THE DESIGN, TRANSMISSION MATRIX AND BEAM EMITTANCE FOR THE INR CYCLOTRON BEAM-EXTRACTION SYSTEM

Zao Xiaofeng Zhang Jian Chen Kuosen

The Institute of Nuclear Research Academia Sinica, Shanghai of China

The design feature of the beam-extration system of the INR cyclotron is described. Here extraction efficiency of the system is about 60 -80% for 10-30MeV proton. A simple harmonic oscillation equation (horizontal and vertical motion) of the electrostatic deflector is derived. The first order transmission matrix for beam-extraction system is given. The method for determining the extracted beam initial emittance ellipse matrix is analyzed.

Design Feature and Operation Results

Design Feature. The INR 120 cm classical cyclotron has been converted to a three sector focusing cyclotron. Its beam extraction system consists of two sets of electrostatic deflector, a magnetic focusing channel and a magnetic shielded channel and steering magnet ($\pm 2^{o}$). Layout of this equipment is shown in Fig.1.

The first deflector with parallel plate is 50 deg in the azimuthal width. The second one with hyperboloid is 52 deg. The entrance of the first deflector is located in the clockwise direction 6 deg behind the hill of spiral-sector magnetic field (on the radius 60 cm). There is a interspace of 8 deg between the two deflectors to interpose a probe. The interspace falls just in the region where the curvature direction of deflected extration orbit turns into the opposite one for beams of various energy. The entrance and the exit gap width of the first deflector are 5 mm and 8 mm, respectively. The positions of entrances and exits can be adjusted within $\pm$ 15 mm along the radial direction by a motor with numerical indicator. The septums consist of about 60 sheets of high-purity graphite. There is a 0.1 mm thick tantalum plate with v-slot of 4 cm length in entrance part of the first septum. The high-voltage electrodes of deflector only 26 mm high. The whole deflector system is mounted on an aluminum base plate which can be pulled out of the vacuum box by means of wheels. Behind the second deflector, a magnetic focusing channel is installed. It consists of three pieces of iron(DT4) of 19 mmx39 mmx140 mm and is similar to a quadrupole lens of 800-900 G/cm gradient which provides the radial focusing for the beams extracted.

The purpose of 350 mm long magnetic shielded channel is to shield the fringing magnetic field and the extracted beam trajectory could be aligned with the axis of the old beam transport system. The double-layer shielded construction, the external layer is DT4 iron of 10 mm thick and the internal layer is 1J79 iron-nikelalloy of 2 mm thick, is applied to provide very high shielding efficiency (better than 98%).

Mechanism of Extraction. The beam extraction takes place at ν_R=0.8. The turn-separation of group of particles before deflection is generated by a first harmonic magnetic field component which causing a coherent ocillation amplitude followed by a precessional motion of the orbit beyond the 580 mm radius where ν_R=1. The calculation showns that a total turn-separation of 7-8 mm at the 620 mm radius for 10-30 MeV proton beam can be obtained with a first harmonic field which is less than 4 G, as shown in Fig.2. The blowing up of the beam because of the $\nu_R=2\nu_z$ coupling resonance the beam has to pass can be avoided if the first harmonic magnetic field is less than 7 G. The calculated acceptance of the system are more than 20 mm•mrad both in the horizontal and vertical plane.

Operation Results. The whole extraction system has been working satisfactorily since september 1983. The shape of the internal side surface of the two deflectors and their positions are optimum for the trajectory of 30MeV proton beam. When other energy or kinds of particle beam is extracted, the positions of the whole system need not change. It is enough that the steering magnet provides varied extracted beam with the steering of less than 1 deg. The extraction voltage V1 and V2 of two deflectors are set around the calculating value.

Simple Harmonic Oscillation Equation

Suppose the whole deflector were subdivided into many small elements (along azimuthal direction), as a first order approximation, we could consider each of the element as a two-electrode system with circular arcs of the same center, so that the equilibrium orbit of the extracted beam is a circular arc with same center as one of the two-electrode system, that is the central ray of the two-electrode system. Then the following equation is obtained for the particle of mass m, charge e, velocity v_o and curvature radius ρ_0 to move on the equilibrium orbit.

$$\frac{e}{c}v_o H_o - eE_o = m\frac{v_o^2}{\rho_o} \qquad (1)$$

where H_0-the magnetic strength (G); E_0-the electric stength (v/cm); C-the light velocity.

