SCALING-LAWS OF WAKE-FIELDS IN OPTICAL STRUCTURES

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Abstract

The electromagnetic wake-fields associated with two simple models are investigated. First, the effect of the radius of curvature as well as the frequency dependence of the dielectric coefficient are addressed, where the radiation characteristics generated by a relativistic line charge moving in the vicinity of a dielectric cylinder are explored; it is shown that the emitted energy increases logarithmically with the kinetic energy $(\gamma-1)$ of the line charge. Secondly, surface roughness effects on a moving electron bunch are examined. Such effects are considered by resorting to a model of a metallic structure with random perturbations on its surface. Based upon this model, analytic expressions have been derived for the average energy emitted per groove and for its standard deviation.

INTRODUCTION

Analysis of wake-fields generated by moving charges in the vicinity of dielectric and metallic bodies is essential for the design of the next generation of optical acceleration structures. On the one hand, such an analysis contributes to the evaluation of the deceleration force facilitating an appropriate design to minimize this force and ensure efficient acceleration process; on the other hand, it enables to determine the impact of beam characteristics and geometrical parameters of the structures on the wake-field, as the latter may be responsible for the increase of beam emittance and/or of excitation of high-order modes. Moreover, in optical accelerators one macro-bunch consists of many hundreds of micro-bunches and, therefore, the stability of a single micro-bunch depends mainly on its interaction with the wake-fields generated by trailing micro-bunches in the same macro-bunch [1],[2].

Future optical acceleration structures are very different from the metallic symmetric structures used so far in the microwave range; it is therefore necessary to investigate a variety of topics that, while being well known for microwave acceleration structures such as wake-fields, surface roughness, emittance growth, are almost unknown in the optical range. The purpose behind this communication is to introduce, based on simple models, some scaling-laws of wake-fields in optical structures in order to explore the following issues: a) the effect of the radius of curvature of a dielectric body on the motion of charges moving in its vicinity [3]; b) frequency dependence of the dielectric coefficient [3]; c) properties of wake-fields generated by moving bunches due to surface roughness in optical structures [4],[5]. The relevant scaling-laws obtained may assist in the design of future optical acceleration structures.

APPROACH TO SOLUTION

In order to have an analytic or quasi-analytic expressions for the wake-field generated by moving particles in different geometries the following approach is utilized.

An electron bunch moving in a free space generates a current density J parallel to the direction of its motion. As this current density is confined to one direction and no additional currents are excited, it suffices to consider only one component of the magnetic vector potential $A^{(p)}$, superscript p indicating this is to be the *primary field*. This potential satisfies the *non-homogeneous* wave equation. The next step in the solution is to determine the *secondary field* (superscript s) due to the presence of an obstacle, e.g. a dielectric cylinder. The secondary field satisfies the *homogeneous* wave equation. Both secondary and primary fields together must satisfy the boundary conditions. Based on the constraints imposed by the boundary conditions the explicit expression for the secondary wake-field is achieved, and accordingly, one may evaluate the total energy emitted by the moving bunch.

DIELECTRIC CYLINDER

Consider a line charge carrying the charge per unit length λ [C/m] and moving in free space (ε_0,μ_0) at a constant velocity v_0 above an infinite dielectric cylinder of radius R and relative dielectric coefficient ε_r. The axis of a cylindrical coordinate system (r,φ,z) coincides with that of the cylinder. The line charge is located at a distance $y=h$ from the cylinder's axis and moves along the x-axis, as illustrated in Fig. 1. The primary magnetic field, at the dielectric cylinder surface i.e. at $r=R$, in a cylindrical coordinate system and in the frequency-domain, reads

$$H_z^{(p)}=\frac{\lambda}{4\pi}e^{-j\frac{\omega}{\mathrm{v}_0}R\cos\varphi+\frac{\omega}{\mathrm{v}_0\gamma}R\sin\varphi-\frac{\omega}{\mathrm{v}_0\gamma}h}. \quad (1)$$

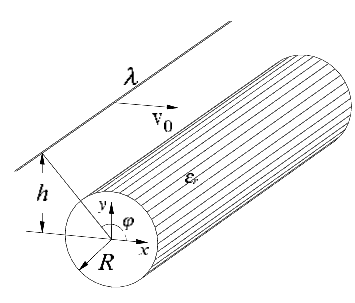

Figure 1: A line charge moving in the vicinity of a dielectric cylinder

where $\gamma = \left(1-\beta^2\right)^{-0.5}$ and $\beta = \mathrm{v}_0/c$. Accordingly, the secondary magnetic field in cylindrical coordinate has the following form

$$H_z^{(s)} = \sum_{n=-\infty}^{\infty} e^{jn\varphi} \begin{cases} A_n H_n^{(2)}\left(\frac{\omega}{c}r\right) & r > R \\ B_n J_n\left(\frac{\omega}{c}\sqrt{\varepsilon_r}r\right) & r < R. \end{cases} \tag{2}$$

