ANALYTICAL AND NUMERICAL STUDY OF SUPERCONDUCTING MINIUNDULATORS*

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Abstract

Based on a wire model of a superconducting miniundulator (supramini), analytical formulae are derived to describe the spatial distribution of the magnetic field as determined by the parameters of the undulator. The location of the field zeros is shown to be shifted compared with the position of the wires influencing the radiation spectrum significantly. After introducing a simple compensation coil, the field quality, locations of zeros, field integrals, and spectrum become satisfactory. Numerical simulations agree well with the analytical results.

INTRODUCTION

The development of superconducting miniundulators (supraminis) is driven by their potential to produce harder radiation for a given beam energy or to save beam energy for a given spectral range. Supraminis are expected to play an important role in upgrade projects of 3rd generation sources. Furthermore, they are believed to be a key component of 4th generation sources including FELs[1-4].

The spatial structure of the magnetic field of a supramini determines the quality of the electron orbit and the radiation emitted. In the case of an infinitely long undulator, the field zeros would coincide with the wire positions, for symmetry. As soon as a finite part is cut out, the field will leak out and the zeros shift with respect to the wire positions. Hence, the spatial structure of the magnetic field will not feature a constant cell length, such as the undulator period λ_u. Instead, the cell length will vary along the undulator and deteriorate the spectral power of the radiation.

In this paper, we derive analytical formulae for the spatial distribution of the magnetic field from a simple wire model. Their results are confirmed by numerical simulation. The positions of the zeros will be analysed and the influence on the spectral power of the radiation simulated. Adding a one-wire coil will be shown to compensate the magnetic field, its zeros, field integrals, and the spectral power such as to deviate negligibly from the ideal case as calculated by numerical simulation.

MODEL AND DISCUSSION

Using the vector potential $\vec{A}(\vec{r})$ and magnetic field $\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$ we derive equation (1) for the vertical magnetic field component that is produced by an array of wire pairs (wp) as sketched in Fig. 1. In a right-handed Cartesian frame where x and y are the horizontal and vertical coordinates transverse to the beam, respectively, and z points along the beam axis of the undulator, z = 0 is the centre point of the undulator, any correction field being ignored for now. A wire pair is formed by the two wires at a given location z with the same amount of current flowing in the same direction.

$$B_y(0,0,z) = B_F \sum_{i=1}^{p} (-1)^i \left[\frac{z_i^2}{(\bar{g}^2 + z_i^2)\sqrt{\hat{g}^2 + z_i^2}} \right] \qquad (1)$$

$$z_i = z + \frac{\lambda_u}{2}(p/2 - i + \frac{1}{2}), \quad p = \begin{cases} 2n & \text{even number of } wp \\ 2n+1 & \text{odd number of } wp \end{cases}$$

where $\bar{g} = g/2 + R$ is the distance from a wire center to the midplane with g the total gap width, $\hat{g}^2 = \bar{g}^2 + a^2$, $B_F = \frac{\mu_0}{4\pi} I 4a$, a the length of a wire in x-direction, I the electric current, p the number of wire pairs (wp) with the integer $n \geq 1$, λ_u /2 the distance between adjacent wire pairs. The absolute value of the magnetic field B_y at z = 0 will peak or be zero for an even or odd number of wp, respectively.

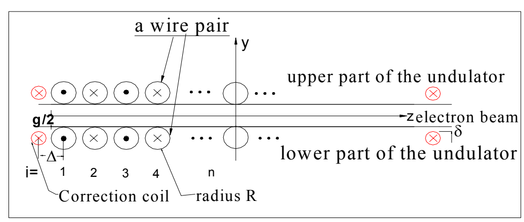

Figure 1: Supramini wire model (number of wp odd)

In Fig. 2, the magnetic field is plotted quantitatively for 10 and 11 wire pairs, λ_u=14mm, I=16mm^2×1000A/mm^2, g=5mm, a=200mm. The field is leaking out from the undulator edges, the zeros are shifted with respect to the wire positions and not equidistant. For symmetry reasons, our further analysis will be restricted to z ≥0.

