COST COMPARISON OF LINEAR AND CIRCULAR ACCELERATORS*

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Abstract

A simple comparison of construction cost between a Linear and a Circular Accelerator is made. Two simplified models are proposed and studied. The comparison is made with the two major magnet and RF cavity components. An approximated criterion is found according to which the Circular Accelerator is indeed the more economical of the two provided that the beam circulates a minimum number of turns.

INTRODUCTION

A Proton Driver of high intensity and sufficiently large energy is sought for a variety of applications, one being the Neutrino Factory. A debate is presently going on about the best accelerator architecture. Apart from the repetition rate and other beam performance requirements, a main issue is certainly the cost. A Linear Accelerator is thought to provide the most efficient method but at large cost; whereas a Circular Accelerator could be less expensive but with some beam performance limitations. In particular an intermediate approach is the re-circulation of the beam in one or more sections of Linear Accelerator joined together by magnet arcs. This could be also the case of a FFAG accelerator. The question is then how many turns the beam is required to re-circulate in order for the approach to be more economically effective.

TWO ACCELERATOR MODELS

Consider two Accelerator Scenarios shown in the Figure 1 below. Scenario A is a Linear Accelerator (**LA**), for instance a Super Conducting Linac (SCL) of length L_A made essentially of RF cavities with fewer magnets for focusing. The **LA** accelerates between E_1 and E_A with an energy increment $\Delta E_A = E_A - E_1$ assumed uniform along the length with a gradient G expressed in MeV/m. The cost is also taken to be constant along the length at the rate of C_{RF} dollars per unit length. Essentially the cost is that of the RF cavity system that can be split in two contributions: that of the inertial components and that proportional to the beam power gain P per unit length.

Scenario B is a Circular Accelerator (**CA**), for instance a Rapid-Cycling Synchrotron (RCS), a Cyclotron, or a Fixed-Field Alternating-Gradient (FFAG) accelerator. The circumference is L_B made of two components: an RF system that accelerates at the rate U, expressed in MeV/revolution, over a shorter length, and a magnetic system with uniform cost along the circumference of C_M dollars per unit length. The **CA** accelerates from the same initial energy E_1 but to a different final value E_B with an energy increment $\Delta E_B = E_B - E_1$. It is thus assumed that the two accelerator scenarios share the same injector at the energy E_1, and that the cost of this is common to both. Moreover the accelerating RF structure in the **CA** is taken to be equal to a small section of the **LA**, though in reality the two structures may differ (more for the RCS than for the FFAG) essentially because the two accelerators would employ different frequencies, and the beam power gain per unit length may also be different.

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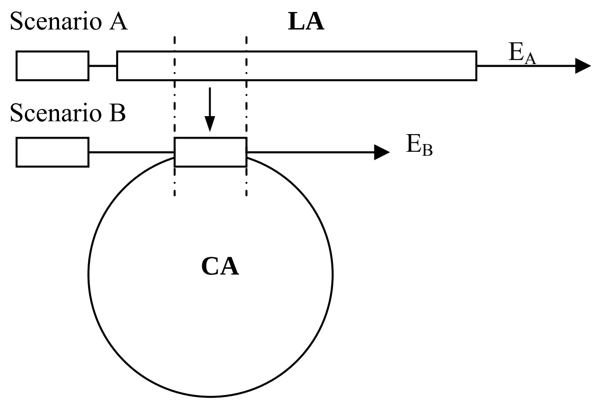

Figure 1. Linear (**LA**) and Circular (**CA**) Accelerator Scenarios for Cost Comparison

COST ESTIMATE

Let C_A and C_B be the cost of the two scenarios accelerators, respectively A and B, and $r = C_B / C_A$ the ratio of the total cost of the later to that of the former scenario. For scenario A we have

$$C_A = L_A C_{RF} = L_A (k_1 + k_2 P_A)$$

Here P_A is the increment of beam power in the **LA** per unit length, k_1 is the cost of the RF structure per unit length, and k_2 the cost of the RF power. Denoting the total beam power gain with $P_{tot} = L_A P_A$

$$C_A = k_1 L_A + k_2 P_{tot}$$

Let S be the length of the RF system in the CA, then the cost of the CA is

$$C_B = L_B C_M + S(k_1 + k_2 P_B)$$

where SP_B is the power gain per revolution in the **CA** that may differ from that in the **LA**. Let n be the average number of revolutions needed to complete the acceleration from E_1 to E_B. Obviously

$$n = P_{tot} / SP_B = \Delta E_B / U$$

and

$$S = UL_A / \Delta E_A$$

so that

$$C_B = L_B C_M + (k_1 \eta L_A + k_2 P_{tot}) / n$$

where $\eta = \Delta E_B / \Delta E_A$, having assumed the same total power P_{tot} in the two scenarios. Conversely