For the particle of deviation X from equilibrium orbit

$$\frac{e}{c}vH - eE = m\frac{v^2}{\rho} - m\frac{d^2x}{dt^2} \qquad (2)$$

As a first order approximation, where $\rho=\rho_0$ +X and

$$H = H_o(1 - M_o\frac{x}{\rho_o}) \qquad (3)$$

$$E = E_o(1 - N_o\frac{x}{\rho_o}) \qquad (4)$$

$$M_o = -\frac{\rho_o}{H_o}\left(\frac{\partial H}{\partial R}\right)_o \cos\alpha_o \qquad (5)$$

$$N_o = -\frac{\rho_o}{E_o}\left(\frac{\partial E}{\partial R}\right)_o \cos\alpha_o \qquad (6)$$

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$$\alpha_0 = \arctan\left[\frac{1}{\rho_0}\left(\frac{dR}{d\theta}\right)_0\right] \quad (7)$$

where we use cylindrical coordinated (θ,R,Z) of which the coordinate origin is chosen at magnetic centre of cyclotron.

For convenience, we assume that the electric potential on the equilibrium orbit is zero, then the increase of the kinetic energy for a particle in electric potential field is equal to the decrease of the potential energy, that is

$$\frac{m}{2}v^2 - e x E_0 = \frac{m}{2} m v_0^2 \quad (8)$$

By using the above equations and the binomial theorm and converting the time independent variable to azimuthal variation, as a first order approximation, the equation of the X-direction motion can be written in the following form

$$\frac{d^2x}{d\theta^2} + [(F_0 - M_0)F_0 - (3-N_0)(F_0-1)]x = 0 \quad (9)$$

which represents a simple harmonic motion about the equilibrium orbit, where $F_0 = H_0/(H_0 - E_0/300\beta_0)$, $\beta_0 = v_0/c$.

In the Z-direction, perpendicular to the plane of the equilibrium orbit, the motion depends on the equation

$$-eE_z + \frac{e}{c}H_R v_0 = m\frac{d^2z}{dt^2} \quad (10)$$

where

$$E_z = E_0(1-N_0)\frac{z}{\rho_0} \quad (11)$$

$$H_R = -M_0\frac{H_0 z}{R} \quad (12)$$

By the same way as one in the X-direction and in first order approximation, the simple harmonic oscillation equation about the median plane can be written in the form

$$\frac{d^2z}{d\theta^2} + [M_0F_0 + (1-N_0)(F_0-1)]z = 0 \quad (13)$$

the equation (9) and (13) indicate clearly that the focusing force acting on the particles in the deflectors consists of electric and magnetic contribution and which depends on the ratio of each contribution to the centripetel force acting on the particle to move along the equilibrium orbit.

Transmission Matrix of Extraction System

In Cartesion co-ordinates with X- and Y-axes orthogonal to the motion direction l of the particle to move on the equilibrium orbit (that is l is the optical axis of the extraction system), assume whole extraction system can be subdivided into several small elements, the field in each element are almost independent of the distance along the equilibrium orbit, so that the first order beam-optics theory could be used for calculating transmission characteristic of the extraction system of INR cyclotron.

Equilibrium Orbit. The equilibrium orbit of the extracted beam is the solution of the following motion equation with two certain boundary conditions (at the entrance and the exit of extraction system) by numerical integration

$$\frac{d^2R}{d\theta^2} - \frac{2}{R}\frac{dR}{d\theta} - R = -\left[R^2 + \left(\frac{dR}{d\theta}\right)^2\right]^{3/2}\left[H(R,\theta) - \frac{E_0(\theta)}{300\beta}\right]/RG \quad (14)$$

where G is the magnetic rigidity of particle, $E_0(\theta)=0$ outside the deflectors. The entrance boundary condition depends on the central ray of precessional extraction beam as "e" in the n=31 grid shown in Fig.2. The exit condition depends on the request of the beam-transport system for cyclotron.