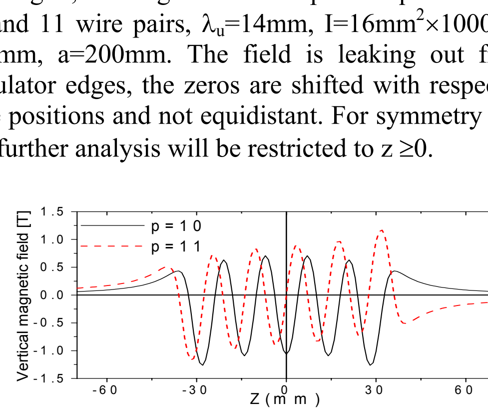

Figure 2: Vertical magnetic field as computed from eq. 1.

We determine the positions of the field zeros from the Taylor expansion of the field. Truncating it after the 2nd order term we can solve the resulting quadratic equation. In detail,

* Work supported by NUS C-380-003-003-001, A*STAR/ MOE RP 3979908M, and A*STAR 012 105 0038 grants.

$$B_y(z) = B_0 + B_0'(z-z_0) + \frac{B_0''}{2}(z-z_0)^2 + \cdots\cdots = 0 \quad (2)$$

where $B_0 = B_y(z_0)$, $B_0' = \left.\frac{\partial B_y}{\partial z}\right|_{z_0}$, $B_0'' = \left.\frac{\partial^2 B_y}{\partial z^2}\right|_{z_0}$, and

$$\frac{\partial B_y}{\partial z} = B_F \sum_{i=1}^{p} (-1)^i \left[\frac{(\bar{g}^2 - z_i^2)\hat{g}^2 - 2z_i^4}{(\bar{g}^2 + z_i^2)\sqrt{\hat{g}^2 + z_i^2}} \right];$$

$$\frac{\partial^2 B_y}{\partial z^2} = B_F \sum_{i=1}^{p} (-1)^i \left[\frac{6z_i^7 + Az_i^5 - 2Bz_i^3\hat{g}^2 - 3Cz_i\bar{g}^2\hat{g}^2}{(\bar{g}^2 + z_i^2)^3(\hat{g}^2 + z_i^2)^{5/2}} \right]$$

Here, $A = 5\bar{g}^2 + 7a^2, B = 6\bar{g}^2 - a^2, C = 3\bar{g}^2 + 2a^2$, and z_0 is the reference value for the expansion, i.e., anyone of the field zeros of an ideal infinitely long undulator which corresponds to a wire pair position. Truncating after the 2nd order, we obtain the solutions of eq. (2) as

$$z = z_0 - \frac{B_0'}{B_0''} \pm \sqrt{\frac{B_0'^2}{B_0''^2} - \frac{2B_0}{B_0''}} \quad (3)$$

In Fig. 3, we plot the difference between the location of a zero and its corresponding wire pair that marks the zero location in case of the infinite undulator, $\Delta z_{1j} = z_j - z_{0j}$, with j counting wire pairs from z = 0 in positive z direction. The shift of the zeros is small near the center of the undulator and increases to about 1 mm at the edge. To compare with the "undulator period" λ_u, we plot the difference $\Delta z_{2j} = (z_j - z_{j-2}) - \lambda_u$ that will bear directly on the spectral output (Fig. 4). We find Δz_{2j} which is an "effective undulator period" deviating significantly from λ_u, to a lesser extent near the center and increasing to about 0.7 mm at the edge. The spectral output will reflect this variation of the effective undulator period.