Imposing the boundary conditions the unknown amplitudes are determined, and therefore, with the explicit expression for the secondary electromagnetic field, it is possible to evaluate the total energy emitted by the line charge due to the wake. Using the following quantities, $\Omega \equiv \frac{\omega}{c}h$, $\bar{A}_n \equiv \frac{A_n}{\lambda/4\pi}$, $\bar{W} \equiv \frac{W/\Delta_z}{\lambda^2/\pi\varepsilon_0}$, the normalized emitted energy $(\bar{W})$ is given by the following term

$$\bar{W} = \int_0^\infty d\Omega \frac{1}{\Omega} \sum_{n=-\infty}^{\infty} \left|\bar{A}_n(\Omega)\right|^2 \tag{3}$$

where the amplitudes $\bar{A}_n$ are given by an analytic expression [3]. Of special interst is also the spectrum of the radiated power or the longitudinal impedance defined as

$$Z_{||}(\omega) = -\frac{1}{\lambda}\int_{-\infty}^{\infty} dx E_x^{(s)}(x,h;\omega) e^{j\frac{\omega}{c\beta}x} . \tag{4}$$

Using the normalized quantities notation the two are related $\frac{d\bar{W}}{d\Omega} = \bar{Z}_{||}$, where $\bar{Z}_{||}$ is the normalized longitudinal impedance, defined by $\bar{Z}_{||} \equiv Z_{||}/2\eta_0 h$. Simulations show that the spectrum main peak increases with the increase of the line charge momentum $(\gamma\beta)$ and the dielectric coefficient (ε_r) of the cylinder as well as the line charge get closer to the cylinder. Moreover, the following scaling-laws were obtained for the normalized longitu dinal impedance $\bar{Z}_{peak} \propto \frac{R}{h}; \bar{Z}_{peak} \propto (\gamma)^{1/3}$, where $\bar{Z}_{peak}$ is the value of the main peak of the longitudinal impedance. The normalized frequency (Ω_{peak}) where the spectrum reaches its largest (and first) peak is inversely proportional to the ratio R/h, specifically, it satisfies the following scaling-law $\Omega_{peak} \propto \sqrt{\frac{h}{R}}$. For high frequencies i.e. $(\Omega R/h \gg 1)$, the spectrum has a high frequency dependence of Ω^{-1}. In addition, it was shown that the width of the main peak of the spectrum increases as the kinetic energy of the line charge increases. In accordance with these results, it is clear that one can design a system that will radiate energy whose spectrum will peak at a desired frequency.

As the spectrum of the emitted energy was analyzed it is possible to proceed one step further and investigate the total emitted energy. For this purpose different simulations were performed. For intermediate energies, the following simple scaling-law illustrating the dependence of the emitted energy on the parameter R/h was obtained $\bar{W} \propto \frac{R}{h}e^{-2(h/R)a}$, where a is a positive parameter determined by the dielectric coefficient of the cylinder and the line charge momentum. For relativistic energies the emitted energies decreases linearly as a function of h/R. As for the dependence of the emitted energy on the kinetic energy of the line charge, it was shown that the overall emitted energy increases logarithmically with the kinetic energy $(\gamma-1)$ of the line charge, particularly, the following scaling-law was revealed $\bar{W} \propto \ln(\gamma-1)$.

Finally, the frequency dependence of the dielectric coefficient of the cylinder is examined. As a simple model of the frequency dependence of ε_r it is assumed that the dielectric coefficient drops from ε_r to unity at a frequency Ω_c. Based on different simulations, it was concluded that the frequency dependence of the dielectric coefficient in the optical regime may be utilized in order to reduce the deceleration force acting on the line charge by its wake-field. Roughly, the emitted energy for ultra-relativistic values of γ varies as $\sqrt{\Omega_c}$.

SURFACE ROUGHNESS

Acceleration structures that operate at a wavelength of a few centimeters i.e., microwave regime, are machined today with an accuracy of microns. In future, it will not be possible to maintain 4-5 orders of magnitude difference between the operating wavelength $(1\mu\mathrm{m})$ and the achievable tolerance since this would entail engineering of a surface at the atomic level. As a result, the size of irregularities may be of the same order of magnitude as the micro-bunches, and they may generate wake-fields that in turn, may alter the dynamics of the electrons bunch. Accordingly, the wake-field due to surface roughness in an optical structure was considered in [4], [5].

Consider a metallic structure consisting of a random number (N) of grooves attached to a waveguide of the above type, as illustrated in Fig. 2. The center of the n^{th} groove is located at the coordinate z_n, its width is denoted by d_n and its external radius by $R_{\mathrm{ext},n}$. An electron bunch of radius R_b, length L_z and total charge Q, is moving along

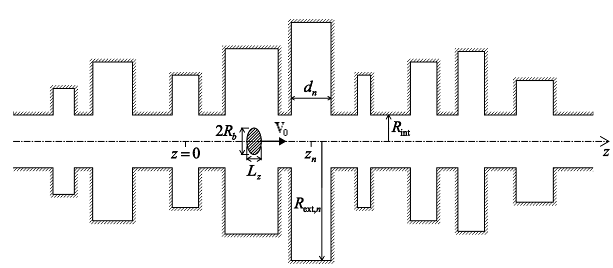

Figure 2: A finite-size bunch moving in vacuum at a constant velocity v_0 along the axis of a structure with random size grooves.

the symmetry axis of the structure at a constant velocity v_0, generating a current density denoted by $J_z(r,z;t)$.

