THE COD INVESTIGATION OF THE EXPERIMENTAL RING CSRe

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Abstract

A new multipurpose experimental ring (CSRe) is being constructed in Lanzhou. It is a heavy ion storage ring used for internal-target experiments or high precision spectroscopy with electron-cooling. In order to obtain large acceptance the closed orbit distortion (COD) of storage ring should be considered. The global COD caused by misalignments and field errors of all magnets in the CSRe will be discussed.

1 INTRODUCTION

The experimental ring CSRe shown in Figure 1 is one storage ring of the HIRFL-CSR complex[1]. HIRFL-CSR, a new accelerator project at the Heavy Ion Research Facility in Lanzhou (HIRFL), consists of a main ring (CSRm) and an experimental ring (CSRe). The heavy ion beams from the HIRFL will be accumulated, cooled and accelerated in the CSRm, and then extracted to produce radioactive ion beams (RIB) or highly charged heavy ions. The secondary beams can be accepted and stored by the CSRe for internal-target experiments or high precision spectroscopy with e-cooling.

2 LATTICE

CSRe is not only an experimental ring with internal target but also a high-resolution spectrometer. In order to meet different experiments needs three lattice modes will be adopt. First one is the internal-target mode with small $\beta$-function in the target point and large transverse acceptance. The second one is the high-resolution mode used for mass spectrometer with large momentum acceptance. The third one is the isochronous mode for the mass measurements of short lifetime nuclei with Time-of-Flight method.

In order to investigate the closed orbit distortion (COD) of CSRe, only the internal-target lattice mode is discussed. The layout of the CSRe is shown in Figure 1. It is a racetrack shape and consists of two quasi-symmetric parts. One is the internal target part and another is the e-cooler part. Each part is a symmetric system and consists of two identical arc sections. Each arc section consists of four dipoles, two triplets or one triplet and one doublet. 11 independent variables for quadruple are used in the CSRe. Table 1 is the lattice parameters of the CSRe for the internal-target mode, and Figure 2 is the distribution of $\beta$-functions and dispersion for this mode.

Table 1: Lattice parameters of the internal-target mode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference (m)</td>
<td>128.80</td>
</tr>
<tr>
<td>$Bp_{\max}$ (Tm)</td>
<td>8.4</td>
</tr>
<tr>
<td>Transition gamma</td>
<td>$\gamma_0 = 2.457$</td>
</tr>
<tr>
<td>Betatron tune values</td>
<td>$Q_x / Q_y = 2.53 / 2.57$</td>
</tr>
<tr>
<td>Natural chromaticity</td>
<td>$Q'_x / Q'_y = -3.70 / -3.55$</td>
</tr>
<tr>
<td>Max. $\beta$-Amplitude</td>
<td>$\beta_x / \beta_y = 25.7/8.7$ m (Dipole) $\beta_x / \beta_y = 43.0/20.4$ m (Quadruple)</td>
</tr>
<tr>
<td>Max. Dispersion</td>
<td>$D_{\max}(x)=7.9$ m (Dipole, $\beta_x=14$m) $D_{\max}(x)=9.4$ m (Quadruple)</td>
</tr>
<tr>
<td>Injection section</td>
<td>$\beta_x = 30.8$ m, $D_x = 0$ m (Septum) $\beta_x = 31.4$ m, $D_x = 0$ m (Quadruple.)</td>
</tr>
<tr>
<td>E-cooler section</td>
<td>$\beta_x / \beta_y = 12.9/16.5$ m, $D_x = 0$</td>
</tr>
</tbody>
</table>

Figure 1: The layout of the CSRe

Figure 2 The distribution of the $\beta$-functions and the dispersion for the internal-target mode.
3 THE COD THEORY OF STORAGE RING

3.1 Derivation of the general COD formula

Considering the magnetic field errors, the Hill equation[2] of the particle motion is the following form,
\[ \frac{d^2y}{ds^2} + K(s)y = \frac{1}{\rho B} \Delta B = F(S) \]  
(1)
where \( Y \) is the horizontal or vertical displacement (x or y) from the ideal equilibrium orbit, \( K(s) \) represents the focusing function \( K_x(s) \) or \( K_y(s) \). \( K(s) \) satisfies the periodicity relation \( K(s+L) = K(s+C) = K(s) \), \( L \) and \( C \) are the periodicity length and circumference of the equilibrium orbit respectively. \( F(s) = k \Delta B / B \) is a measure of the deviation of the field on the ideal orbit from its ideal value \( B \), and \( \rho \) is the radius of curvature of the equilibrium orbit respectively. \( F(s) = k \Delta B / B \) is a measure of the deviation of the field on the ideal orbit from its ideal value \( B \).

