SUBPICOSECOND ELECTRON BEAM DIAGNOSTICS BY
COHERENT TRANSITION RADIATION INTERFEROMETER

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Abstract
Subpicosecond and picosecond electron single pulses have been measured by the coherent transition radiation interferometer at the S-band twin linac at Nuclear Engineering Research Laboratory, University of Tokyo. The results were compared with those obtained by the femtosecond streak camera. From the comparison, the reliability of the method to measure subpicosecond electron pulses that would be close to the time resolution of the femtosecond streak camera (200 fs at FWHM) has been discussed.

1 INTRODUCTION
We aim to produce and measure the femtosecond pulse which pulse length is shorter than a time resolution of the femtosecond streak camera (200 fs at FWHM) in near future. Now in the Nuclear Engineering Research Laboratory of University of Tokyo, the shortest bunch that can be generated is close to the time resolution. There are two promising methods to evaluate pulse shape of femtosecond electron bunch. The first one is to measure Cherenkov radiation or optical transition radiation emitted by the electron bunch by the femtosecond streak camera. The second one is the coherent far-infrared transition radiation interferometry [1,2,3]. It is important to compare the results by the two methods in order to confirm the precision of both methods[4,5,6].

In this paper, we start from the construction of the Michelson coherent transition radiation interferometer and measure subpicosecond and picosecond electron pulses which are longer than the time resolution of the streak camera. Furthermore, the results are compared with each other and the reliability and improvement of the method is discussed.

2 MICHELSON INTERFEROMETER
Radiations from a relativistic electron bunch such as synchrotron radiation, transition radiation, Cherenkov radiation etc. have broad spectrum. In case that the wavelength of the radiation is shorter than the electron bunch length, the phase of radiation emitted by electrons is different from one another so that the radiation is incoherent. On the other hand, in case that the wavelength is longer than the bunch length, the phase becomes almost the same so that the radiation is coherent. This is called the temporal coherence of radiation. The coherent radiation shows the interferogram when we use an interferometer such as the Michelson interferometer. The information of the electron bunch can be deduced from the interferogram. Another important feature of coherent radiation is the dependence of the power on the number of electrons in the bunch. The following theory shows that the power of the incoherent radiation is linear to the number while that of the coherent radiation is linear to the square. From the interferogram of the light intensity of interfered two coherent radiation pulses, the longitudinal bunch distribution are given in the following procedure.

When the cross section of the beam is small and the observation point is far from the source point, the intensity of the transition radiation is expressed by the analogy of the intensity of coherent synchrotron radiation as,

\[ I_{\text{trans}}(v) = N \left[ 1 + (N - 1) f(v) \right] I_e(v), \]

where \( N \) is the number of electrons in the bunch, \( V \) is the wave number which is the inverse of the wavelength of the transition radiation and \( I_e(v) \) is the transition radiation intensity emitted from a single electron. The first term of Eq.(1) expresses the incoherent transition radiation and the second term the coherent transition radiation. The quantity \( f(V) \) is the bunch form factor which is given by the Fourier transform of the distribution function, \( S(\vec{x}) \), of the electron in the bunch,

\[ f(v) = \left| \int S(\vec{x}) \exp \left[ 2\pi (\vec{n} \cdot \vec{x}) v \right] d\vec{x} \right|^2, \]

where \( \vec{n} \) is the unit vector directed from the center of the bunch to the observation point and \( \vec{x} \) is the position vector of the electron relative to the bunch center. Since \( N \gg 1 \), we approximately have,

\[ I_{\text{trans}} \approx N^2 f(v) I_e(v), \]
The form factor $f(v)$ can be divided into two parts, the longitudinal bunch form factor $f_L(v)$ and the transverse bunch form factor $f_T(v)$ as follows,

$$f_L(v) = \int h(z) \exp(i2\pi z \cos \theta/v) dz,$$

(4)

$$f_T(v) = 2\pi \int g(\rho) I_f(2\pi \rho \sin \theta \nu) d\rho,$$

(5)

where $h(z)$ and $g(\rho)$ are the longitudinal ($z$) and transverse ($\rho$) distribution function of the electron bunch, respectively. The transverse bunch form factor is obtained by measuring the transverse distribution of the electron bunch. When we observe the transition radiation from the on-axis or nearly on-axis direction, i.e., $\theta^2 \gg 1$, $\cos \theta$ and $\sin \theta$ can be unity and zero, respectively.

From the experiment, the interferogram of the light intensity of the two interfered coherent transition radiation pulses as a function of the moving mirror position of the interferometer are obtained. By definition the interferogram can be written,

$$S(\delta) = \frac{4\pi}{RT} \left| \widetilde{E}(\omega) \right|^2 e^{-i2\pi \delta c/v} d\nu,$$

(6)

where $S(\delta)$ is the intensity of the recombined radiation intensity at the detector which expressed in the time domain with an additional time delay $\delta/c$ for the movable mirror minus the intensity at $\delta \to \pm \infty$, $\widetilde{E}(\omega)$ is the Fourier transform of the electrical field of the transition radiation and $R,T$ are the coefficients of reflection and transmission at the beam splitter, respectively.

