',\theta,\theta',z)} \qquad (A1)$$

where $D(r,r',\theta,\theta',z) = \left| r^2+r'^2-2rr'\cos(\theta-\theta')+Z^2 \right|^{1/2}$

When calculating the contribution of pole-tips to the magnetic induction, two approximations are commonly used :

- the magnetization M is assumed to be uniform, and equal to its saturation value,
- the geometry is approximated by a set of plane surfaces, described here as P-surfaces, parallel to the median plane.

The overall induction $B_z(r,\theta)$ is obtained by performing a numerical integration of (A1) over each P-surface, and summing over the whole set. The desired azimuthal harmonics are then deduced by Fourier analysis. An alternative way consists in reversing the order of these operations, and choosing an appropriate shape for the P-surfaces :

The Fourrier expansion of (A1) gives :

$$d^2B_z(r,\theta) = \frac{MZ}{2\pi^2}\,r'dr'd\theta' \sum_{m=\infty}^{+\infty} e^{im(\theta-\theta')} \int_0^{\pi} \frac{\cos m\phi}{D^3(r,r',\phi,z)}\,d\phi$$

where $D(r,r',\theta,z) = \left| r^2+r'^2-2rr'\cos\phi + z^2 \right|^{1/2}$

If the pair of pole-tip elements is azimuthaly periodic with period $\frac{2\pi}{N}$, we get :

$$d^2B_z(r,\theta) = \frac{NMZ}{2\pi^2}\,r'dr'd\theta' \sum_{p=0}^{\infty} \varepsilon_p \cos pN(\theta-\theta') \int_0^{\pi} \frac{\cos pN\phi}{D^3(r,r',\phi,z)}\,d\phi$$

where $\varepsilon_p = 1$ for $p = 0$

$\qquad = 2$ for $p > 0$

the integration and sum are then to be performed over one period.

For a moderate spiralling, the P-surfaces may be delimited by radii and arcs :

$$\theta_j - \delta\theta_j \leq \theta' \leq \theta_j + \delta\theta_j$$

$$r_j - \delta r_j \leq r' \leq r_j + \delta r_j$$

and we get for the integration over θ' and r' :

$$\int_{\theta_j-\delta\theta}^{\theta_j+\delta\theta_j} \cos pN(\theta-\theta')\,d\theta' = 2\,\frac{\sin pN\delta\theta_j}{pN}\,\cos pN(\theta-\theta_j)$$

$$\int_{r_j-\delta r_j}^{r_j+\delta r_j} \frac{r'dr'}{D^3(r,r',\phi,z)} = - \left[\frac{D+r'}{D(D+r'-r\cos\phi)} \right]_{r_j-\delta r_j}^{r_j+\delta r'_j}$$

The contribution of the j^{th} set of P-surfaces is then :

$$B_{zj} = - \frac{NMZ_j}{\pi^2} \sum_{p=o}^{\infty} \varepsilon_p \frac{\sin pN\delta\theta}{pN} \cos pN(\theta - \theta_j) \int_o^{\pi} \left[\frac{D+r'}{D(D+r'-r\cos\phi)} \right]_{r_j-\delta r_j}^{r_j+\delta r_j} \cos pN\phi . d\phi$$

The p^{th} harmonic of the induction is then obtained by performing a numerical integration over ϕ, and summing over the P-surfaces composing one period .

"ALADIN" has been developped from this method. The numerical integration, probably not optimized, is based on Gauss-Legrendre integration.

For the given example, the mean value and the 3 firsts harmonics of the induction are computed for 16 values of the radius in 1 mn of Univac 1110 time. In comparison with classical programs (ALACER and derivations), the benefit is more than an order of magnitude in computing time for quite similar results (typically 1,0 Gauss for amplitudes, 1 mrad for phases).

References

1. H.G. Blosser and D.A. Johnson, Focusing Properties of Superconducting Cyclotron Magnets, NIM 121, (1974) 301-306.

2. G. Bellomo and F. Resmini, A Method for minimizing Trim Coil Power Requirements in Superconducting Cyclotrons, 8th International Conference on Cyclotrons and their Applications 1978, (2095-2100).

3. A. Laisné, IPN-Orsay Project, First Machine Design Studies, 9th International Conference on Cyclotrons and their Applications, CAEN-1981 (203-208).