COMPARISON OF CODES FOR SMITH-PURCELL FEL *

Vinit Kumar#, RRCAT, Indore, M.P. 452013, India
Kwang-Je Kim, ANL, Argonne, IL 60439, USA
Dazhi Li, ILT, Suita, Osaka 565-0871, Japan

Abstract

Smith-Purcell (SP) free-electron lasers (FELs) using low energy electron beam are being seen as attractive option for a compact source of coherent terahertz radiation. Recently, Kumar and Kim [1] have performed numerical simulation of SP-FELs based on a computer code using Maxwell-Lorentz equations. Li et al. [2,3], and Donohue et al. [4] have performed calculations using particle in cell (PIC) codes. In this paper, we present a comparison of these methods and compare results obtained using different codes.

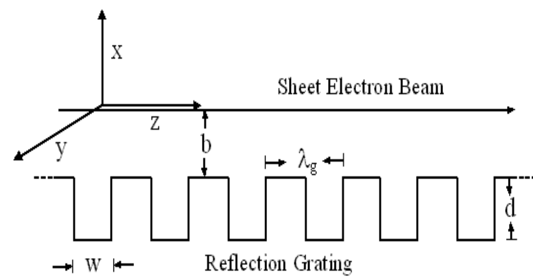

Figure 1: Schematic of an SP-FEL using a sheet electron beam. The sheet electron beam is in the plane x = 0.

INTRODUCTION

Recently, there has been a lot of interest in the analysis and simulation of Smith-Purcell (SP) free-electron laser (FEL) [1-6]. The possibility of a SP-FEL based on low energy electron beam is seen as an attractive option for compact terahertz (THz) source of coherent radiation. The SP-FEL is a backward wave oscillator (BWO) for low energy electron beam [1,5]. In a BWO, like any oscillator system, the electron beam current needs to be higher than a threshold value, known as the start current, in order to produce coherent electromagnetic oscillation. If the electron beam current is higher than the start current, the coherent electromagnetic oscillations start growing and then saturate due to nonlinearity. In order to build such a device, it is important to study the start current and the saturation behaviour of SP-FEL. Kumar and Kim [1] have performed an analysis and numerical simulation of a SP-FEL system using Maxwell-Lorentz equations, where they have studied the growth of power, the efficiency at saturation and the start current in a SP-FEL. Li et al. [2] and Donohue et al. [4] have performed a more detailed 2D simulation of a SP-FEL system using a computer code MAGIC [7] which is a particle in cell (PIC) code. In Ref. 3, Li et al. have performed three-dimensional simulation of SP-FEL system using MAGIC. A natural question then arises that how do these different codes compare with each other. In this paper, we present a comparison of the results obtained using different computer codes for simulating a SP-FEL. We find that the results of the fast, 2-D simulation using Maxwell-Lorentz equation developed in Ref. 1 agree well with other more elaborate simulations.

In the next section, we discuss the numerical simulation and calculations that we have performed based on Maxwell-Lorentz equations. We then discuss the comparison of results obtained using this approach with the already published results [2-4] obtained using PIC code in the following section and conclude.

DETAILS OF MAXWELL-LORENTZ SIMULATION

We start with a brief description of the SP-FEL system. Figure 1 shows the schematic of the SP-FEL setup. We assume the system to have translational invariance in the y-direction and hence ours is a 2D analysis. We assume a sheet electron beam which travels with a speed βc along the z-axis, at a height b above the grating of length L, having grooves of depth d, width w and period λ_g. Here, c is the speed of light in vacuum.

In a SP-FEL, the electron beam interacts with the co-propagating surface electromagnetic mode supported by the grating. As shown in Refs. 1 and 5, the co-propagating surface mode has a group velocity in the direction opposite to the electron beam for low electron beam energy. The backward surface mode supported by the grating is a linear combination of infinite number of Floquet-Bloch harmonics having the z-component of propagation vectors differing from each other by an integral multiple of k_g, where $k_g = 2\pi/\lambda_g$. The y-component H_y of the magnetic field of the backward surface mode can be written as $\sum A_n \exp(ik_0 z + ink_g z - \Gamma_n x - i\omega t)$, where the summation is implied over all n from -∞ to +∞ [1]. Here, ω is the frequency, $k_0 = \omega/c\beta$ is the propagation vector of the backward surface mode and $\Gamma_n = \sqrt{(k_0 + nk_g)^2 - \omega^2/c^2}$. The zeroth-order component of this mode has the longitudinal electric field given by $E_0(z,t)\exp(ik_0 z - i\omega t)$ at $x = 0$. The amplitude of all other components of the backward surface mode have to maintain a fixed ratio with the amplitude of the zeroth-order component such that the elec-