We now focus on a supramini with p=101 wire pairs keeping all other parameters the same as above. Figs. 5-7 show the magnetic field, Δz_{1j}, and Δz_{2j}, the latter two in comparison with the numerical simulation using RADIA [5]. Analytical results and numerical simulations are in good agreement. The deviations of the field, its zeros, and of the "effective period length" are as expected. Their

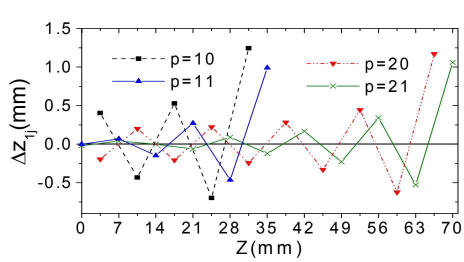

Figure 3: Δz_{1j} vs z for odd and even numbers of wire pairs.

influence on the spectral output is depicted in Fig. 8 where the dash-dotted line displays the tenfold of the flux in the first harmonic of the p=101 supramini while the solid line represents an ideal 50 period undulator. The curves were calculated using the SPECTRA7.0 code [6] assuming an electron beam energy of 120MeV, 100mA current, normalised emittances of $\varepsilon^n_x = \varepsilon^n_y = 4$mm mrad, and beta functions β_x=6m, and β_y=2m. Compared with the ideal case, the peak flux density without compensation is

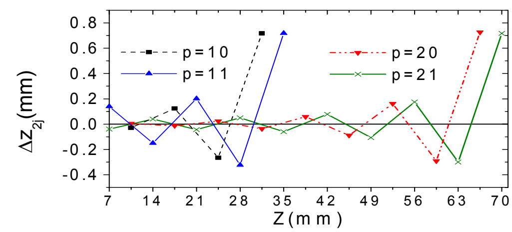

Figure 4: Δz_{2j} vs z for odd and even numbers of wire pairs.

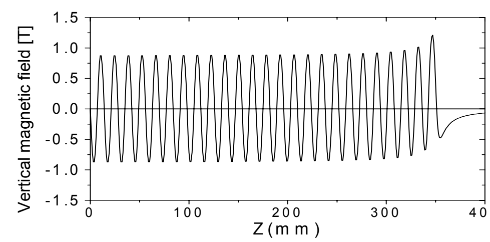

Figure 5: Vertical magnetic field as computed from eq. 1 for p=101 and other parameters as given above.

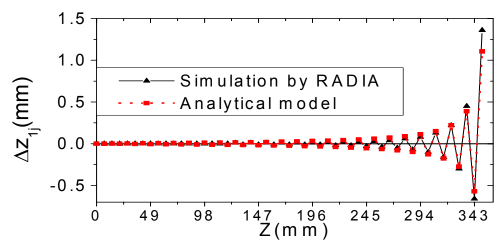

Figure 6: Δz_{1j} vs z from the analytical model and RADIA.

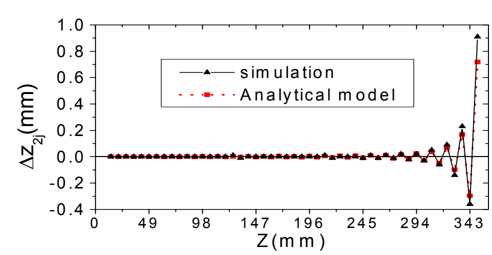

Figure 7: Δz_{2j} vs z from the analytical model and RADIA.

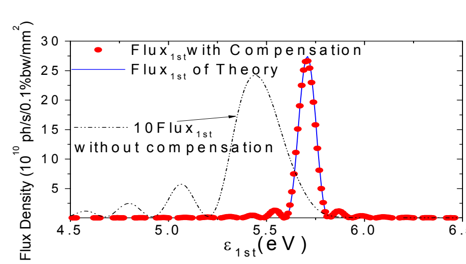

Figure 8: Fundamental radiation spectrum for the undulator with (circles) and without (dash-dotted) compensation coil in comparison to an ideal undulator (solid).

more than one order of magnitude smaller, the spectral width (FWHM) is about three times larger and the energy dependence is asymmetric.

