IMPEDANCE REDUCTION OF MA-LOADED RF CAVITY DUE TO THE PRESENCE OF FRINGING FIELD

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Abstract
Based on the studies of compact medical Fixed-Field Alternating-Gradient (FFAG) accelerators, the elements within the accelerator exhibit strong magnetic interference, which is due to its tight lattice structure. One of the notable problems for a compact-size machine is the reduction of the rf cavity’s shunt impedance due to the presence of a fringing field. Although the excitation properties of the rf core under a static bias field can be measured, the magnetic field distribution within the rf core may not be uniform in realistic settings. The field distribution inside rf core must be considered when designing and estimating the performance of the rf cavities exposed to non-uniform magnetic field. In this report, a simple numerical model has been formulated to take into account the fringing effect and its results are compared to actual measurements.

INTRODUCTION
There has been a growing demand for compact accelerators, especially in medical and industrial applications. In the medical field, the heavy-ion accelerators have shown their effectiveness for curing localized cancer by accurately delivering the appropriate doses of ion beam to the cancer cells and eradicating them with little damage to adjacent normal cells. The design and development of compact and low-cost accelerator can possibly promote the widespread adoption of heavy-ion radiotherapy.

Referring to the recent design studies [1,2] of compact medical accelerators based on FFAG principle, compact designs immediately lead to a limited drift space for placing rf cavities. Thus, rf cavities with compactness and ease of use are essential for compact machines.

To achieve compactness and ease of use, magnetic alloy (MA), which is a soft magnetic material composed of nano-scale crystalline, has been recently realized for rf cavities. One example of MA materials is FINEMET which possesses a high saturation magnetic flux density and excellent soft magnetic properties such as high permeability and low quality factor. Such MA-loaded cavities assure a high gradient accelerating field and a broad-band feature that requires no tuner. However, because of its high permeability, MA core is susceptible to the fringing field, whose effects cannot be ignored especially for a compact machine.

In this report, the magnetic susceptibility of the FINEMET core (or FINEMET-loaded cavities) due to the presence of a fringing field is estimated by a simple model and compared to actual measurements.

IMPEDANCE PROPERTIES OF MA CORE AND BIAS-FIELD DEPENDENCE

Permeability of MA Core
By measuring the complex impedance of the FINEMET core (its B-H curve is given in Fig. 1), we deduced the complex permeability of FINEMET, \( \mu = \mu' - j\mu'' \). Fig. 2 shows a nice logarithmic dependence of its permeability as a function of frequency. Here, the real and imaginary parts of permeability correspond to the core inductance and core loss respectively, as expressed by

\[
Z_{\text{core}} = j\omega L_0 = j\omega \mu' L_0 + \mu'' \omega L_0 \equiv R + jX, \quad (1)
\]

where \( L_0 \) can be written in terms of the core’s dimension of thickness \( \tau \), inner radius \( r_1 \) and outer radius \( r_2 \) by

\[
L_0 = \frac{\mu'' \tau}{2\pi} \log \frac{r_2}{r_1}. \quad (2)
\]

Fig. 1. B-H curve of FINEMET.
Saturation flux density of FINEMET is about 1.23 T.

Fig. 2. Deduced Complex permeability of FINEMET.

Bias-Field Dependence of Core Impedance

In order to account for the fringing field, we measured the magnetic susceptibility of FINEMET by introducing a bias field. Fig. 3 illustrates the complex permeability of FINEMET as a function of bias field \( H \).
INFLUENCE OF FRINGING FIELD

General Consideration

In general, fixed-field machines such as FFAGs are likely to have a wide horizontal aperture in general. When a rf cavity is placed in a short straight section bounded by the main magnets, the magnetic flux tends to flow into the MA core from sides (A) and passes through a narrow cross-sectional area (B), as shown in Fig. 4. Since the total magnetic fluxes on areas A and B are conserved, the magnetic flux density is expected to be much higher in B than A due to the difference in tangential cross-sectional area. Indeed, our previous report has indicated that the magnetic field inside the core nearby the main magnets is non-uniform, and flux saturation was observed around area B [1].

The results of the TOSCA calculation show that a major magnetic field component inside the core is pointing in the direction parallel to the circumference of the core. We also note that the magnetic field components perpendicular to the circumference of the core rarely play a role because of the strong influence of diamagnetism. Therefore, only the bias-field dependence of Fig. 3, in which the direction of magnetic flux inside the core is pointing along the core’s circumference, is enough to be considered for the estimation of cavity performance.

