ESTIMATION OF ERRORS IN DEFINITION OF CENTRAL MASS ENERGY IN HIGH PRECISION EXPERIMENTS ON COLLIDING BEAMS*

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Abstract

The new experiments on high precision mass measurements of J/Ψ and Ψ' mesons have been performed in BINP. The resonant depolarization technique was used for energy calibration. To increase the accuracy of the experiments the more thorough analysis of possible errors has been done. The present work examines the errors in definition of central mass energy.

INTRODUCTION.

The experiments on high precision mass measurements of narrow resonances produced in electron positron collisions have been performed in BINP (Novosibirsk). The last series of these experiments on mass measurement of J/Ψ and Ψ'-mesons was finished in 2002 [1]. Comparing with our old measurements [2] and PDG [3] data the more precise values of masses has been obtained ($M_{J/\Psi} = 3096.917 \pm 0.010 \pm 0.007$ MeV, $M_{\Psi'} = 3686.111 \pm 0.025 \pm 0.009$ MeV), the error is approximately 3 times smaller then in previous experiments. It should be noted, that the errors are about 100 times smaller than the value of the energy spread. Achievement of such an accuracy needs a thorough analysis of possible errors and corrections.

The important part of these experiments is an energy calibration by resonant depolarization technique [4, 5], based on measurement of the spin precession frequency. In the absence of non-vertical magnetic fields the energy E weighted on particles ensemble along the closed trajectory unambiguously related with spin tune ν_0 by known expression: $\nu_0 = E[MeV]/440.6486(1)[MeV]$. Any non-vertical magnetic fields disturb this relation and calculations of average energy using spin tune is in need of corresponding corrections [6]. Energies of electrons and positrons could be different due to existence of electrical fields and different orbits. Therefore it is necessary to consider corresponding corrections or to perform independent energy calibrations of each beam.

In precise experiments on colliding beams it is necessary to know average energy of interactions in the central mass system. However, simple kinematic addition of average particles momentums in IP could result in significant errors. Also, it is worthy to note that the average energy of the beam along the closed orbit is different from the average energy in the interaction point (IP). The analysis of these effects has been done in [6], but the last investigations revealed additional factors not included before. The goal of this work is a thorough analysis of all primary errors and corrections, related to the central mass energy definition in high accuracy experiments. This work presents results of errors analysis related to the last experiments on VEPP-4M collider.

EXCITATION OF DISPERSION OF OPPOSITE SIGNS FOR ELECTRONS AND POSITRONS.

The electrostatical separation of the beams in parasitic interaction points can excite a dispersion, which will have different sign for electrons and positrons due to opposite deflections of the beams. Existence of such a dispersion will disturb the energy distribution of luminosity which will result in the difference of the luminosity weighted central mass energy E_t and the double mean energy of the beams E_0 in the IP. Using the following symbols

- x, y, E– particles coordinates in transverse plane and energy,
- $\psi_{x,y}$– x,y dispersion function,
- $\varphi_{x,y}$– x,y perturbation of dispersion function of opposite signs for electrons and positrons,
- $\delta = (E - E_0)/E_0$– relative energy deviation,
- E_t– summarized energy of collided particles,
- $\delta_t = (Et - 2E_0)/E_0$– relative center of mass energy deviation,
- $\sigma_{x,y,\delta}$– spatial and energy RMS of the beam,
- $d_{x,y}$– half of beam separation in x, y directions,
- L– luminosity,

considering particles energy and spatial distribution to be Gaussian and making necessary convolutions one obtains

$$\frac{dL}{d\delta_t} \propto \exp\left[-\frac{(\varphi_x\delta_t + 2d_x)^2}{4\sigma_x^2} - \frac{(\varphi_y\delta_t + 2d_y)^2}{4\sigma_y^2} - \frac{\delta_t^2}{4\sigma_\delta^2} + \frac{\left(\dfrac{\psi_x(\varphi_x\delta_t + 2d_x)}{\sigma_x^2} + \dfrac{\psi_y(\varphi_y\delta_t + 2d_y)}{\sigma_y^2}\right)^2}{4\left(\dfrac{1}{\sigma_\delta^2} + \dfrac{\psi_x^2}{\sigma_x^2} + \dfrac{\psi_y^2}{\sigma_y^2}\right)}\right], \quad (1)$$

only terms containing δ_t are written. The full luminosity depends on separation by the following formula

$$L \propto \frac{1}{4\pi\sigma_x\sigma_y} \exp\left(-\frac{d_x^2}{4\sigma_x^2}\right) \exp\left(-\frac{d_y^2}{4\sigma_y^2}\right). \quad (2)$$

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As one can see from the equations above, the average δ_t of interactions is not zero only in presence of beams separations in the IP. During the statistics acquisition in mass measurement experiments there still could be parasitic separations of the beams resulting in energy bias.

