A HAMILTONIAN FOR WAVE LENGTH SHIFTER AND ITS STUDIES ON INDUS-1

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Abstract

The INDUS-1 is a 450 MeV synchrotron radiation source for the production of VUV radiation. In order to produce the radiation of shorter wavelengths (λ_c = 31 A°), a superconducting wavelength shifter (WLS) with peak field of 3T is being considered for Indus-1. In this paper, L. Smith's Hamiltonian for Halbach's magnetic field model has been re-derived to estimate focussing component under the compensated electron beam trajectory transformation. Various linear compensation schemes are presented to minimize the linear effects of WLS and its effects on machine operation are also theoretically studied.

INTRODUCTION

It is well known that an insertion device perturbs [2,3] the motion of an electron in storage rings. It produces the linear and non-linear beam dynamics effects. In the case of the WLS nonlinear contribution would be small but due to its linear effect, linear optics will be distorted and it will disturb the nonlinear optics of the ring. So it is necessary to predict precisely quadrupole component for WLS. For this purpose, Hamiltonian for WLS has been rederived for compensated electron bump trajectory transformation and its effects on Indus-1 machine operation are studied theoretically.

A HAMILTOINAN FOR WAVELENGTH SHIFTER

Magnetic Field

The components of the transverse magnetic field in a WLS with finite pole width may be obtained from Halbach's[1] expression.

$$B_x = \frac{k_{mx}}{k_{my}} B_m \sinh k_{mx}x \sinh k_{my}y \sin k_m(z-c_m)$$

$$B_y = B_m \cosh k_{mx}x \cosh k_{my}y \sin k_m(z-c_m)$$

$$B_z = -\frac{k_m}{k_{my}} B_m \cosh k_{mx}x \sinh k_{my}y \cos k_m(z-c_m)$$

$$k_{mx}^2 + k_{my}^2 = k_m^2 = (\pi/d_m)^2$$

If k_{mx}^2 is small then $k_{my}^2 \sim k_m^2 \sim (\frac{\pi}{d_m})^2$ and $B_1 = B_3 = -\frac{B_2 d_2}{2d_1}$

Where subscripts m denotes pole number 1,2, 3, B_m is used to denote peak magnetic field, d_m is used to represent corresponding pole length and c_m denotes phase adjustment (c_1=0, c_2=d_1 c_3=d_1 +d_2) for different pole. In present case d_1= d_2= d_3 & $k_{x1} = k_{x2}$. B_2 is the its' peak magnetic field and x, y and z are horizontal, vertical and beam directions respectively.

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Electron beam trajectory

The equation of motion for the first pole in the horizontal plane, with $\cosh k_x x \sim 1$ and y=0. is :

$$\frac{d^2x}{ds^2} = \frac{B_2}{2B\rho} \sin k(s-c_1)$$

Where ρ =radius of curvature in the field B and ds = vdt
The first integral of equation is

$$\frac{dx}{ds} = -\frac{B_2}{2kB\rho}\left[\cos k(s-c_1) - 1\right]$$

By using the relation $z_e'^2 = 1 - x_e'^2 \sim 1$

After second integration

$$x = -\frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_1)}{k} - s\right]$$

Similarly for the second pole and third pole, we can write

$$x = \frac{B_2}{k_2 B\rho}\left[\frac{\sin k(s-c_2)}{k} + \frac{d}{2}\right]$$

$$x = -\frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_3)}{k} + s - 3d\right]$$

In above equations on right hand side second additional term is due to compensated electron bump trajectory.

