NONLINEAR DYNAMICS STUDY OF STORAGE RING WITH SUPER-PERIODS *

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Abstract

Many modern light source storage rings use a magnetic structure consisting of a number of repetitive lattice cells, the super-periods. The study of one super-period, the smallest unit of a periodic magnetic lattice, can reveal the dynamical proprieties of the storage ring with more clarity and with a much reduced computational effort. In this work, using particle tracking and frequency analysis, beam dynamics of a typical triple-bend achromat, the ALS lattice, have been studied. The dynamic aperture scaling with the sextupole strength is confirmed numerically to yield additional insight. Extending the ALS lattice to different numbers of super-periods, we have found a simple scaling relationship between the dynamic aperture and the number of super-periods. This scaling relationship allows us to compare dynamics performance of different lattices from 2nd and 3rd generation light source storage rings.

INTRODUCTION

The study of nonlinear beam dynamics is critical in designing a storage ring based synchrotron radiation light source. The dynamic aperture (DA), a stable region of particle motion in phase space, is usually used to evaluate the nonlinear dynamics performance of a storage ring. A large dynamic aperture is required to achieve a high injection efficiency and a good beam lifetime. From current understanding of nonlinear dynamics, the dynamic aperture is mainly determined by two factors: (1) the tune shift with amplitude, which describes the change of betatron oscillation frequency of particles as their oscillation amplitudes vary; (2) the overlap of resonance layers [1], which separates the stable region of beam dynamics from the unstable region. Due to the difficulties of calculating the dynamic aperture analytically, the dynamic aperture is usually computed by tracking a set of particles in the storage ring over a large number of turns and observing survival particles.

Modern storage ring based light sources are usually designed with a number of identical magnetic lattice cells referred to as super-periods (SPs). In some literature [2], it is believed that nonlinear dynamics performance of a storage ring can be improved with a higher degree of periodicity to suppress nonlinear resonance terms. However, since the super-period is the smallest repetitive structure of a storage ring, in absence of errors which could break the periodicity, beam dynamics characteristics of a storage ring with $N_{\rm SP}$ super-periods should be completely determined by features of one super-period. Dynamics analysis based upon one super-period has the following benefits: (1) it can reduce the computational effort of particle tracking; (2) it can avoid resonance folding in analyzing one-turn tracking data of a storage ring lattice with multiple super-periods.

One of challenges for designing a low-emittance 3rd generation light source storage ring is its smaller dynamic aperture compared with the 2nd generation light source storage ring. Traditionally, the small dynamic aperture of a 3rd generation light source storage ring is directly attributed to the use of strong linear focusing which in terms requires the use of strong sextupoles to compensate the large natural chromaticity. However, the requirement of a small beam emittance, therefore, a small η function, in 3rd generation light source rings also results in the need of strong sextupoles for compensation of chromatic effects. In both scenarios, the dynamic aperture reduction of the low-emittance 3rd generation light source storage ring is the result of using strong sextupoles.

In late 1980s, it was recognized using an analytic argument that for a storage ring with only sextupole magnets, the dynamic aperture should scale inversely with the square of the sextupole strength [3]. Emery used the FODO lattice as an example to study this scaling relationship [3, 4].

In this paper, the relationship between the dynamic aperture and sextupole strength is studied using particle tracking and Frequency Map Analysis techniques. This study is carried out using one super-period of a well-known triple-bend achromat (TBA) lattice, the Advance Light Source (ALS) lattice. We have first explored the dynamic aperture scaling with the sextupole strength. Extending the ALS lattice to different numbers of super-periods while keeping the linear focusing mostly unchanged, we have found a simple scaling relationship between the dynamic aperture and the number of super-periods. This relationship confirms that with a fixed linear focusing, the dynamic aperture reduction occurs as the strength of sextupoles increases as needed for chromatic compensation in a low-emittance lattice.

DYNAMIC APERTURE SCALING WITH SEXTUPOLE STRENGTH

Let us consider a time-dependent system containing distributed sextupoles, its Hamiltonian can be written as

$$H(q,p;s) = \frac{p^2}{2} + \frac{k(s)}{2}q^2 + \frac{1}{3}S(s)q^3, \qquad (1)$$

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where (q, p) is a pair of canonical variables, $k(s)$ and $S(s)$ are the strengths of location-dependent quadrupoles and sextupoles, respectively. Using the following canonical transformation,

$$q \to \bar{q} = \lambda q, \quad p \to \bar{p} = \lambda p, \tag{2}$$
$$H \to K = \lambda^2 H, \tag{3}$$

and the scaled sextupole strength $\bar{S}(s) = S(s)/\lambda$, the new Hamiltonian can be expressed as

$$K(\bar{q}, \bar{p}; s) = \frac{\bar{p}^2}{2} + \frac{k(s)}{2}\bar{q}^2 + \frac{1}{3}\bar{S}(s)\bar{q}^3. \tag{4}$$

In Eq. 4, the new dynamic variables $(\bar{q}, \bar{p})$ scale linearly with original variables (q, p), and the form of the new Hamiltonian $K(\bar{q}, \bar{p}; k, \bar{S}; s)$ is the same as the old Hamiltonian $H(q, p; k, S; s)$. It indicates that the characteristics of the particle motion of $H(q, p; k_0, S_0; s)$ in phase space is the same as that of $K(\bar{q}, \bar{p}; k_0, S_0; s)$ with the same quadrupole and sextupole strengths, k_0 and S_0. By comparing these two Hamiltonians, the ratio of the dynamic aperture between systems H and K is $1/\lambda^2$, therefore the dynamic aperture of system $H(q, p; k, S; s)$ is inversely proportional to the square of the sextupole strength.