In case of VEPP-4M collider $\psi_y = 0$, $\varphi_x = 0$ the luminosity weighted relative energy deviation of interactions will be

$$<\delta_t>= \frac{2\varphi_y d_y \sigma_\delta^2}{\varphi_y^2 \sigma_\delta^2 + \sigma_y^2}$$

Sources of parasitic beam separation in IP and numerical estimations.

- Beams separation in IP could arise from orbit distortions due to orbital bumps, beam separations in parasitic IPs. During the accelerator tuning for maximum luminosity the remained separation will be defined by an accuracy of luminosity measurement. In case of VEPP-4M collider for $E_0 = 1850$ MeV (Ψ'), $\varphi_y = -800$ μm, $d_y = 1.2$ μm, $\sigma_\delta = 5 \cdot 10^{-4}$, $\Delta L/L \sim 2\%$, $|E_t - 2E_0| = 8.8$ keV. To operate

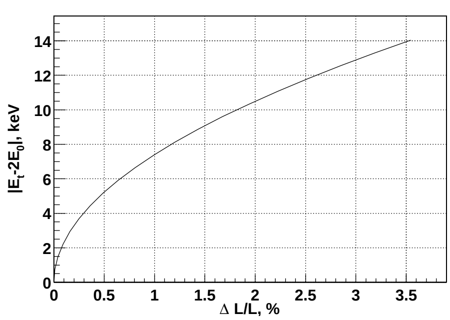

Figure 1: Invariant mass shift versus luminosity deviation from maximum. $E_0 = 1850$ MeV (Ψ'), $\varphi_y = -800$ μm, $\sigma_\delta = 5 \cdot 10^{-4}$.

  in mode with two bunches of electrons and positrons additional electrostatical separation in arcs is used at VEPP-4M collider. Choosing the right sign of the arc bumps the excited vertical dispersion could be reduced to $\varphi_y = -380$ μm. The energy bias than is $|E_t - 2E_0| = 4.2$ keV, for $\Delta L/L \sim 2\%$.
- Separated beams in parasitic IPs experience each other as additional correctors causing orbital distortions of opposite signs for electrons and positrons. In case of VEPP-4M collider additional separation $\Delta y = 0.4$ μm for 2 mA of beam current, $E_0 = 1850$ MeV resulting in $E_t - 2E_0 = 3$ keV and $(L - L_{max})/L_{max} = 0.2\%$.
- Beam-beam effects in presence of small beam separation in IP could increase or decrease beam separation. Additional separation Δy equals

$$\Delta y = -\frac{4\pi\xi_y \cot(\pi\nu_y)}{1 + 4\pi\xi_y \cot(\pi\nu_y)} d_{y0}.$$

  In case of VEPP-4M $\nu_y = 7.57945$, $I = 2$ mA, $\xi_y = 0.072$ additional separation $\Delta y = 0.2\, d_{y0}$, where d_{y0} is beam separation without beam-beam effects.

During the statistics aqcuisition the beam separation is periodically adjusted to provide maximum of luminosity. The average value of the energy shift is suppressed by square root of number of runs.

CHROMATICITY OF OPTICAL FUNCTIONS IN IP.

In presence of horizontal dispersion the particles energy and spatial distribution could be written as

$$n(x,y,\delta) = \frac{1}{(2\pi)^{\frac{3}{2}} \sigma_{\beta x}\sigma_{\beta y}\sigma_\delta} \exp\left[-\frac{(x-\psi\delta)^2}{2\sigma_{\beta x}^2}\right] \exp\left[-\frac{y^2}{2\sigma_{\beta y}^2}\right] \exp\left[-\frac{\delta^2}{2\sigma_\delta^2}\right] \quad (3)$$

Taking into account chromatisity of optical functions $\beta_{1x(y)}$, ψ_1 for ensemble of particles with energy deviation δ the spatial RMS are

$$\sigma_{\beta x} = \sqrt{\epsilon_x(\beta_{0x} + \beta_{1x}\delta)}$$
$$\sigma_{\beta y} = \sqrt{\epsilon_y(\beta_{0y} + \beta_{1y}\delta)},$$

where $\epsilon_{x(y)}$ is emmitances in $x(y)$ plane. Introducing $\delta_t = (E_t - 2E_0)/E_0$, the luminosity energy derivative is

$$\frac{dL}{d\delta_t}(\delta_t) \propto \frac{1}{\sqrt{\pi}2\sigma_\delta^2} \exp\left[-\frac{\delta_t^2}{4\sigma_\delta^2}\right] \frac{1}{2\pi} \frac{1}{\sqrt{\epsilon_y(2\beta_{0y} + \beta_{1y}\delta_t)}} \frac{1}{\sqrt{\epsilon_x(2\beta_{0x} + \beta_{1x}\delta_r) + (\psi_0 + \psi_1\delta_t)^2 2\sigma_\delta^2}}. \quad (4)$$

As it could be seen the energy distribution of luminosity is no longer symmetrical thus causing an energy shift of luminosity weighted energy. The measurements of β_{1x} and β_{1y} in VEPP-4M collider gave the shift of -4 ± 2 keV for J/Ψ and $+5 \pm 2.5$ keV for Ψ'.