A Hamiltonian for Betatron motion

The Hamiltonian of the motion of an electron under above magnetic field[2] can be written as,

$$H = \frac{1}{2}\left[p_z^2 + (p_x - A_x \cos k_m(z-c_m))^2 + (p_y - A_y \cos k_m(z-c_m))^2\right]$$

Where: $A_{mx} = -\frac{B_m}{k_m B\rho} \cosh k_{mx}x \cosh k_{my}y$

$$A_{my} = \frac{B_m k_{mx}}{k_{my}} \frac{\sinh k_{mx}x \sinh k_{my}y}{k_m B\rho}$$

A canonical transformation is required to change variables from (x, y, z) to (x_β, y_β, s) where s is distance along the equilibrium orbit, x_β is a displacement in the (x, z) plane perpendicular to the equilibrium orbit and $y_\beta = y$ is vertical displacement from the equilibrium orbit. Transformation between variables for first pole, second pole and third pole can be written as

$$x = x_e + z_e' x_\beta = x_e - \frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_1)}{k} - s\right] \sim x - a_1$$

$$z = s - x_e' x_\beta = s - x_e' \frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_1)}{k} - s\right] \sim s$$

For the second pole

$$x \sim x_e + \frac{B_2}{kB\rho}\left[\frac{\sin k(s-c_2)}{k} + \frac{d}{2}\right] \sim x_e - a_2$$

$$z = s - \frac{B_2}{kB\rho}\left[\frac{\sin k(s-c_2)}{k} + \frac{d}{2}\right] x_e^{'} \sim s$$

For the third pole

$$x \sim x_e - \frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_3)}{k} + s - 3d\right] \sim x_e - a_3$$

$$z = z_e - \frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_3)}{k} + s - 2.d_1 - d2\right] x_e^{'} \sim s$$

Where

$$a_1 = \frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_1)}{k} - s\right], \; a_2 = -\frac{B_2}{kB\rho}\left[\frac{\sin k(s-c_2)}{k} + \frac{d}{2}\right]$$

& $a_3 = \frac{B_2}{2kB\rho}\left[\frac{\sin k(s-c_3)}{k} + s - 3d\right]$

and the canonical momentum transformation can be written as

$$p_x = z_e^{'} p_{x_\beta} + \frac{x_e^{/}}{(1+\Omega x_\beta)} p_s$$

$$p_z = \frac{z_e^{/}}{(1+\Omega x_\beta)} p_s - x_e^{/} p_{x_\beta}$$

$$p_y = p_{y_\beta}$$

After transformation, Hamiltonian, the betatron equations of motion can be obtained by Hamiltonian' s equations.

$$x'' = -\frac{B^2{}_m k_{mx}^2}{2k_m{}^2 B^2 \rho^2}\left[(x-a_m) + \frac{1}{3}k_{mx}^2 (x-a_m)^3 + k_{my}^2 (x-a_m) y^2 - \frac{1}{2}k_{mx}^2 (x-a_m) y^2\right]$$

$$+ \frac{B_m \sin k_m (s-c_m)}{B\rho}\left[\begin{array}{l} \frac{k_{my}^2 y^2}{2} + \frac{k_{my}^4 y^4}{24} + \frac{k_{mx}^2 (x-a_m)^2}{2} + \frac{k_{mx}^2 k_{my}^2 (x-a_m)^2 y^2}{4} \\ + \frac{k_{mx}^4 (x-a_m)^4}{24} \end{array}\right]$$

$$+ \frac{B_m \cos k_m (s-c_m)}{k_m B\rho} y y^{/} k_{mx}^2 \left[a_m + \frac{k_{my}^2 y^2}{6} + \frac{k_{mx}^2 (x-a_m)^2}{2}\right]$$

$$y'' = -\frac{B^2{}_m k_{my}^2}{2k_m^2 B^2 \rho^2}\left[\begin{array}{l} y + k_{mx}^2 (x-a_m)^2 y + \frac{1}{3}k_{mx}^4 y (x-a_m)^3 \\ + \frac{1}{6}k_{my}^2 y^3 \end{array}\right]$$

$$- \frac{B_m \cos k_m (s-c_m) k_{my}^2 y x^{/}}{k_m B\rho}\left[1 + \frac{1}{2}k_{my}^2 y + \frac{1}{2}k_{mx}^2 (x-a_m)^2\right]$$

$$- \frac{B_m \sin k_m (s-c_m)}{B\rho} k_{mx}^2 y \left[\begin{array}{l} (x-a_m) + \frac{1}{6}k_{my}^2 (x-a_m) y^3 \\ + \frac{1}{6}k_{mx}^4 (x-a_m)^3 y \end{array}\right]$$