\[ 3.1 \text{ Derivation of the general COD formula} \]

The solutions of the homogeneous equation of Eq. (1) are
\[ y(s) = e^{Qs} \phi(s) \]  
(2)
where
\[ \phi(s) = \int ds / \rho B(s) \]  
(3)
and
\[ Q = \int \frac{ds}{\rho B(s)} = \frac{N\mu}{2\pi} \]  
(4)
Introduce a new variable \( V = \beta^2 Y \)  
(5)
Eq. (1) becomes
\[ \frac{d^2V}{dF^2} + Q^2 V = Q^2 f(\phi) \]  
(6)
where
\[ f(\phi) = \beta^2 F(s) = \beta^2 \frac{\Delta B}{B} \]  
(7)
By using the above transform, the forced oscillations of the Hill equation (1) can be reduced to the forced oscillations of a harmonic oscillator. The periodic solution of Eq. (6) is
\[ V(\phi) = \frac{Q^2}{2\sin \pi Q} \left[ f(\phi) \cos Q(\pi + \phi - \phi_0) d\phi \right] \]  
(8)
From Eq.(8), the square of the amplitude of \( V(\phi) \) is
\[ V^2(\phi) = \frac{Q^2}{4\sin^2 \pi Q} \left[ f(\phi) f(\phi_0) \cos Q(\phi_0 - \phi_0) d\phi d\phi_0 \right] \]  
(9)
and its expectation value is
\[ \langle V^2(\phi) \rangle = \frac{Q^2}{4\sin^2 \pi Q} \left[ f(\phi_0) f(\phi_0_0) \cos Q(\phi_0_0 - \phi_0_0) d\phi_0 d\phi_0_0 \right] \]  
(10)
In practice the perturbing function \( F(s) \) in detail can’t be know, but some of its statistical characteristics are known. The distribution of \( \Delta B / B \) in all magnets follows the statistical rule, and those errors were actually caused by misalignments or manufacture of magnets.

The errors in different magnets are uncorrelated, if \( \phi_1 \) and \( \phi_2 \) do not locate in the same magnet, then
\[ \langle f(\phi_1) f(\phi_2) \rangle = 0 \]  
(11)
For the magnet numbered \( i \), its length is \( L_i \), and \( L_i \) is small compared to the wave length(\( \beta \)) of betatron oscillations. When \( \phi_i \) and \( \phi_2 \) are in the same magnet,
\[ \cos Q(\phi_i - \phi_2) = 0 \]  
(12)
and
\[ \Delta \phi_i(s) = \int^s_0 d\phi / \rho B(s) = \frac{L_i}{\beta \rho} \]  
(13)
then
\[ \langle V^2(\phi) \rangle = \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( \frac{\Delta B}{B} \right)^2 \]  
(14)
From Eq. (5) and Eq. (14), we can get the mean square of the closed orbit distortion (COD) in the magnet numbered \( j \),
\[ \langle \text{Y_{cod}}^2 \rangle_j = \beta_j^2 \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( \frac{\Delta B}{B} \right)^2 \]  
(15)
3.2 The most important errors in storage ring

For storage ring, there are many errors by manufacture and misalignment, but only those three kinds of errors are most serious. One is the dipole field error \( \Delta B_x \), caused by manufacture, this error will result in a horizontal COD,
\[ \langle x_{i,\beta}^2 \rangle = \langle x_{i,\beta}^2 \rangle - \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( \frac{\Delta B}{B} \right)^2 \]  
(16)
and the vertical COD
\[ \langle y_{i,\beta}^2 \rangle = \langle y_{i,\beta}^2 \rangle - \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( \frac{\Delta B}{B} \right)^2 \]  
(17)
Then, \( \Delta \psi_D \) will cause a vertical COD
\[ \langle x_{i,\beta}^2 \rangle = \langle x_{i,\beta}^2 \rangle - \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( \frac{\Delta B}{B} \right)^2 \]  
(18)
and the third one is the transverse displacement \( \Delta Y_q \) (\( \Delta x_q \)) of quadruple. The displacement \( \Delta Y_q \) will cause one dipole field error \( \Delta B_x \), namely,
\[ \Delta B_x = B_p K_x \Delta Y_q \]  
(19)
Thus the COD caused by \( \Delta Y_q \) will be
\[ \langle x_{i,\beta}^2 \rangle = \langle x_{i,\beta}^2 \rangle - \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( K_x \Delta Y_q \right)^2 \]  
(20)
and
\[ \langle y_{i,\beta}^2 \rangle = \langle y_{i,\beta}^2 \rangle - \frac{1}{4\sin^2 \pi Q} \sum \beta_i^2 \left( K_x \Delta Y_q \right)^2 \]  
(21)
According to Eq.(16), Eq.(18), Eq.(20) and Eq.(21), the mean expectation value of COD for a ring can be estimated as the following,
the horizontal COD \( x_i = \sqrt{\langle x_{i,\beta}^2 \rangle + \langle x_{i,\beta}^2 \rangle} \)  
(22)
the vertical COD \( y_i = \sqrt{\langle y_{i,\beta}^2 \rangle + \langle y_{i,\beta}^2 \rangle} \)  
(23)
by project will be less than $\Delta B / B = \pm 5 \times 10^{-4}$, and $\Delta y_q = 0.15$ mm, $\Delta \psi_q = 0.5$ mrad.