Numerical Approach

To take into account the non-uniform field distribution inside the core (average radius of $r_\text{avg}$), we divide the toroidal core into $m$ small rings and assume each small ring consists of $n$ segments divided in angular direction, as indicated in Fig. 5. Letting a local permeability of one angular segment $i$ at radius $r_j$ to be $\mu_i(r_j)$, the total magnetic flux density $B$ can be defined as

$$B = \Phi = \frac{NI}{\sum_{j} \mu_i(r_j)}.$$

The magnetic flux density inside a toroidal core of permeability $\mu$ and circumference $r_\text{avg}$ is given by

$$B = \mu NI \frac{1}{r_\text{avg}}.$$

By comparing Eqs. 3 and 4, we deduce an effective permeability $\mu_e$ in the following form.

$$\mu_e = \frac{r_\text{avg}}{m} \sum_{j} \left( \sum_{i} \mu_i(r_j) \right)^{-1}.$$

Here, each $\mu_i$ can be determined using values in Figs. 1 and 3 and from the magnetic field data obtained from the TOSCA calculation.

Experimental Settings and TOSCA Calculation

To replicate the influence of the fringing field, we prepared a small experimental setting, in which a steering magnet as the main magnet and a small toroidal FINEMET core (outer radius: 141 mm, inner radius: 89 mm, thickness: 10 mm) were placed in 10 mm apart (see Fig. 6). A dialogue box in Fig. 6 shows a typical magnetic field distribution inside the core (calculation by TOSCA).
field distribution inside the core as calculated by TOSCA. Here, the non-uniform field distribution is expected.

Fig. 7a (excitation current of 0.5 A) shows both the radial and the angular variations of the magnetic field inside the core. The maximum and minimum values of magnetic field at a radius of 45 mm are plotted as a function of the excitation current in Fig. 7b. When the excitation current of a magnet is above 1 A, the maximum field inside the core is expected to exceed half the saturation flux density.

Based on the magnetic field values obtained by TOSCA, the corresponding magnetic field strength $H$ can be determined by the $B$-$H$ curve in Fig. 1. Since the $H$-dependence of the core’s permeability is known from Fig. 3, an effective permeability under the influence of fringing field can now be estimated from Eq. 5.

Fig. 7. Magnetic field distribution inside the core (TOSCA calculation). (a) Angular distribution at $I = 0.5$ A. (b) Maximum and minimum magnetic field at the core’s radius of 45 mm.

Measurements and Analysis

We have measured the core’s impedance by a network analyser and compared it with the calculations specified above (see Fig. 8). In our calculations, the effective permeability is computed from TOSCA magnetic data together with Figs. 1 and 3, and then is converted it into the complex impedance of Eq. 1. These calculation results are indicated by square marks (or “cal(1)”) in the figure. Our measured results are lower than expected. If we only consider the maximum values of magnetic field at each radius as in Fig. 7b and ignore the angular dependence to find the corresponding permeability from Figs. 1 and 3, an expected impedance (triangles or “cal(2)” in the figure) seems to coincide fairly well with measurements for excitation current below 1.0 A (or magnetic field of less than half the saturation flux density).

For magnetic fields larger than 0.6 T, both computational methods overestimate the actual impedance. We believe the inaccuracy of the TOSCA data as not the source of these discrepancies, since the calculated magnetic field at the pole gap of a steering magnet is consistent with the measured values. They are most likely caused by the error in estimating the bias-dependent permeability using a simple relation of air-core inductance given in Eq. 2, because the magnetic flux density may reach its saturation flux density around inner radius for large $H$.

Fig. 8. Calculated and measured impedance of FINEMET core under the influence of the fringing field. The excitation current of steering magnet is (a) 0.5 A, (b) 1.0 A, and (c) 1.3 A, respectively. Measured values in circles are compared with calculations described in the text.

SUMMARY

We have investigated the performance of MA-loaded cavities under the influence of a fringing field by measuring the impedance properties of MA (FINEMET in this report) core placed nearby a steering magnet.

Our measurements indicated that the reduction of core permeability can be well predicted by simply considering the maximum magnetic field inside the core, when the magnetic field inside is lower than half the saturation flux density. They have also shown that, for internal magnetic flux density larger than half the saturation flux density, the reduction of core’s permeability is expected to be severe than the estimation based on the bias-field measurement of Fig. 3.

REFERENCES