Solving for $\left| \widetilde{E}(\omega) \right|^2$ yields,

$$\left| \widetilde{E}(\nu) \right|^2 = \frac{1}{4\pi |RT|^2} \int_{-\infty}^{\infty} S(\delta) e^{-i2\pi \delta c/v} d\delta.$$

(7)

Using Eq.(3) and the relation $I_{\text{tot}}(\nu) = |\widetilde{E}(2\pi \nu)|^2$, the bunch form factor can be obtained by,

$$f(\nu) = \frac{1}{4\pi |RT|^2 N^2 I_f(v)} \int_{-\infty}^{\infty} S(\delta) e^{-i2\pi \delta /v} d\delta,$$

(8)

Finally the Kramers-Kronig relation and inverse Fourier transform gives the longitudinal bunch distribution $h(z)$ from the longitudinal bunch form factor as follows[13],

$$h(z) = \int g(v) \exp \left[ i(\phi_g(v) - 2\pi v) \right] dv,$$

(9)

$$\phi_g(v) = -2\nu \int_{0}^{+\infty} \ln \left[ \frac{g(v')}{g(v)} \right] dv'.$$

3 EXPERIMENT

3.1 Experimental setup

We performed this comparison at the 35L linac where the achromatic-arc-type magnetic pulse compressor was installed. In the experiment the longitudinal bunch distribution was controlled by tuning the energy modulation of the bunch in the accelerating tube for the magnetic pulse compression. We chose femtosecond and picosecond (FWHM) pulse widths and performed the comparison between the femtosecond streak camera and the Michelson coherent transition radiation interferometry measurement as shown in Fig. 1. We measured the transition radiation in the far-infrared region emitted by an electron bunch at the Al-foil put in air after the 50 µm thick Ti window at the end of the 35L linac. We used liquid-He-cooled Si bolometers as a detector for the far-infrared radiation. The major beam parameters are as follows: the energy was 32 MeV, the pulse length is 800 fs to 1.7 ps (FWHM) and the electron charge per single pulse is 30 to 250 pC.

![Fig. 1 Experimental setup](image)

3.2 Procedure of analysis

On the bases of the procedure of analysis as be seen in chapter 2, we have analyzed these pulses from the interferograms which we have got by the Michelson interferometer. Because of nonuniform transparency of the 100 µm-thick Mylar beam splitter and diffraction loss of long wavelength components, the bunch form factor was obtained within rather limited range. Therefore we had to use theoretical bunch form factor assuming the Gaussian or exponential distribution out of the range.
4 RESULTS AND DISCUSSION

The interferogram of the subpicosecond electron pulse is shown in Fig.2.

Fig. 2 Interferogram of the subpicosecond electron pulse.

The experimental result of the bunch form factor is shown by the solid curve and that of theoretical by dashed curve in Fig. 3. In the figure, we chose the Gaussian distribution as the theoretical curve, since the exponential function has unphysical long tails in both sizes and the simultaneous observation of the bunch shapes by the streak camera indicates that the Gaussian is closer to the real bunch distribution.

Fig. 3 Bunch form factor

The dashed curves in Fig. 3 represent those of three bunch length (400, 500 and 600 fs at FWHM). We used the measured bunch form factor from 9.5 to 18 cm\(^{-1}\) range and the theoretical bunch form factor out of the range for the analysis. In this case, we adopt the bunch form factor of 500 fs bunch length and extrapolate this to the range under 9.5 cm\(^{-1}\) and over 18.0 cm\(^{-1}\).

Finally, we got the bunch distribution derived by the interferometry as shown by the solid curve in Fig. 4. The dashed curve in the figure is one of the pulse shape taken by the streak camera. The result by the interferometry gives 550 fs bunch length at FWHM while that by the streak camera becomes 650 fs. The calibration of the camera was also performed by using a Ti:Sapphire laser. Then the error at FWHM was found out to be 370 fs assuming the law of error propagation. After the above error is subtracted, the net pulse length becomes 550 fs. These results agree with each other and it is therefore clear that the reliability of the method to measure subpicosecond pulse has been confirmed.

With the choice of thinner beam splitter which determines the appropriate spectrum window of the interferometer and the improvement in the optical alignment, we expect the method is promising even for the shorter bunch (100 - 200 fs FWHM) with better resolution. This is also because the spectrum shifts from the far-infrared region to the infrared region where the sensitivity of the Si-bolometer becomes better.

Fig. 4 Bunch distribution by the interferometry (solid curve) and that by the streak camera (dashed curve)

5 CONCLUSION

From the comparison of the diagnostics by the interferometry with that by the streak camera, the reliability of the interferometric method to measure subpicosecond electron pulses that are close to the time resolution of the femtosecond streak camera was confirmed. The design for 100 - 200 fs has started.

5 REFERENCES