Transmission Matrix. The transfer matrix in fringing magnetic field is known. Note that as the matricex M_{xi} and M_{yi} for the "i" are represented by the following formulas

$$[m_{xi}] = \begin{bmatrix} \cos k_{xi}l_i & \sin k_{xi}l_i & \frac{1-\cos k_{xi}l_i}{\rho_i k_{xi}^2} \\ -k\sin k_{xi}l_i & \cos k_{xi}l_i & \frac{\sin k_{xi}l_i}{\rho_i k_{xi}} \\ 0 & 0 & 1 \end{bmatrix} \quad (15)$$

$$[m_{yi}] = \begin{bmatrix} \cos k_{yi}l_i & \sin k_{yi}l_i/k_{yi} \\ -k_{yi}\sin k_{yi}l_i & \cos k_{yi}l_i \end{bmatrix} \quad (16)$$

The elements K_{xi}, K_{yi} and l_i are as follow

$$k_{xi} = \sqrt{1-M_{oi}}/\rho_{oi} \quad (17)$$

$$k_{yi} = \sqrt{M_{oi}}/\rho_{oi} \quad (18)$$

$$l_i = \sqrt{\left(\frac{dR}{d\theta}\right)_{oi}^2 + R_{oi}^2} \times \Delta\theta_i \quad (19)$$

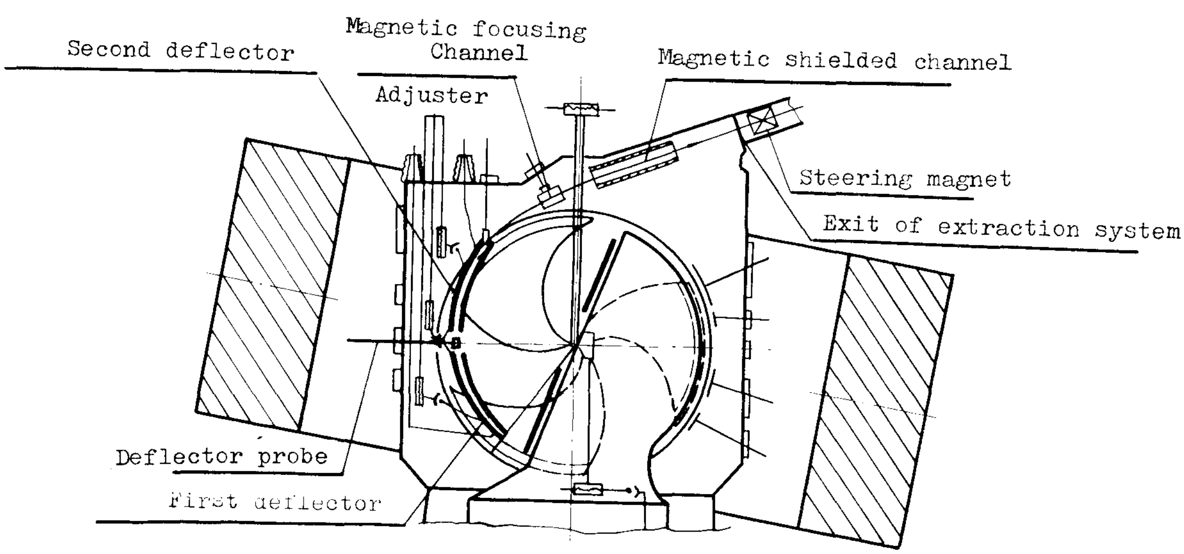

Fig.1. Layout of the extraction system

where $\Delta\theta_i$ is the step of integration numerically. $(\frac{dR}{d\theta})_{oi}$ and R_{oi} are the value of extracted beam in the equilibrium orbit for the element "i".

The transfer matrix in the magnetic focusing channel is the same as one of a quadrupole magnetic lens.

The transfer matrix in the drift space is used for calculation of the magnetic shielded channel because the magnetic field is almost zero.

