STUDY OF THE DIFFUSION PROCESSES CAUSED BY THE BEAM-BEAM INTERACTIONS

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Abstract

We discuss diffusion caused by the beam-beam interactions. Slow diffusion has been studied to estimate beam life time. Fast diffusion, which affects luminosity, is discussed in this paper. Dynamics of colliding beam-beam system is strongly nonlinear and have many degree of freedom. In the case without linear coupling, a beam-beam limit [1] is caused by a distortion of beam distribution. In this case, nonlinear diffusion is very weak, but synchrotron radiation plays an important role. While linear coupling at the collision point, namely x-y coupling, dispersion and crossing angle, induces a diffusion, with the result that luminosity is degraded. Diffusion rate due to the linear coupling is estimated by a weak-strong simulation.

INTRODUCTION

We consider a physical system with a stochastic kick for the dynamical variable ($x(s)$) as

$$x(s+\epsilon) = x(s) + \Delta x(s)\eta(s), \tag{1}$$

where $\eta(s)$ is random variable which takes 0 or 1 for s. If the stochastic kick for x do not have correlation between s, the average of the kick is expressed by

$$\langle \Delta x(s)\Delta x(s')\eta(s)\eta(s')\rangle = B\delta(s-s'). \tag{2}$$

where for simplicity, B, which is called the diffusion coefficient, is assumed to be a constant.

For a physical system in which particles obey $x(s) = const$ except for the stochastic kick, we have so-called the diffusion equation for the particle distribution,

$$\frac{\partial}{\partial s}\Psi(x,s) = B\frac{\partial^2}{\partial x^2}\Psi(x,s) \tag{3}$$

For initial condition, $\Psi(x,0) = \delta(x-x_0)$, the solution is given as

$$\Psi = \frac{1}{\sqrt{4\pi Bs}}\exp\left[-\frac{(x-x_0)^2}{4Bs}\right]. \tag{4}$$

This equation means that Ψ is Gaussian and its rms value increase as $\sigma^2 = 2Bs$. When the initial distribution is Gaussian with rms value of σ_0, the rms value evolutes as

$$\sigma^2(s) = \sigma_0^2 + 2Bs. \tag{5}$$

When particles obeys an equation which represents damping of $x(s)$,

$$\frac{dx}{ds} = -Dx, \tag{6}$$

the diffusion equation is replaced by Fokker-Plank equation as

$$\frac{\partial}{\partial s}\Psi = Dx\frac{\partial}{\partial x}\Psi + B\frac{\partial^2}{\partial x^2}\Psi. \tag{7}$$

We obtain an equilibrium solution $\partial\Psi/\partial s = 0$,

$$\Psi = \exp\left[-\frac{x^2}{2B/D}\right]. \tag{8}$$

As is well-known, the equilibrium distribution does not depend on initial one.

The betatron motion is discarded in these discussions. In most of the circular accelerators, the betatron motion is much faster than the diffusion and damping, therefore the same discussion is applied by using the adiabatic invariant ($\sqrt{J_x}$) instead of x.

We consider a colliding two-beam system in a circular accelerator. Synchrotron radiation gives random diffusion due to the quantum excitation. The diffusion is represented by the above model.

As is well-known, nonlinear dynamical system in multi-dimension has a kind of diffusion nature. A time dependent one dimensional system gives chaotic behavior. Arnold diffusion occurs for more dimensional system.

If there are some diffusion mechanisms, which are independent each other, the total diffusion coefficient is summation of each diffusion coefficient ($B = \sum B_i$).

Many studies had been done for a halo formation with a weak-strong model [2, 3, 4]. They treated a slow diffusion which forms beam halo. As shown later, beam-beam system without linear coupling have little diffusion.

We discuss a fast diffusion of beam-beam system which affects the equilibrium beam size in this paper. The diffusion rate is evaluated by the evolution of the beam size using a weak-strong simulation. A strong beam in the weak-strong simulation was given by a distorted distribution determined by a strong-strong simulation [1] or Gaussian distribution with rms values determined by the emittances. The beam-beam force was calculated by the particle in cell method for the distorted distribution and the Gaussian distribution. An exact formula was also used for the Gaussian distribution.

We discuss the diffusion characteristics for head-on and crossing collisions in Sec.2 and 3, respectively.