* Work supported by U.S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.
#vinit@cat.ernet.in

tromagnetic field satisfies the required boundary conditions at the grating surface. Hence, as the zeroth-order component of the surface mode evolves due to interaction with co-propagating electron beam, the amplitude of all other components also evolve proportionately.

The evolution of the backward surface mode due to interaction with the co-propagating electron beam can be described using following Maxwell-Lorentz equations [1].

$$\frac{\partial \mathcal{E}}{\partial \tau} - \frac{\partial \mathcal{E}}{\partial \zeta} = -\mathcal{J}\langle e^{-i\psi}\rangle, \tag{1}$$

$$\frac{\partial \eta_i}{\partial \zeta} = (\mathcal{E} + \mathcal{E}_{sc})e^{i\psi_i} + c.c., \tag{2}$$

$$\frac{\partial \psi_i}{\partial \zeta} = \eta_i, \tag{3}$$

$$\mathcal{E}_{sc} = i\frac{\mathcal{J}}{\chi L}(\chi_1 - e^{2\Gamma_0 b})\langle e^{-i\psi}\rangle, \tag{4}$$

where, the notations used are described in Ref. 1. Here, $\mathcal{E}$ is the dimensionless electric field, $\mathcal{E}_{sc}$ is the dimensionless space charge field, $\mathcal{J}$ is the dimensionless beam current, τ is the dimensionless time, $\zeta = z/L$ is the normalised distance along the grating, ψ_i is the phase of i^{th} electron, η_i is the dimensionless relative energy of the i^{th} electron and χ and χ_1 are related to the singularity associated with the surface mode as defined in Ref. 1. The formula for conversion from E_0 to $\mathcal{E}$ is the following

$$\mathcal{E} = \frac{4\pi}{I_A Z_0}\frac{kL^2}{\beta^2\gamma^3}E_0, \tag{5}$$

where Z_0 = 377 Ω is the characteristic impedance of free space, I_A = 17 kA is the Alfvén current and γ is the electron energy in the unit of its rest mass energy.

Eqs (1-4) can be used to perform 2D simulation of the SP-FEL. For given grating parameters and electron beam parameters, one has to first evaluate the resonant wavelength $\lambda = 2\pi c/\omega$, the group velocity v_g of the surface mode in the backward direction and the parameters χ and χ_1 as per the procedure described in Ref. 1. We then proceed for numerical solution of Eqs. (1-4) for which we use the approach used by Ginzburg et al. [8] and later also by Levush et al. [9] for BWO. The electron dynamics equations for a given field distribution along the interaction region are solved by the predictor-corrector method. Then, knowing the modified electron distribution in phase space, the field distribution at the next time step is obtained by solving the partial differential equations (Eq. 1) by the finite difference method. The method is stable for $\Delta\tau < \Delta\zeta$. Here, $\Delta\tau$ and $\Delta\zeta$ are the step sizes in τ and ζ respectively, used in the finite difference method. For initializing the electron beam in phase space, we simulate the shot noise using the algorithm given by Penman and McNeil [10], which is commonly used in FEL codes. In our simulation, we have used 1024 test particles and the step sizes are $\Delta\tau$ = 0.01 and $\Delta\zeta$ = 0.02. We have checked for the convergence of the solution with these simulation parameters. One can analyse the growth of power in the backward surface mode and thus study the saturated electric and magnetic field in the surface mode as well as the start current using this simulation [1].