Obviously, the field needs to be compensated. To this end, we add one wire pair at each supramini end (Fig. 1). Their field can be described as

$$B^{c}{}_{y}(0,0,z) = f_1 \; B_F\left[\frac{z_c^+}{((\bar{g}+\delta)^2+(z_c^+)^2)\sqrt{(\bar{g}+\delta)^2+(f_2 a)^2+(z_c^+)^2}} \pm \frac{z_c^-}{((\bar{g}+\delta)^2+(z_c^-)^2)\sqrt{(\bar{g}+\delta)^2+(f_2 a)^2+(z_c^-)^2}}\right] \quad (4)$$

with the + sign holding for an odd and the − sign for an even number of wire pairs, respectively. Here, r is the radius of the compensation wire, $f_2 a$ is the length of the compensation wires, $I^c = f_1 I$ the current flowing, and

$$f_1 B_F = \frac{\mu_0}{4\pi} I^c \; 4 f_2 a,$$

$$z_c^+ = z + \frac{\lambda_u}{2}(p/2-\Delta), \quad z_c^- = z - \frac{\lambda_u}{2}(p/2-\Delta).$$

We can optimize the field by changing f_1, f_2, Δ and δ. The compensation coils may be connected to individual power supplies. Then, the total magnetic field is,

$$B^{total}{}_{y}(0,0,z) = B_y(0,0,z) + B^{c}{}_{y}(0,0,z) \quad (5)$$

Upon compensation, the field is significantly improved. For $f_1 = 0.495$, $f_2 = 1$, $\Delta = 3.325$mm, and δ=0, the vertical magnetic field, the horizontal transverse velocity and orbit are shown in Figs. 9, 10 and 11. The initial conditions in the center of the supramini are x'_0=5.02mrad, x_0=0μm for a 120 MeV electron beam.

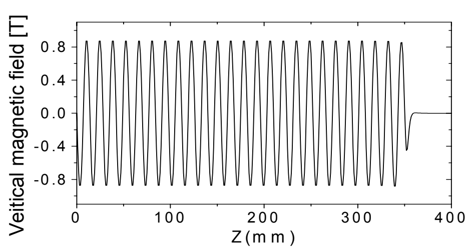

Figure 9: Vertical component of the magnetic field as computed from eq. 5 including the compensation field.

The spatial structure of the magnetic field has become much better with the compensation. The mutual distances of adjacent field zeros Δz_{1j} are very uniform inside the supramini with minor deviations for the last four wire pairs at the edge (Fig. 12). The maximum deviation is 0.147mm as compared with 1.106mm without compensation. The circle scatter in Fig. 8 give the spectrum of the compensated supramini. It agrees within 97% with the ideal one.

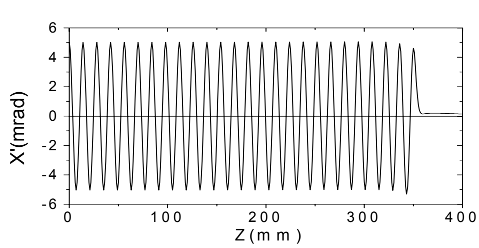

Figure 10: Horizontal transverse velocity component x'

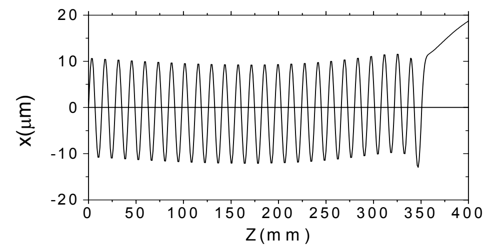

Figure 11: Beam trajectory along the undulator

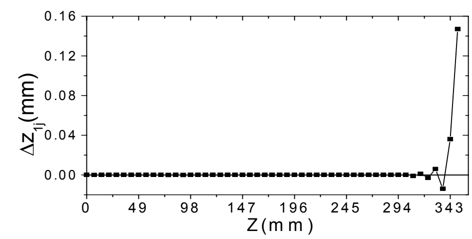

Figure 12: Mutual distance of adjacent field zeros Δz_{1j} for the compensated field of the p=101 supramini.

CONCLUSION

Using an analytical wire model for the supramini, we can show that the field zeros are neither equidistant nor coincident with the values expected from the positions of the wire pairs. This will adversely affect the radiation spectrum. A simple compensation method is presented which improves the spatial structure of the magnetic field significantly and moves the field zeros close to their ideal position. The peak spectral power in the fundamental then coincides within 97% with the ideal undulator.

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