STARTING TIME EFFECTS ON THE AXIAL EMITTANCE OF THE MSU CYCLOTRON*

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Abstract

Normally if a uniform source output and adiabatic conditions are assumed, it is found that the axial phase space at extraction is independent of starting time. However, we have found that the changes in the axial phase space as the beam passes through the $\nu_r=2\nu_z$ resonance depend on the centering of the particles as they pass through the stop band, which is correlated with the starting time of those particles. The result of this time dependence is to increase the time averaged emittance (the quantity normally measured) by a significant amount over that of a single starting time.

INTRODUCTION

Previous estimates of the axial emittance of the extracted beam of the K500 cyclotron suggested that it would be of the order of 5 mm-mrad (un-normalized). These estimates were primarily based on source output data[1] for PIG sources similar to our design[2]. If the acceleration process beyond the central region is reasonably adiabatic, as is normally true for an accelerator of this type, and the source (central region) output approximates eigen-ellipse, then the normalized emittance will be the same after extraction. Early measurements of the K500 emittance suggested that the actual value at the exit of the cyclotron was considerably larger than these arguments would suggest[3]. The question then arose, were the measured values physically possible and if so what part of the aforementioned argument is in error. Aside from questioning the source data itself the only obvious source for error was the assumption that the acceleration was always adiabatic.

NUMERICAL STUDY

Since both the measurements and a recent comprehensive orbit study were performed with a 30 MeV/A $^{12}C^{4+}$ beam we decided to check the adiabatic assumption using this case also. As can been seen in figure 1 the beam must cross both the $\nu_r=1$ and the $\nu_r=2\nu_z$ resonances before reaching extraction energy. It was felt that perhaps the beam would experience some non-adiabatic effects as the beam passed through these stop bands, which are enlarged by the presence of first and second harmonics of the magnetic field. To check this hypothesis we choose a group of 41 central rays which populate the starting times between 230° and 250° in rf time. These rays were positioned to be 50 turns before the deflector with the orbit parameters chosen to conform with the selection process used in the centering studies[4]. With each of these 41 particles we associated an ellipse which corresponded to a 5 mm-mrad axial phase space eigen-ellipse for the τ_0 =240° ray. Using the spiral gap[5] code which assumes linear z motion, but allows the radial motion to couple into the axial, we tracked all 41 orbits forward 50 turns and then on thru the extraction system up to the edge of the yoke. In figure 2 we have plotted the axial phase space ellipse for all the rays at several points along the trajectory. As can be seen in this figure, up to turn 26 the ellipse all have roughly the same orientation, as one would expect if the acceleration were adiabatic, since we started on an eigen-ellipse. Around turn 26 the particles pass through the $\nu_r=2\nu_z$ resonance, which because of the enlarged $\nu_r=1$ stop band, coincides with the $2\nu_z=1$ resonance. After this point each of the ellipses begin to rotate at different rates indicating that they are no longer eigen-ellipses. After another 24 turns, when these particles reach the deflector entrance, there are significant differences in the orientation and aspect ratios of the ellipses, as is shown in the plot marked '50' in figure 3. Also shown in this plot is an ellipse which encompasses 95% of the points contained in the boundary of the other 41 ellipses. Presumably if one were to measure the emittance of the beam at this point it would be the area of this encompassing ellipse which one would find. In this case the area is 22 mm-mrad, and a similar ellipse at the exit of the yoke (turn 51) has an equivalent area.