Once the dimensionless electric field $\mathcal{E}$ at the location of the sheet beam is known using this simulation, one can calculate the amplitude of the total electric and magnetic field in the surface mode. The total magnetic field at the grating surface and the total electric field at the location of the sheet beam are given by the following expressions

$$H_y(x=-b,z,t) = \frac{I_A}{4\pi}\frac{\beta^4\gamma^4}{kL^2}e^{\Gamma_0 b}\mathcal{E}\mathcal{T}_1, \tag{6}$$

$$E_z(x=0,z,t) = Z_0\frac{I_A}{4\pi}\frac{\beta^3\gamma^3}{kL^2}\mathcal{E}\mathcal{T}_2, \tag{7}$$

where $\mathcal{T}_1$ and $\mathcal{T}_2$ are given by the following series

$$\mathcal{T}_1 = \sum_{n=-\infty}^{n=+\infty}\frac{A_n}{A_0}e^{(ik_0 z + ink_g z - i\omega t)}, \tag{8}$$

$$\mathcal{T}_2 = \sum_{n=-\infty}^{n=+\infty}\frac{\Gamma_n}{\Gamma_0}\frac{A_n}{A_0}\frac{e^{-\Gamma_n b}}{e^{-\Gamma_0 b}}e^{(ik_0 z + ink_g z - i\omega t)}. \tag{9}$$

Note that the coefficients A_n/A_0 for given grating parameters and the phase velocity of the surface mode can be calculated by satisfying the boundary conditions at the grating surface [1,5].

Eqs. (1-4) can be solved analytically in the linear regime and we can derive the following expression for the start current density [1]

$$\frac{dI_s}{dy} = 7.68\frac{I_A}{2\pi\chi}\frac{\beta^4\gamma^4}{kL^3}e^{2\Gamma_0 b}. \tag{10}$$

In the next section, we will use the numerical simulation based on Eqs. (1-4) and then calculate the total electromagnetic field at saturation and the start current using Eqs. (6-10) to compare with results obtained using PIC simulations.

COMPARISON WITH PIC SIMULATIONS

A more powerful, but computer intensive approach to simulate a SP-FEL is to use a PIC code. Donohue et al. [4] and Li et al. [2,3] have recently used a commercially available code MAGIC [7] to perform 2D/3D simulation of SP-FEL. MAGIC is an electromagnetic PIC code, i.e., a finte-difference, time domain code for simulating plasma physics processes. The full set of Maxwell's time-dependent equation is solved to obtain electromagnetic fields. Similarly, the complete Lorentz force equation is solved to obtain relativistic particle trajectories, and the continuity equation is solved to provide current and charge densities for Maxwell's equations.

First, we compare our results for parameters discussed by Donohue et al. in Ref. 4. They have performed 2D simulation using a sheet electron beam. The parameters used are: beam energy = 50 keV, dI/dy = 1000 A/m, λ_g

= 173 μm, d = 100 μm, w = 62 μm, N = 35 and B_z = 2 T. Here, N is the number of grating periods and B_z is external magnetic field used for guiding. The beam thickness is the x direction is taken to be 20 μm and the edge of the beam is taken to be at a height of 20 μm from the grating top surface. With these parameters, the amplitude $|B_y|$ of the total magnetic field at the grating top surface ($x = -b$) in the surface mode at saturation has been obtained to be 0.01 T in Ref. 4 using 2D MAGIC simulation. In our model, since we take a sheet beam of zero thickness, we choose the equivalent height b of the sheet beam from the grating top surface to be 30 μm. For these parameters, we obtain λ = 631 μm, v_g/c = 0.39, χ = 28 per cm and χ_1 = 1.9. We have then run the simulation based on Maxwell-Lorentz equation discussed in the previous section using these parameters and find that at saturation, the amplitude $|\mathcal{E}|$ of dimensionless electric field is 5.34. Next, by solving the boundary value problem at the grating surface [1,5], we evaluated the coefficients in the surface mode as A_2/A_0 = -0.14 + i 0.175, A_1/A_0 = 1.31 + i 2.74 and A_{-1}/A_0 = -0.09 - i 0.18. We find that other coefficients are very small and not significant. Putting these coefficients in the series in Eq. (8), we find that peak value of $|\mathcal{T}_1|$ is 4.41. Putting all these in Eq. (6), we evaluate the amplitude of $|H_y|$ at saturation to be 7.13 $\times$ 10^3 A/m which gives us $|B_y|$ = 0.009 T. This agrees quite well with the result obtained using MAGIC.