In averaging procedure $\sin(k_m s)$ and $\cos(k_m s)$ term are retained due to compensated electron bump trajectory After averaging the electron trajectory in combination with different magnetic field component gives net linear and non-linear effect. There is a linear effect, which is equivalent to that of a horizontally and vertically focusing quadrupole, whose strengths are given by,

$$x'' = -\frac{3}{4}\frac{B_m^2 k_{mx}^2}{(B\rho)^2 k_m{}^2} x - \frac{5}{4}\frac{B_m^2}{(B\rho)^2}\frac{k_{mx}^2}{k_m^2} x$$

$$y^{"} = -\frac{3}{4}\frac{B_m^2 k_{my}^2}{(B\rho)^2 k_m{}^2} y + \frac{5}{4}\frac{B_m^2}{(B\rho)^2}\frac{k_{mx}^2}{k_m^2} y$$

The term in k_x is due to collapse of the field in x direction due to finite width of magnetic poles and first term in k_y is due to the edge focusing of the magnet. In the case of k_x^2 =0.0, WLS will act as a focussing device in the vertical plane and oscillating sin & cos term in the Hamiltonian will not give any contribution to the quadrupole component. If WLS has finite coupling then oscillating term in the hamiltoinian cannot be neglected and orbit offset in the sextupole component will give quadrupole component in addition to the edge focussing. This shows that for WLS, coupling coefficients are very stringent. In the equation second term represents the quadrupole component, which arises due to orbit offset in the sextupole component.

STUDIES FOR INDUS-1

In Indus-1, WLS can be operated in two options: a) It is switched on after injection. b) It is kept on during injection. Since, beam lifetime is short, therefore second option is preferred. The parameters of WLS are B = 3T, number of poles (n) =3, total magnetic length =0.54m. Further effects of coupling are studied by introducing coupling $(k_x/k_y)^2$ = -0.03. A series of hard-edged rectangular magnets was used to model the horizontal & vertical focussing caused by the WLS. In horizontal plane focussing due to finite k_x is modelled by defining field index into the dipole. The linear as well as non-linear effects are proportional to β_y at its location and inversely proportional to the square of beam energy. In Indus-1, β_y at WLS is small (0.7m – 1.2m), but due to its low energy, its linear effects are significant, however non-linear effects of WLS will not be stringent. In this paper a study, has been carried out for present operating tune point (1.69, 1.28).

Linear effect compensation

It is required to correct both tunes and β-asymmetries for smooth operation. The various options of compensations such as local beta/global tunes, global beta and tunes using 8 and 16 QMs were studied using computer program BURHANI [5] it is found that it is not possible to compensate both simultaneously. The β-asymmetries can be compensated completely by α -matching, except in region of the device using QF and QD families of quadrupoles placed in the neighbourhood of WLS. But it changes the linear tunes by a large amount. Therefore, the scheme in which first α-matching are done by using neighbouring QF and QD families and then linear tunes are compensated globally with remaining families of QF and QD was studied (fig.1 & 2). It offers a promising solution. The vertical β-asymmetries is corrected from 49% to 21 % and are reduced proportionally further if tunes are corrected partially. This scheme can help to achieve acceptable level of linear distortions for operation of WLS in a real machine. It is noted that only tunes correction is not a good solution as it leaves the large β-asymmetries.