HEAD-ON COLLISION

In the head-on collision, the beam-beam limit is caused by distortion of beam distribution [1]. The kurtosis of the distribution was $k_y = \langle y^4\rangle/3\langle y^2\rangle \approx 4$ in an operating point $(\nu_x,\nu_y) = (0.51, 0.58)$. The operating point, which

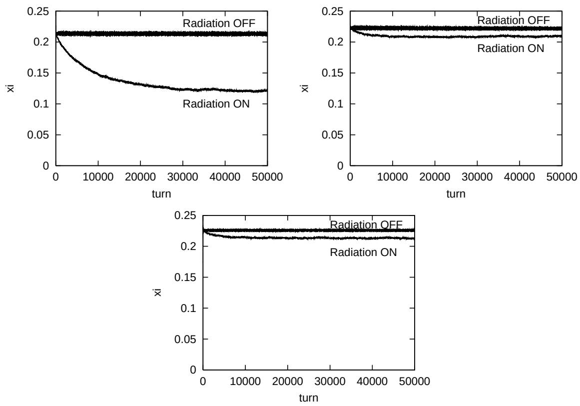

Figure 1: Evolution of the beam-beam parameter given by the two dimensional simulation. Two lines which correspond to ON/OFF of the synchrotron radiation are depicted. Picture (a) is obtained by PIC method for the distorted distribution. Pictures (b) and (c) are obtained by PIC method and exact error function formula, respectively, for Gaussian distribution.

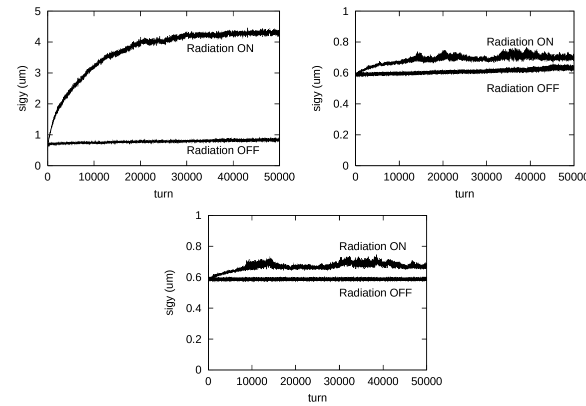

Figure 2: Evolution of the vertical beam size given by the two dimensional simulation. Two lines which correspond to ON/OFF of the synchrotron radiation are plotted. Picture (a) is obtained by PIC method for the distorted distribution. Pictures (b) and (c) are obtained by PIC method and exact error function formula, respectively, for Gaussian distribution.

was used in some machines, CESR, KEKB and PEP-II, gave the best luminosity performance in the tune space.

We first discuss the diffusion characteristics for the distorted beam ($k_y \sim 4$) and the Gaussian beam ($k_y = 1$) in the head-on collision. Figures 1 and 2 show evolution for the beam-beam parameter and vertical beam size, respectively. In each pictures, the evolutions in the case of including the damping and excitation of synchrotron radiation and of Hamiltonian system without the radiation are depicted. Three pictures in each figure are depicted for distorted distribution obtained by the strong-strong simulation, Gaussian distribution whose force are calculated by PIC method, and Gaussian whose force is calculated by exact formula with complex error function. For the beam-beam system without synchrotron radiation, luminosity and beam size are kept to be an initial value: namely, diffusion due to nonlinearity is weak. The behaviors in the case of including the radiation are different each other. Synchrotron radiation plays an important role for the beam-beam limit. Structure of the phase space for the distorted distribution is sensitive for the diffusion.

CROSSING ANGLE

In some strong-strong simulations [6], the luminosity is degraded in the collision with finite crossing angle. The reason why the crossing angle makes worse the luminosity is discussed. We focus the nonlinear diffusion caused by the collision with crossing angle. The weak-strong simulation with Gaussian model is used here. Needless to say, it is better to use the distribution obtained by the strong-strong simulation. A 3D PIC weak-strong code is under-construction. Figure 3 shows evolution of the beam-beam parameter for zero and finite crossing angles. Pictures (a) and (b) are depicted beam-beam parameter evolutions with/without synchrotron radiation damping, respectively, for tune operating point of (0.508,0.55). The operating point is closed to that of LER for the present KEKB. For zero crossing angle, diffusion due to nonlinearity is not seen as is discussed in previous section. For finite crossing angle, diffusion is clearly seen. Including the synchrotron radiation, the diffusion is emphasized further more. Pictures (c) and (d) are depicted for the tune operating point (0.518,0.58). The operating point is closed to that of HER for the present KEKB. The diffusion rate due to nonlinearity, which depends on the operating point, is worse than previous operating point. The beam-beam limit or total diffusion including synchrotron radiation is also worse than the previous point.