Next, we compare our results with those obtained by Li et al. in Ref. 2. Here, they have used electron beam energy = 40 keV and the same grating parameters as in the previous example except that N = 50. They have used the electron beam thickness in the x direction as 24 μm and the height of the edge of the electron beam from the grating top surface as 34 μm. For these parameters they obtain the start current density as 600 A/m. In this case, we obtain λ = 663.67 μm and χ = 15 per cm. Assuming an equivalent b = 46 μm in our sheet beam model and using Eq. (10), we obtain start current density as 520 A/m. This is around 13% smaller than the value obtained using MAGIC.

Finally, we compare our results with 3D MAGIC simulations reported recently by Li et al. in Ref. 3. Here, they have used electron beam energy = 100 keV, I = 0.8 A, λ_g = 2 cm, d = 1 cm, w = 1 cm, N = 46 and B_z = 2 T. A cylindrical electron beam of radius 2.5 mm is assumed and the edge of the beam is taken to be at a height of 2 mm from the grating top surface. For simulating this case, we take b = 4.5 mm in our sheet beam model. We obtain λ = 64.55 mm, v_g/c = 0.16, χ = 1.21 per cm and χ_1 = 5.5 for this case. Using Eq. (10), we find that dI_s/dy = 1.6 A/m for these parameters. Li et. al. have studied the growth of electromagnetic oscillation for this case using MAGIC and obtain the start current to be 0.5 A. Note that since our analysis is 2D, we can calculate the start current density dI_s/dy and not the total start current I_s. If the width of the surface mode in y direction is Δy, the start current $I_s = dI_s/dy \times \Delta y$. The width of the mode Δy is an unknown parameter in our 2D analysis and can be calculated only in a 3D analysis. However, here we can evaluate the equivalent Δy by dividing the start current obtained using MAGIC by start current density obtained using our 2D analysis. This way, we obtain the equivalent Δy = 25 cm for this case. Hence, the equivalent surface current density for the beam current of 0.8 A is 3.2 A/m in our 2D simulation. For these parameters, we have performed the numerical simulation based on Maxwell-Lorentz equations and find that $|\mathcal{E}|$ = 4.39 at saturation. Next, in order to evaluate the amplitude $|E_z|$ of the total electric field in the surface mode at the location of the electron beam ($x = 0$) using Eq. (7), we need to evaluate the series in Eq. (9). We find that all the terms except n = 1 and 0 can be neglected in the series and A_1/A_0 = i 1.53. Hence, we find the peak value of $|\mathcal{T}_2|$ to be 2.25. Putting these numbers in Eq. (7), we obtain $|E_z|$ = 1.7 $\times 10^4$ V/m which compares well with 1.45 $\times 10^4$ V/m as obtained using MAGIC.

CONCLUSIONS

We have compared the results of 2D simulation of SP-FEL using Maxwell-Lorentz equation with 2D/3D MAGIC simulations and found the comparison to be good. Simulations based on Maxwell-Lorentz equation are fast and the reported simulations typically take few tens of minutes whereas the MAGIC simulations take longer time. 3D MAGIC simulations reported here typically take several days. We plan to perform detailed 3D MAGIC simulations in future to study the 3D mode size and structure.

REFERENCES

[1] V. Kumar and K.-J. Kim, Phys. Rev. E 73 (2006) 026501.

[2] D. Li et al., Phys. Rev. ST Accel. Beams, 9 (2006) 040701.

[3] D. Li et al., Proceedings of FEL conference, (2006) 439.

[4] J. T. Donohue and J. Gardelle, Phys. Rev. ST Accel. Beams, 9 (2006) 060701.

[5] H. L. Andrews and C. A. Brau, Phys. Rev. ST Accel. Beams, 7 (2004) 070701.

[6] H. L. Andrews et al., Phys. Rev. ST Accel. Beams, 8 (2005) 050703.

[7] D. Smithe and T. M. Abu-Elfadi, Comput. Phys. Commun. 95 (1997) 106.

[8] N. S. Gingburg, S. P. Kuznetov, and T. N. Fedoseeva, Sov. Radiophys. Electron., 21 (1979) 728.

[9] B. Levush et al., IEEE Trans. Plasma Sci. 20, (1992) 263.

[10] C. Penman and B. W. J. McNeil, Opt. Commun. 90 (1992) 82.