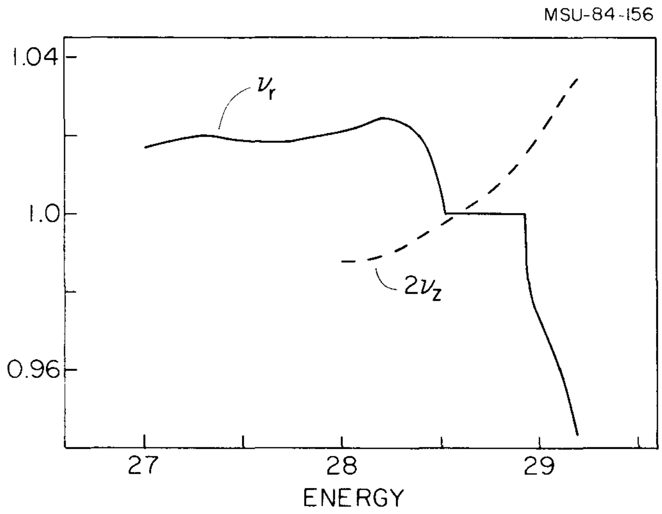

Figure 1. The radial and vertical focusing frequencies ν_r and ν_z as a function of energy for the compensated $^{12}C^{4+}$ 30 MeV/A field. The beam crosses $\nu_r=2\nu_z$ and $2\nu_z=1$ simultaneously during the $\nu_r=1$ stop band, at about 28.4 MeV/A.

Now that we have found a mechanism which could give an effective broadening of the axial emittance we need to check that it can be extracted. In figure 3 we have replotted all 41 of the ellipses at the entrance to the deflector with a larger ellipse which shows the maximum possible acceptance of the extraction system.

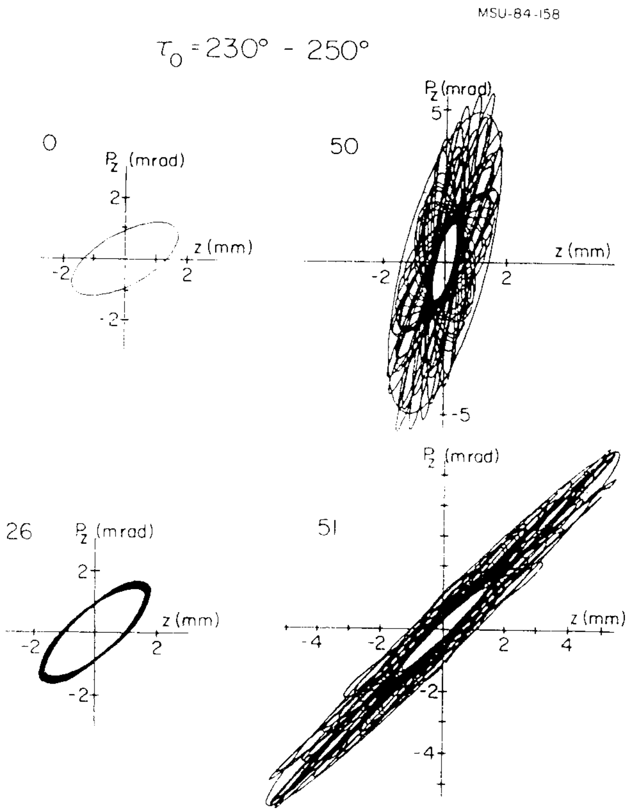

Figure 2. Axial phase space ellipses for the 41 central rays which populate the starting times between 230° and 250° rf time, at 4 different turns. The plot labeled '0' shows the ellipses at the start, 50 turns before extraction. Turn 26 corresponds to the point at which the beam is passing through the $\nu_r=2\nu_z$ resonance. Turn 50 is the entrance to the deflector, while turn 51 is at the exit of the yoke. Included in the plot at turn 50 is an ellipse which contains 95% of the boundry points in the other 41 ellipses. This ellipse has as area four times larger than the starting ellipse suggesting that the observed emittance would be considerably larger than the internal emittance.

This acceptance was found by computing a transfer matrix for the 240° central ray at 2 degree intervals along the extraction trajectory and then using this to determine the limitations on z and p_z at the entrance imposed by the mechanical shape of the channel at each point. The minimum of the values found is then the largest possible acceptance. It should be noted that this procedure ignores the fact the the electric and magnetic fields at the edge of the aperture will be highly non-uniform and therefore unpredictable, thus making much of this area "unusable", but how much is uncertain. Never-the-less this approach would suggest that most of this broadened axial phase space should pass through the extraction system. Since 22 mm-mrad is larger than the value which has been measured the measurements appear to be within reason. Besides the non-linear effects the actual value of the emittance would be smaller if, the extreme starting times aren't populated, the eigen-ellipse before the resonance is smaller than 5 mm-mrad, or some of the central rays go outside the "useful" physical aperture of the extraction system (this was not checked).