$$C_B = L_B C_M + [C_A + k_1 (\eta - 1) L_A] / n$$

and the cost ratio

$$r = C_B / C_A = r_0 + \alpha / n$$

with

$$r_0 = (L_B / L_A)(C_M / C_{RF})$$

that is the product of the ratio of the two accelerator lengths with the ratio of the cost per unit length of the magnet system to that of the RF in the LA including beam power, and

$$\alpha = 1 + (\eta - 1)\ k_1 / C_{RF}$$

The situation is illustrated in Figure 2 that plots the cost ratio versus the number of revolutions in the CA. There are two solutions: (1) $r_0 > 1$ in which case the less expensive scenario is A, and (2) $r_0 < 1$ in which case there are two possibilities: if the number of revolution $n < n_0 = \alpha / (1 - r_0)$ again scenario A is the less expensive, but if the number of revolution $n > n_0$ than the **CA** is less expensive.

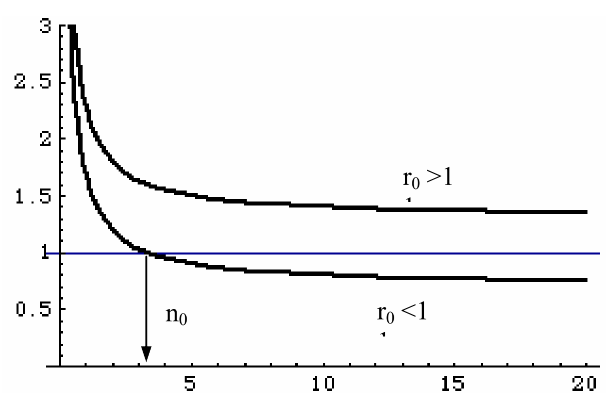

Figure 2. Cost ratio r vs. number of revolutions in the CA

We plotted curves for equal cost r = 1 in Figure 3. The length ratio $\lambda = L_B / L_A$ is on the ordinate axis and the cost ratio $\rho = C_M / C_{RF}$ on the abscissa. The curves correspond to different threshold values n_0. A chosen **CA** is described by a point on the diagram with coordinates (ρ, λ) that lies on a curve corresponding to a threshold value n_0 for which the two scenarios A and B have about the same cost. If the actual number of revolution n is larger than n_0 than the **CA** is most cost advantageous (left bottom corner), otherwise if $n < n_0$ than the **LA** is more cost effective (right top corner). As examples we have shown the corresponding locations for CEBAF and the FFAG ring for acceleration of muons from 10 to 20 GeV.

Finally in Table 1 we give a provocative and subjective comparison of the performance of 3 different types of accelerators.

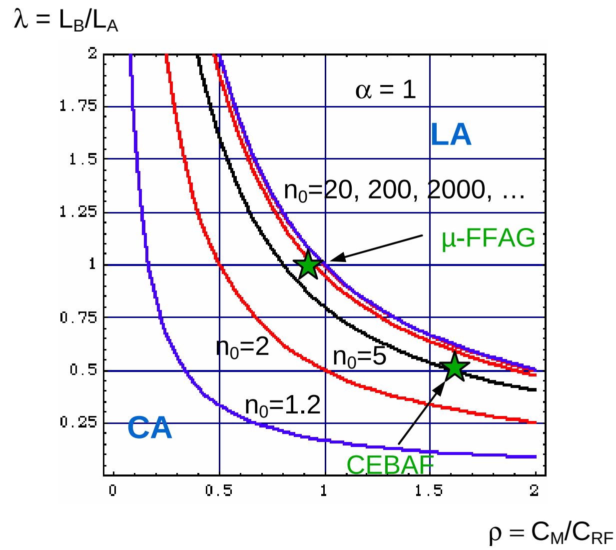

Figure 3. Plot of length ratio $\lambda = L_B/L_A$ versus cost ratio $\rho = C_M/C_{RF}$ for different threshold value of number of revolutions n_0 for which the two scenarios A and B have about the same cost.

Table 1. Subjective Comparison of 3 Accelerators.

	Linac	RCS	FFAG
Magnets	none	ramped	constant
Rep. Rate	> 100 Hz	< 100 Hz	1 kHz
CW-mode	yes	no	maybe
RF Power	large	modest	medium
RF freq.	GHz	MHz	M-GHz
Space-Charge	none	yes	(yes)
Cost	expensive	large	modest
Max Energy	5 GeV	50 GeV	10 GeV
Beam Intensity	low	high	medium
Accel . Period	fast	long	short
Efficiency	40%	<10%	20-30%

Red = bad, Green = good, Black = neutral

CONCLUSION

Provided that the cost per unit length of the magnet system does not exceed that of the RF system, a Circular Accelerator is more economical to construct than a Linear Accelerator when the number of revolutions for the entire acceleration cycle is larger than a threshold number.