As the only component of the current density is parallel to the z-axis, it suffices to consider the longitudinal magnetic vector-potential A_z which satisfies the non-homogeneous wave equation. Its solution has two components: the so-called *primary* field determined by the current density with the metallic structure absent, and the so-called *secondary* field accounting for the impact of the structure. Taken together, these fields satisfy the boundary-conditions. The emitted energy is given by

$$W = \frac{Q^2}{4\pi\varepsilon_0 R_{\rm int}} \mathrm{Re}\left[\int_0^\infty d\Omega S(\Omega)\right] \equiv \frac{Q^2}{4\pi\varepsilon_0 R_{\rm int}} \bar{W}\,; \quad (5)$$

$S(\Omega)$ representing the normalized spectrum and $\bar{W}$ the normalized energy. As already indicated, the geometrical parameters of all grooves are random and of the same order of magnitude. Explicitly, they are given by $\bar{R}_{{\rm ext},n} \equiv \frac{R_{{\rm ext},n}}{R_{\rm int}} = 1 + \bar{g}_n, \bar{d}_n \equiv \frac{d_n}{R_{\rm int}} = \bar{g}_n$, where $\bar{g}_n$ is a random variable uniformly distributed between 0 and $\bar{\delta}$; $\bar{\delta}$ will be referred to as the *normalized roughness parameter*. The center of the first groove ($\bar{z}_1 \equiv z_1/R_{\rm int}$) is chosen as point of reference and accordingly the location of the $n^{th}(n = 2,3,...,N)$ groove is given by $\bar{z}_{n+1} = \bar{z}_n + \bar{d}_n/2 + \bar{d}_{n+1}/2 + \bar{g}_n$. It should be pointed out that each geometric parameter is assumed to have its own random perturbation, but all have the same distribution. Therefore, the radius of the groove and its width are not correlated, as can be seen from Fig. 2.

The discussion that follows is limited to relatively small values of $\bar{\delta}$, since if $R_{\rm int}$ is of the order of $0.5\mu m$ the typical roughness is not expected to exceed $0.1\mu m$; therefore we consider $0 \le \bar{\delta} \le 0.2$. Moreover, the accelerated bunch is expected to be of the order $0.16 \le \bar{L}_z \equiv L_z/R_{\rm int} \le 0.25$; the normalized radius of the bunch is chosen to be $\bar{R}_b \equiv R_b/R_{\rm int} = 0.5$.

As a first step we focus on the *total energy* emitted, establishing its dependence on the roughness parameter $(\bar{\delta})$ with $\bar{L}_z$ and γ as parameters. According to simulations results, two facts are evident: first, the average and normalized energy per groove $\left(\tilde{W} \equiv \bar{W}/N\right)$ increases with the increase of $\bar{\delta}$. Secondly, $\tilde{W}$ increases as the length of the bunch ($\bar{L}_z$) decreases. The average emitted energy can be roughly approximated by

$$\frac{\langle W\rangle}{\frac{Q^2}{4\pi\varepsilon_0 R_{\rm int}} \times N} \simeq$$
$$0.57\tanh\left(\frac{45\frac{\langle g\rangle}{R_{\rm int}}}{1+20.72\frac{L_z}{R_{\rm int}}}\right) + \frac{1.429}{1+20.72\frac{L_z}{R_{\rm int}}}\,. \quad (6)$$

A second important feature of the emitted energy is its (normalized) standard deviation. A best fit of the simulation results shows that the standard deviation may be approximated by

$$\frac{\sqrt{\langle W^2\rangle - \langle W\rangle^2}}{\frac{Q^2}{4\pi\varepsilon_0 R_{\rm int}} \times N} \simeq 0.15\left(\frac{\Delta g}{R_{\rm int}}\right)^{0.25} \tanh\left(121.2\,\frac{\Delta g}{R_{\rm int}}\right)$$
$$\times\left[0.57\tanh\left(\frac{45\,\frac{\langle g\rangle}{R_{\rm int}}}{1+20.72\,\frac{L_z}{R_{\rm int}}}\right) + \frac{1.429}{1+20.72\,\frac{L_z}{R_{\rm int}}}\right]. \quad (7)$$

To summarize, in future, when optical accelerator structures will be designed, the surface roughness effects on the moving electron bunch and its wake-field should be accounted for, as these effects may affect both the longitudinal and the transverse beam dynamics.

CONCLUSION

In the present communication we have investigated the characteristics of wake-fields generated by charged particles moving in the vicinity of different metallic or dielectric optical structures. Specifically, a set of scaling-laws regrading the wake-fields in optical structures and the energy emitted by the moving particles was introduced. The scaling-laws obtained may assist in the design of future optical accelerators.

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