Inside the first deflector, the transfer matrix is the same as one in fringing magnetic field and $N_{oi}=1$. But in the second deflector, the transfer matrix is the same as one of a quadrupole lens with the X-direction focusing, because the hyperboloid electrode will provide a very high electric field gradient and be similar to a electric quadrupole lens ($N_{oi}=\frac{2aR_o}{d(\theta)_i}$, where a is the inhomogeneous electric field factor of the hyperboloid electrode, a=4 in our design; $d(\theta)i$ is the separation between the septums and the deflector electrodes). There are following formulas in two sets of deflectors.

$$k_{xi}^2 = [(F_{oi}-M_{oi})F_{oi}-(3-N_{oi})(F_{oi}-1)]/\rho_{oi}^2 \qquad (20)$$

$$k_{yi}^2 = [M_{oi}F_{oi}+(1-N_{oi})(F_{oi}-1)]/\rho_{oi}^2 \qquad (21)$$

Numerical calculation of the emittance

In fact, the acceptance of the extration system is just the total acceptance of two sets of deflector. In order to calculate the total acceptance in phase space, it is necessary to limit the maximum of extracted beam envelope in the deflectors on the following condition

$$x_{i\,max},\ y_{i\,max} \leq d(\theta)_i/2 \qquad (22)$$

The full curves in Fig.3 shows an example of calculating results about the total acceptance at the entrance of the first deflector.

It is obvious that a beam of particles that occupies less emittance in phase space than the total acceptance of deflector could be extrated both in X- and Y-direction.

In order to determine the initial emittance of extracted beam, a grid of particles of 5 mm×5mrad/π in the (R,P_R) plane, in the (Z,P_Z) plane lying on a circle of π×1mm×1mrad (both being acceptable value for our cyclotron) have been accelerated (31 revolutions) by numerical integration motion equation with a first harmonic field of 1.2G amplitude (cf. Fig.2 and Fig. 4). The radial starting conditions are to locate the central ray "e" at equilibrium orbit 8 revolutions before the $\nu_R=1$ resonance. In the (Z,P_Z) plane, the particles with initial value around Z_0=1mm, P_{Z0}=0 and $Z_0=1/2\sqrt{2}$mm, $P_{Z0}=1/2\sqrt{2}$mrad, and the opposite ones. The traversion of the $\nu_R=1$ and $\nu_R=2\,\nu_Z$ resonance has been considered.

Because both P_x- and X-amplitude increase with the decrease of values of near the entrance of the first deflector (Fig.2), that is to say an extracted beam always gets divergence, it would be reasonance to take the left upper ellipse shown in Fig.3 as the initial emittance ellipse.

In the (Z,P_Z) plane, by transformation nine ellipses shown in Fig.4 into the same phase space and drawing up their common envelope ellipse as shown in Fig.5, it could be realized that extracted beam gets convergence near the entrance of the first deflector, so we conclude that right upper ellipse shown in Fig.3 is the initial emittance ellipse.

According to the formulas

$$[\sigma_K]_{x,y} = ([m_n][m_{n-1}]\cdots[m_2][m_1])_{x,y}\cdot[\sigma_0]_{x,y}\cdot([m_n][m_{n-1}]\cdots[m_2][m_1])^T_{x,y}$$

where n is the number of elements included in the whole extraction system, the emittance ellipse of the extracted beam at the exit of the extraction system can be obtained from the lower ellipse shown in Fig.3.

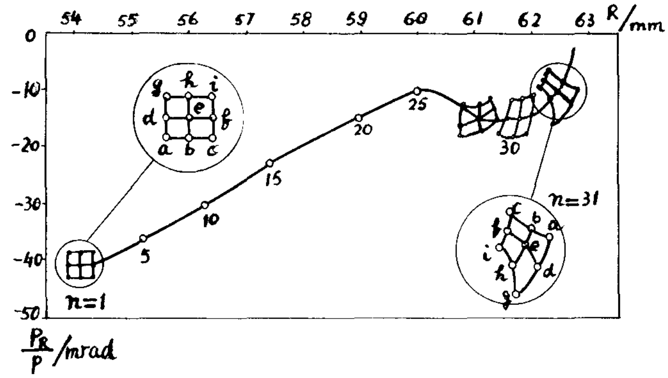

Fig.2. The motion of a grid of particles in radial phase space at an azimuthal position of the entrance of the first deflector, the 31st turn will be extacted.