To check the scaling relation between the dynamic aperture and sextupole strength, the magnetic lattice of the ALS, a very successful 3rd generation light source storage ring, is used as an example. The ALS lattice is composed of 12 TBA super-periods with a horizontal natural emittance of 5.5 nmrad at 1.9 GeV. Two families of sextupoles, SF (focusing) and SD (defocusing), are used for chromaticity compensation. First, we define the dynamic aperture scaling coefficients with the focusing sextupole strength,

$$A_{x,y} = \alpha_{x,y}/S_{\mathrm{F}}^2, \tag{5}$$

where $A_{x,y}$, the dynamic apertures in the phase space, are defined as $A_x = x_{\max}^2/\beta_x$, $A_y = y_{\max}^2/\beta_y$, $\beta_{x,y}$ are the β functions at the observation locations and S_{F} is the strength of the focusing sextupole SF. A sextupole strength vector for zero-chromaticity compensation is defined as $\vec{S_0} = (S_{\mathrm{F}}, S_{\mathrm{D}})_{\xi=0}$. Using particle tracking, the dynamic aperture of the ALS lattice for on-momentum particles is computed for different sextupole strength settings, $\lambda\vec{S_0}$, by varying the scaling coefficient λ from 0.5 to 5 (see Fig. 1). From Fig. 1, it is obvious that the dynamic aperture scales linearly with the sextupole strength, confirming the analytical result. Using a linear fit, the slopes of two curves are

$$\alpha_x = 0.123, \quad \alpha_y = 0.103. \tag{6}$$

These two coefficients are essentially determined by linear and nonlinear configurations of the super-period lattice. $\alpha_{x,y}$ can be used to compare the nonlinear dynamics performance of the same types of lattices, e.g. different DBA lattices or different TBA lattices.

By studying the tune shift with amplitude properties of the ALS lattice shown in Fig. 2, $\alpha_{x,y}$ are found to be related to the stable region boundary in the tune space. It

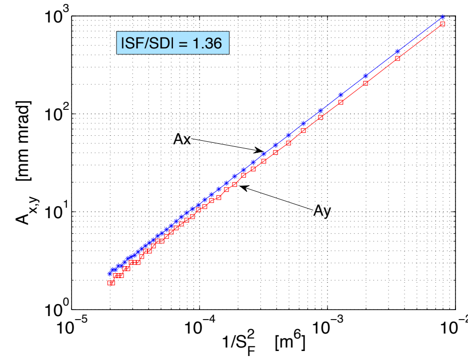

Figure 1: The horizontal and vertical dynamic apertures of the ALS lattice, $A_{x,y}$, are plotted as a function of the strength of the focusing sextupole, S_F. The sextupole strength vector $\vec{S_0} = \lambda(S_{\mathrm{F}}, S_{\mathrm{D}})$ is changed by varying the scaling coefficient λ.

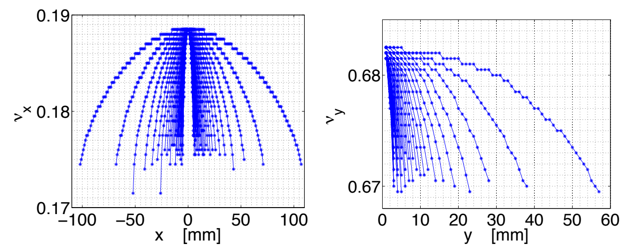

Figure 2: Tune shift with amplitude relationship of the ALS lattice for different sextupole strength settings $\lambda\vec{S_0}$ where λ is from 0.5 to 5. Tunes are computed from the tracking data of one super-period. The horizontal or vertical tunes (ν_x, ν_y) are plotted as a function of displacement (x, y). Different lines correspond to different values of λ.

shows that for different sextupole strengths, the stable region boundaries of the particle motion in the tune space are nearly the same – for ν_x, the stable region boundary is around 0.175; for ν_y, it is around 0.670. It is also noticed that the tune shift with amplitude is dominated by a term proportional to the square of the sextupole strength, $\mathrm{d}\nu_{x,y}/\mathrm{d}J \propto S^2$. Therefore, for the ALS lattice, the dynamic aperture scaling with the sextupole strength can be fully explained by the tune shift with amplitude with a fixed dynamics stability boundary in the tune space, independent of the sextupole strength.