INFLUENCE OF THE POTENTIAL OF THE COLLIDING BEAMS.

Influence of the own beam potential [7].

In the laboratory system the electric potential on the beam axis $(x, y = 0)$ at location s and position $z = s - vt$ within bunch is [8]

$$e\Phi(0,0,z,s) = e\lambda(z)\left(C + \ln 2 - 2\ln\left(\frac{\sigma_x(s) + \sigma_y(s)}{R(s)}\right)\right), \quad (5)$$

where $\lambda(z)$ is the charge line density at longitudinal position z within the bunch, e is the electron charge, $C \simeq$

$0.577...$ denotes Euler's constant, $R(s)$ is the beam-pipe radius, $\sigma_x(s)$ and $\sigma_y(s)$ are the horizontal and vertical RMS beam sizes respectively. The longitudinal electric field is calculated in [8]

$$\varepsilon_s = -\frac{\partial \Phi}{\partial s} - \frac{1}{\gamma^2}\frac{\partial \Phi}{\partial z}, \tag{6}$$

γ is Lorentz factor. The total energy of the beam is constant along the closed orbit, but it is redistributed between potential and kinetic energies depending on beam size and beam-pipe radius (for $\gamma >> 1$), therefore the average energy of the electron is

$$E_0 = \bar{E}_{mech} + \frac{e\bar{\Phi}}{2} = const,$$

where $^-$ denotes averaging over the bunch. This energy determines the curvature radius in the bending magnets, the revolution and the spin precession frequencies[9]. It is measured by resonant depolarization technique. The mass of the produced system (in the case of resonance in rest) in electron-positron anihilation process is

$$Mc^2 = 2\left(\bar{E}_{mech} + e\bar{\Phi}\right) = 2E_0 + e\bar{\Phi}_{IP},$$

note that mechanical and binding energies are converted to the mass of the resonance. The resulted correction is

$$\Delta Mc^2 = e\bar{\Phi}_{IP} \sim \frac{e^2 N}{\sqrt{\pi}\sigma_z} \ln \frac{R_{IP}}{\sigma_{x,IP}}$$

It is about 1.2 keV for $N = 10^{10}$, the longitudinal beam size RMS is $\sigma_z = 3$ cm.

Influence of the incoming beam (longitudinal beam-beam effects).

The longitudinal force from incoming bunch is suppressed by $1/\gamma^2$ in the case of beam with parallel particles trajectories. Taking into account inclined trajectories in the beam the corresponding longitudinal electric field might be estimated from (6) or by approach in [10]. The latter is based on consideration of the transverse electric field projection on longitudinal axis. The value of this effect depends on beam length, beta-functions in IP, beam population and is about 50 eV for 10^{10} particles.

EXAMPLE OF CESR [11].

CESR is an example of accelerator with strong influence of the electrostatical beam separation on energy in IP. The radial beam separation with amplitude of 20 mm is done by two pairs of electrostatical separators giving a so-called pretzeled orbit. This separation results in energy shift with opposite sign for electrons and positrons $\Delta E/E = \pm\sum_i \psi_i\chi_i/\alpha\Pi \sim \pm 10^{-3}$ [6] and the shift of the invariant mass is quadratically small $\Delta M/M = -\Delta E^2/2E^2 \sim 5\cdot 10^{-7}$, where ψ_i is a dispersion at the position of the separator, χ_i is a deflecting angle of the separator, α is a momentum compaction factor, Π is a circumference.

CESR operates with beams crossing at angle of $\sim$ 3 mrad which brings a known error in definition of invariant mass of 15 keV for J/Ψ- meson. We want to take readers attention to another error of the same order of value. The vertical beam separation in parasitic IP together with the pretzeled orbit and skew quadrupole system for detector field influence compensation excites vertical dispersion of $\varphi_y(e^+) = -0.01$ mm for positrons and $\varphi_y(e^-) = 3$ mm [12] for electrons. Giving $\sigma_y = 4\cdot 10^{-3}$ mm— vertical beam size at IP, $\sigma_\delta = 7\cdot 10^{-4}$— energy RMS, $d_y = 0.048$ μm, the shift of the invariant mass is $|E_t - 2E_0| = 14$ keV while $\Delta L/L \sim 2\%$ calculated in assumption of equal vertical dispersion for electrons and positrons $\varphi_y(e^+) = \varphi_y(e^-) = 3$ mm.

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SUMMARY.

The basic errors and corrections in determination of the center mass energy in the high precision mass measurement experiments are discussed. The effects of chromaticity of optical functions in IP and influence of collective fields in the bunches are taken into account. Dispersion of different signs for electrons and positrons in IP is one of the most essential errors that demands the steady control of the colliding beams relative positioning.

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