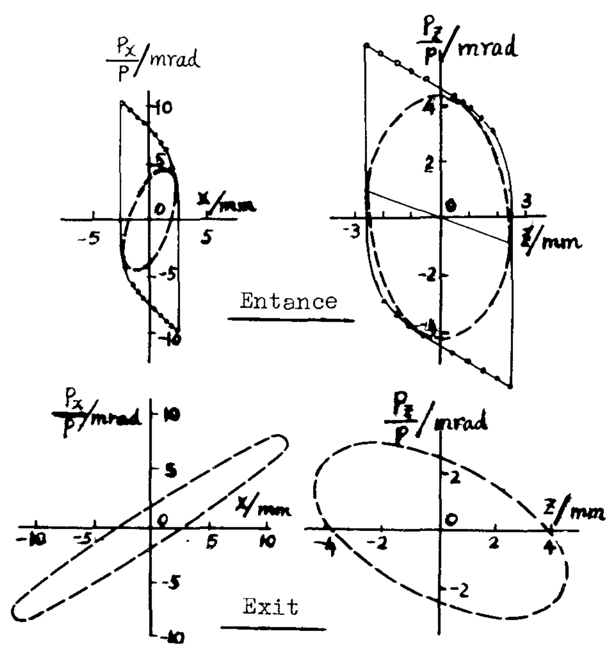

Fig.3. The acceptance of the extraction system at the entrance of the first deflector and emittance ellipse of the extracted beam

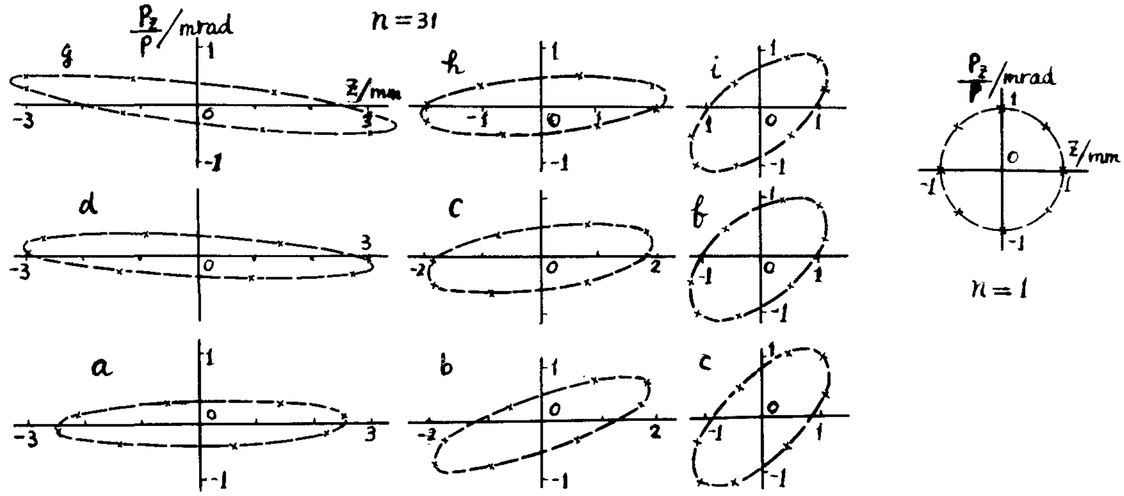

Fig.4. In axial phase space nine emittance ellipse with regard to the n=31 grid of particles in Fig.2. The start condition are a circle of $\pi \times 1$mm$\times 1$mrad.

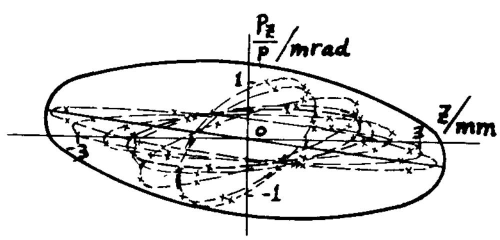

Fig.5. The transformation nine emittance into the same phase space.