SCALING BETWEEN DYNAMIC APERTURE AND N_{SP}

One commonly used method to minimize the emittance of a storage ring is the use of small bending-angle dipoles. This leads to the need for use a large number of super-periods in the storage ring. From the discussion in the Introduction section, this will cause the reduction of the dynamic aperture. To study the relationship between the dynamic aperture and the number of super-period, N_{SP}, a

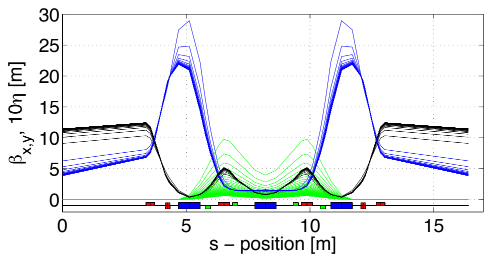

Figure 3: β and η functions of one super-period of ALS-like lattices with different numbers of super-periods from 3 to 50. Black lines are β_x, blue lines are β_y, and green lines are η functions.

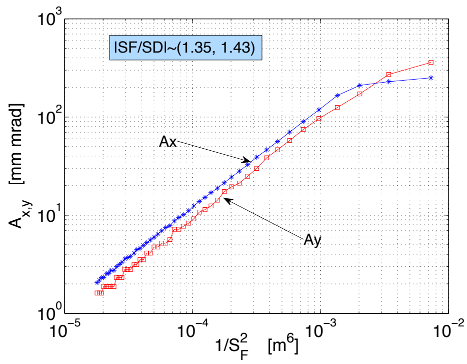

Figure 4: $A_{x,y}$ vs. $1/S_{\mathrm{F}}^2$ for ALS-like lattices with different N_{SP} from 3 to 50. For each lattice, the sextupole strengths are chosen for zero-chromaticity. The ratios between S_{F} and S_{D} are not exactly the same due to small variations of linear lattice functions. Because of large variations of β functions when N_{SP} is small, the linear relation does not extend to cases of a small N_{SP}, i.e. the top-right region in the plot.

set of new lattices with different N_{SP} from 3 to 50 are created based upon the ALS lattice by changing the bending angle of the dipoles. These lattices have the same phase advances, $\nu_{x,y}$, per super-period, and nearly the same β functions as the ALS lattice. β and η functions of these lattices are shown in Fig. 3. It is easy to see that the η function is inversely proportional to N_{SP}. Consequently, the horizontal emittance ϵ_x and zero-chromaticity sextupole strength $\vec{S}_0$ have the following simple scaling relationships,

$$\epsilon_x \propto 1/N_{\mathrm{SP}}^3, \quad \vec{S}_0 \propto N_{\mathrm{SP}}. \tag{7}$$

The dynamic aperture for lattices with different numbers of super-periods is then computed using on-momentum particle tracking for the zero-chromaticity sextupole settings $\vec{S}_0$ and plotted in Fig. 4. It is clear that the dynamic aperture of these lattices scales linearly with $1/S_{\mathrm{F}}^2$, and the corresponding scaling coefficients are

$$\alpha_x = 0.123, \quad \alpha_y = 0.101. \tag{8}$$

These coefficients are nearly the same as those of the ALS lattice. Therefore, these ALS-like lattices with different N_{SP} but with similar linear focusing have the similar nonlinear properties as the ALS lattice. Furthermore, from Eq. 7, the dynamic aperture can be related to N_{SP} as

$$A_{x,y} \quad \propto \quad 1/N_{\mathrm{SP}}^2. \tag{9}$$

Using Eq. 9, dynamics performance of similar lattices can be compared by scaling storage rings to the similar circumference by varying N_{SP} while keeping the linear focusing unchanged.

SUMMARY AND DISCUSSION

In this paper, we have explored a new method to study charged particle nonlinear dynamics using a super-period lattice. With the triple-bend achromat ALS lattice as an example, the dynamic aperture scaling with the sextupole strength, a known analytic result, has been checked using particle tracking. The simulation study reveals that the dynamic aperture of the ALS lattice is mainly limited by the tune-shift with amplitude. Among the ALS-like lattices, the dynamic aperture is found to scale with the number of super-periods as $A_{x,y} \propto N_{\mathrm{SP}}^{-2}$.

Dynamic aperture scaling with the sextupole strength and the number of super-periods has also been confirmed using a double-bend achromat lattice, a version of NSLS-II lattice [5]. The dynamic aperture of the NSLS-II is found to be well optimized using chromatic and harmonic sextupoles.

For future studies, we will focus our dynamics studies in three areas. First, we will compare nonlinear dynamics performance of 2nd and 3rd generation light source rings. Second, we would like to understand the mechanism of the dynamic aperture degradation for off-momentum particles. Third, we will explore the dependency of the dynamic aperture on chromaticity when the strength of individual sextupoles in the sextupole vector is altered.

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