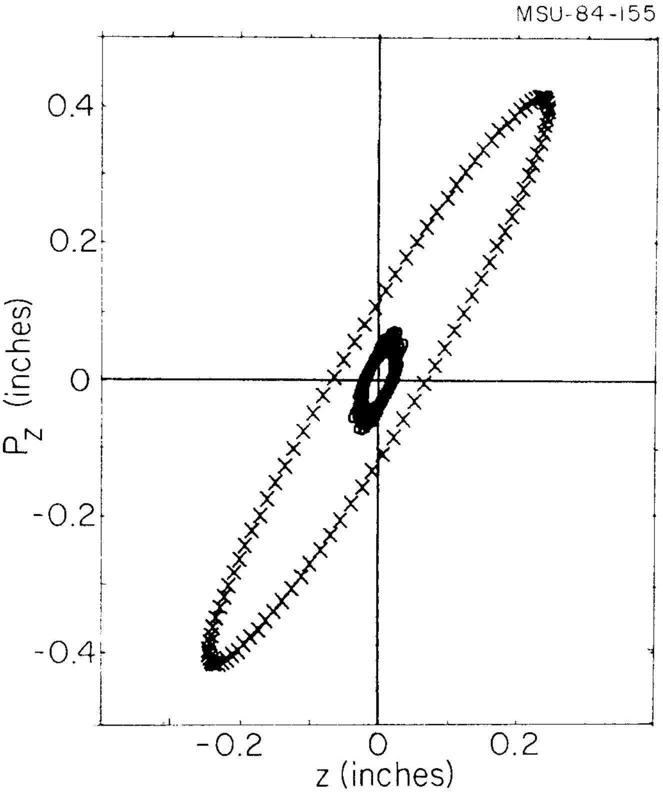

Figure 3. The 41 central ray axial phase space ellipses shown in figure 2 are replotted here with a larger ellipse showing the maximum possible acceptance of the extraction system. This indicates that the extraction system is indeed capable of handling the broadened axial phase space area.

In view of our plans to implement phase slits in the k500 cyclotron to allow for phase selection[6], we were interested in how large the axial emittance might be with a restricted set of starting times. As shown in figure 4 even if the starting times are limited to ±1.5° the effect is still significant, in this case the bounding ellipse has an area of 9 mm-mrad.

EXPLANATION

Transit time effects in the source to puller region caused the particles with different starting time to have different centers. These differences are insignificant except when the particles pass through the stop bands where the small stability region results in enhanced sensitivity to centering. As can be seen in figure 1 the beam crosses the $\nu_r=2\nu_z$ resonance at a point near (or as in this case at) the $\nu_r=1$ resonance. This closeness of the two resonances prevents passing through either one of them centered,since the conditions going in to this region must be chosen to produce the least harmful combined effect. The consequence of this is that all the particles will experience some distortion of the axial phase space, but the effects will depend heavily on radial position which as previously shown varies with starting time. In such a situation even assuming linear z motion, and filling of an axial eigen-ellipse at the center, one will find that after crossing the resonance each starting time's ellipse will tumble at a different rate. This effect wouldn't be noticeable except that it is allowed to continue for another 24 turns before the beam is extracted. After this long a flight path the small differences in tumbling rates result in large differences in orientation, and thus the behavior observed in figure 2.