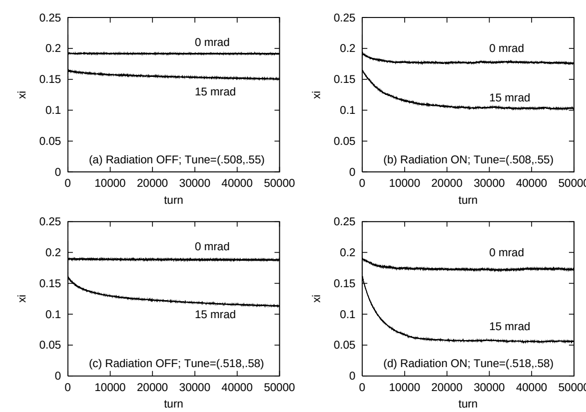

Figure 3: Diffusion due to crossing angle. Pictures (a) and (b) are depicted luminosity evolutions with/without synchrotron radiation damping, respectively, for tune operating point of (0.508,0.55). Pictures (c) and (d) are depicted with/without synchrotron radiation damping, respectively, for the tune operating point (0.518,0.58).

The crossing angle makes an occurrence of the nonlinear diffusion in both of vertical and horizontal beam sizes. Figure 4 shows the diffusion of the horizontal and vertical beam sizes due to the crossing angle. It is interesting that the crossing angle which causes linear $x - z$ coupling also affects the vertical diffusion. A slight diffusion is seen for zero crossing angle. It is not seen at all in two-dimensional simulation as shown in Figure 4 (c).

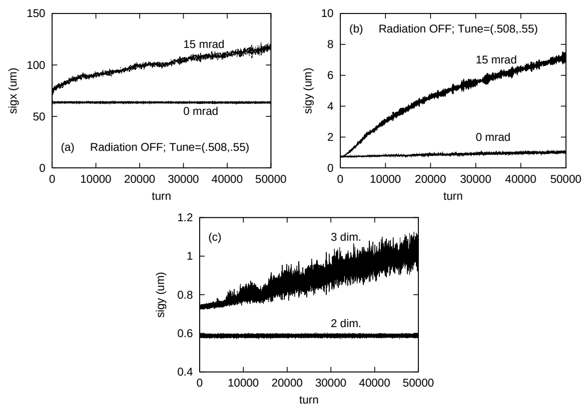

Figure 4: Diffusion of horizontal and vertical beam size due to crossing angle. Pictures (a) and (b) are depicted evolutions of horizontal and vertical beam sizes, respectively. Picture (c) is depicted diffusion seen in two and three dimensional simulations for zero-crossing angle and no synchrotron radiation.

X-Y COUPLING

Similar diffusion is also caused by $x-y$ coupling. Figure 5 shows that diffusion seen in the beam-beam parameter with $x - y$ coupling, $r_4 = 0.4$. Nonlinear diffusion seen in picture (a) shows an interesting feature. The strength of the diffusion is different between two and three dimensional simulation: i.e. it is week for 2 dim. but is clear for 3 dim. Number of dimension affects the diffusion.

The same calculation was done for weak vertical dispersion $\eta_y = 1$ mm. It showed a diffusion characteristics in the vertical beam size again.

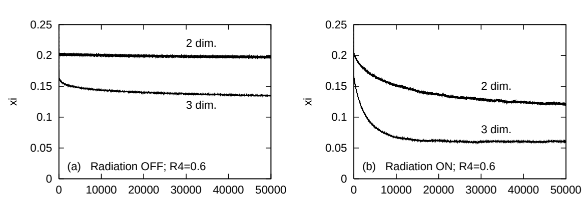

Figure 5: Diffusion of luminosity horizontal and vertical beam size due to crossing angle. Pictures (a) and (b) are depicted evolutions of the beam-beam parameter with and without synchrotron radiation, respectively. In each picture, two lines correspond to two and three dimensional simulations.

SUMMARY

We investigated diffusion rate using a weak-strong simulation. The strong beam is Gaussian or distorted distribution given by a strong-strong simulation. Particle in cell method and exact formula were used to estimate the beam-beam force. Clear diffusion was not observed in the case of collision with zero crossing angle for not only Gaussian strong beam but also distorted beam. The diffusion at zero crossing angle was enhanced by synchrotron radiation strongly for the distorted distribution.

Diffusion caused by linear coupling ($x - y$, $x - z$ and $y - z$) at the collision point is estimated by a weak-strong simulation with Gaussian strong beam. Linear coupling induced a monotonically increase of the vertical beam size and luminosity degradation. We are considering that the increase of the beam size is caused by Arnold diffusion.

It is important to understand the beam-beam limit in hadron colliders. If the beam-beam limit in proton colliders is determined by the diffusion, it is essential to include errors, crossing angle and longitudinal dimension in the simulations.

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