For CSRe

* the number of dipole $N_D = 16$,
* the arc length of dipole $L_D = 2.3562$ m,
* the bending radius of dipole $\rho_D = 6.0$ m,
* the number of quadruple $N_Q = 22$,
* the average length of quadruple $L_Q = 0.65$ m,
* the average gradient of quadruple $K_Q = 0.5$ T/m.

According to the internal-target lattice mode of the CSRe,

* the tune value of the machine $Q_x / Q_y = 2.53 / 2.57$,
* the max. $\beta_x$ and $\beta_y$ values in dipole and quadruple are listed in Table 1,
* the average $\beta_x$ in all dipoles $\bar{\beta}_x = 13$ m,
* the average $\beta_x$ in all quadruples $\bar{\beta}_x = 12$ m.

Then the formulae (16), (18), (20) and (21) can be simplified as following,

$$\Delta \approx \frac{B}{B} D x \bar{\beta}_x c N \left( \frac{L}{P} \right)^2 N_x \beta_x \sin \frac{\psi}{\beta_x} \frac{\pi}{2} \left( \frac{\Delta}{\beta_x} \right)$$

$$\Delta \approx \frac{B}{B} D y \bar{\beta}_y c N \left( \frac{L}{P} \right)^2 N_y \beta_y \sin \frac{\psi}{\beta_y} \frac{\pi}{2} \left( \frac{\Delta}{\beta_y} \right)$$

According to the above four formulae and (22), (23), we can obtain the estimated COD of the mean expectation value for the CSRe as shown in Table 2.

Table 2: Estimated COD of the mean expectation value

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_{x_{1}}$</td>
<td>$x_1$</td>
</tr>
<tr>
<td>Dipole</td>
<td>14</td>
<td>5.3</td>
</tr>
<tr>
<td>Quadruple</td>
<td>43</td>
<td>9.4</td>
</tr>
</tbody>
</table>

5 SIMULATION RESULTS OF COD

For the alignment of magnet, 6 errors should be taken into account. Namely 3 position errors ($\Delta x, \Delta y, \Delta z$) and 3 angle errors ($\Delta \phi, \Delta \theta, \Delta \psi$). As shown in Fig. 3.

Figure 3 The local reference coordinate system.

$\phi$ is rotation angle around the x-axis, $\theta$ is rotation angle around the y-axis, and $\psi$ is rotation angle around the z-axis. All rotations obey the right-hand screw rule with the axes. For CSRe project the required misalignments are listed in Table 3.

Table 3: The magnet misalignments of the CSRe

<table>
<thead>
<tr>
<th>Error</th>
<th>B (mm)</th>
<th>Q (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta X$</td>
<td>0.5</td>
<td>0.15</td>
</tr>
<tr>
<td>$\Delta Y$</td>
<td>0.5</td>
<td>0.15</td>
</tr>
<tr>
<td>$\Delta Z$</td>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Delta \phi$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Delta \theta$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Delta \psi$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Assuming that the distribution of every error is a $\pm 2\sigma$ Gauss distribution, and the random seed number is 40. Then we can do the COD simulation of the CSRe by means of the lattice calculations with errors. The simulation results are shown in Fig. 4.

6 CONCLUSION

According to the estimation and simulation, we can get the result that the simulated max. COD is nearly $3 / 2$ times of the estimated expectation value. Thus defining the aperture of vacuum chamber, the following COD should be considered,

- $x_{COD} (D) = 10$ mm, $y_{COD} (D) = 7$ mm,
- $x_{COD} (Q) = 15$ mm, $y_{COD} (Q) = 10$ mm.

For any two points in storage ring the relation of COD and $\beta$-function can be obtained from Eq. (15),

$$\text{COD}_1 / \text{COD}_2 = (\beta_1 / \beta_2)^{1/2}$$

Example, in the entry of the e-cooler section, the $\beta_y = 18$ m, comparing to the quadruple with max. $\beta$-function, the $y_{COD}$ value of the entry can be calculated from Eq. (25), namely $y_{COD} = 10$ mm.

REFERENCE