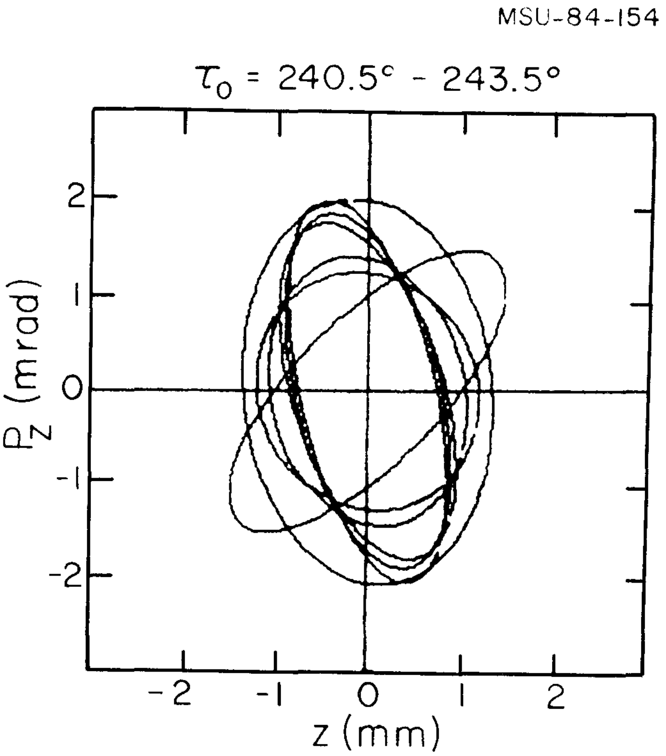

Figure 4. Axial phase space ellipse at the entrance to the extraction system for a group of particles which populate the starting times between 240.5° and 243.5° in rf time. Again an ellipse which encloses 95% of the others is drawn. In this case the encompassing ellipse has an area of 9 mm-mrad.

If the process were adiabatic then the action ,J, as defined by[7],

$$2J=\gamma_\theta z^2+\beta_\theta p_z{}^2+2\alpha_\theta z p_z$$

$$\text{where, } \beta_\theta\gamma_\theta-\alpha_r{}^2=1$$

would be an adiabatic invariant (α_θ , β_θ are the Courant-Snyder parameters evaluated at angle θ). In figure 5 we have plotted the value of J as a function of energy once per turn on a spiral line. As expected it has small fluctuations about a value of .17 until it reaches an energy of about 28.4 MeV , which is the energy of $\nu_r=2\nu_z$, at which point it jumps by a factor of two. Detailed analytical analysis[8] indicates that the action should make such a jump during acceleration through the $\nu_r=2\nu_z$ resonance. The fact that the action jumps by a factor of two says that a broadening by 2 of the phase space area is to be expected if all the particles had similar centering's, as in the phase selected case. To arrive at a result which would be more representative of the broad starting time case it would be necessary to compute the action for all the different starting times at each of the two possible linearly independent z,p_z cases.

Further work is required in order to check the specific agreement between the theory and the numerical calculation. The numerical work presented here is done in a field with existing imperfections in the magnetic field. The presence of a gradient in the first harmonic at the $2\nu_z=1$ (at the same place as $\nu_r=1$) which could have similar effects, and the large $\nu_r=1$ stop band, complicate the issues, so a similar computation in a perfect field is in order. It would also be appropriate to investigate the effect of the non-linear z terms in the hamiltonian, and to allow coupling of the axial motion back into the radial motion.

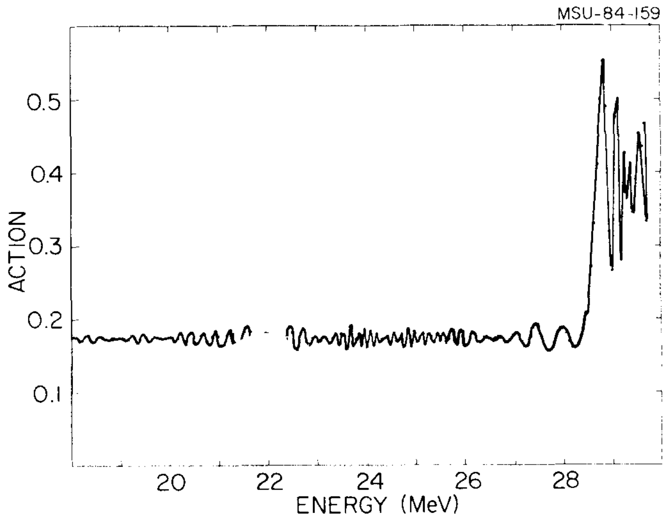

Figure 5. The action as a function of energy for the τ_0 = 242° central ray. As expected J is an adiabatic invariant up to 28.4 MeV, the energy of the $\nu_r=2\nu_z$ resonance. At this energy the action grows by a factor of two indicating a two-fold growth in axial phase space area. The increased fluxuations in J after the resonance are caused by the rapidly changing ν_z in this region.

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