Table 1: β-asymmetry correction results

	$\Delta\nu_X$	$\Delta\nu_Y$	$(\delta\beta/\beta)_{xrms}$	$(\delta\beta/\beta)_{yrms}$
Before correction	-0.02	0.06	10.39	27.47
Only Tune	0.0	0.0	9.36	27.47
α matching. & Tune	0.0	0.0	6.78	23.87
α matching &partial tune	-0.02	0.09	5.24	20.48

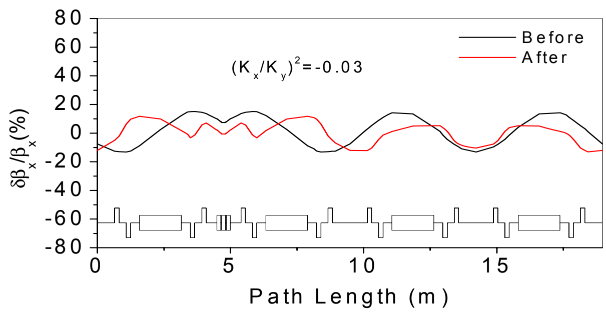

Figure 1

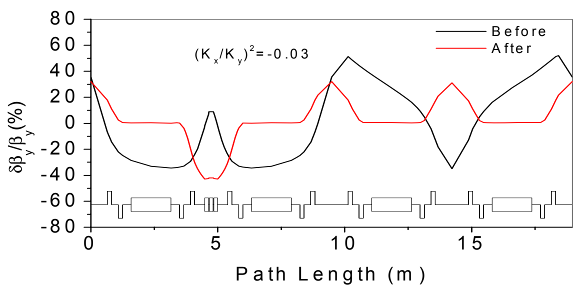

Figure 2

Dynamic aperture

In the presence of WLS, dynamic aperture is reduced mainly due to its linear effects. In the fig .3 dynamic apertures are plotted for on momentum particle after 100,000 turns for a finite k_x. It indicates that after α-matching and globally tunes correction; dynamic aperture improves in vertical plane by 33% and in horizontal plane by 18%. In comparison to only global tune correction. In both cases, dynamic aperture shrinks in comparison to the ideal lattice.

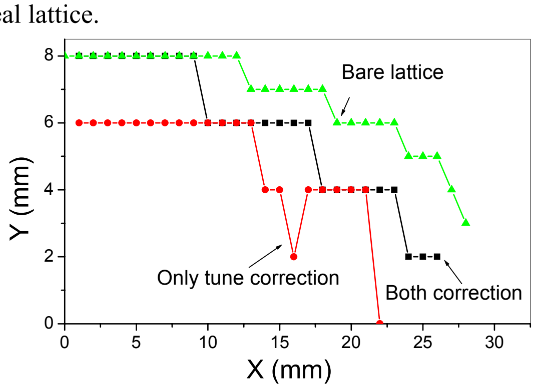

Figure 3

Injection simulation

In the Indus –1, one-kicker horizontal multi-turn injection scheme is adopted. In this scheme, the septum and kicker magnets are located at symmetry points of the ring and diametrically opposite to each other so that maximum of orbit bump (X_{bump}) parallel to the design orbit can be produced at the exit of the septum magnet. As a result, residual betatron oscillation will propagate all over the ring. So if injection is carried out in the presence of the WLS (having finite K_x), then β-asymmetries in the horizontal plane will deteriorate the injection efficiency. At the septum location (located at 21mm from design orbit) maximum oscillation amplitude of injected and stored bunches are tabulated in table 2 for the bare lattice, after tune correction and after both α-matching and globally tune correction. For above calculation 16mm orbit bump is produced using a single kicker magnet.

Table 2: Max. Beam oscillation at septum location

	Injected	stored
Bare lattice	16.85	16.8
Tune correction	18.07	15.8
α matching & Globally tune	16.63	17.43

The above results indicate that injection is possible in all cases; its injection efficiency may be poor. If injection or dynamic apertures are still not sufficient then after α matching, tune correction can be done partially, so that β-asymmetries are reduced to acceptable levels.

CONCLUSIONS

The Hamiltonian for the WLS gives an extra term for the quadrupole component, which arises due to orbit offset in sextupole magnet. To operate WLS in INDUS-1, local β correction/ partial or total global tunes correction may be required. It requires two additional power supplies for the independent variations of QF and QD families nearest to the WLS.

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