

Computation of Eigenmodes in Long and Complex Accelerating Structures by Means of Concatenation Strategies

Thomas Flisgen, Johann Heller and Ursula van Rienen

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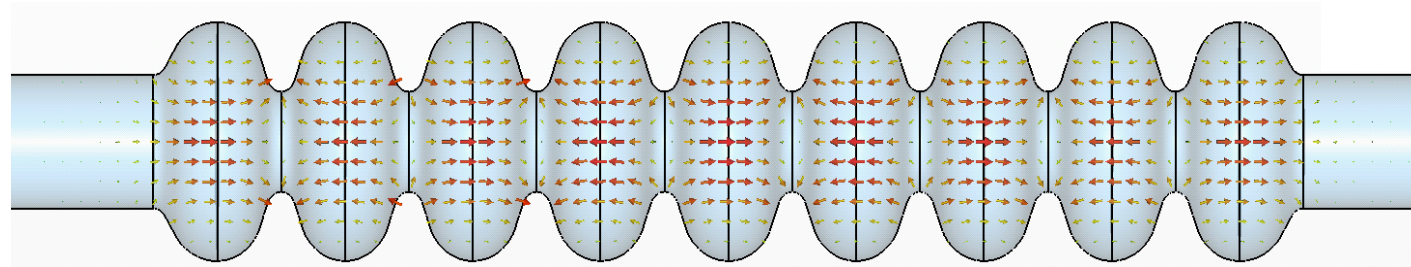
Outline

- Introduction, motivation and fundamental terms
- Challenges in computation of eigenmodes and R/Q parameters and Q_{ext} factors in long cavity chains
- Actual approach: State-Space Concatenations
- Proof of principle: analysis of multi-cavity monopole modes in chains of third harmonic cavities
- Conclusions and outlook

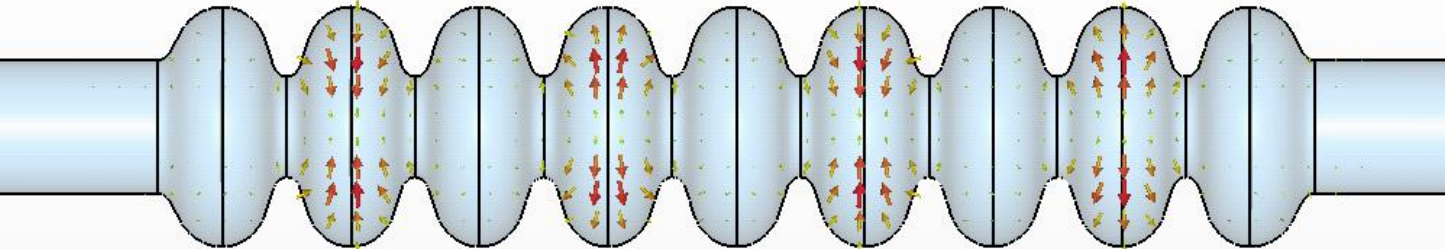
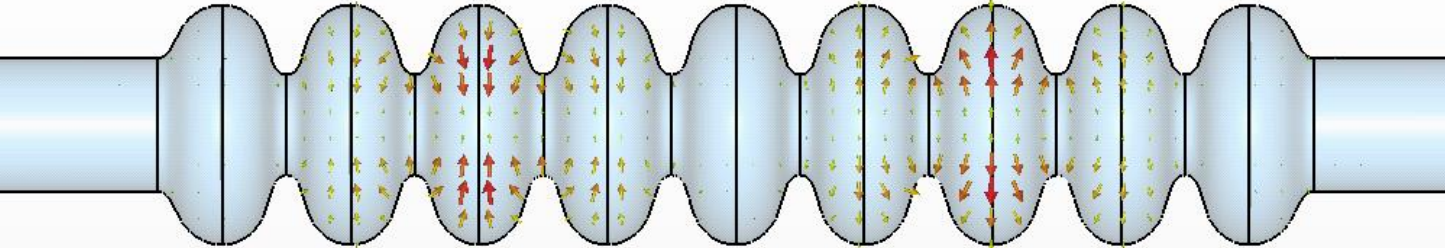
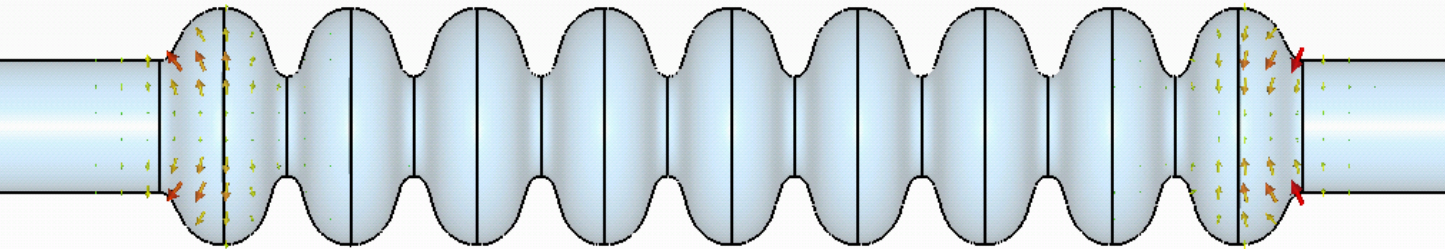


Introduction, Motivation and a few Fundamental Terms

Π -Mode and other Eigenmodes of a Cavity



Π -mode used for
particle acceleration



three arbitrarily
chosen Higher Order
Modes (HOMs)

Two Quantities* of Interest for Modal Analysis

$$Q_{\text{ext},\nu} = \frac{\tilde{\omega}_\nu W_{\text{stored}}}{P_{\text{ports},\nu}}$$

factor describes the energy loss through ports (Q_{ext} factor should be large for Π -mode and small for other modes)

$$\frac{R_\nu}{Q_{\text{beam},\nu}} = \frac{|V_\nu|^2}{\tilde{\omega}_\nu W_{\text{stored},\nu}}$$

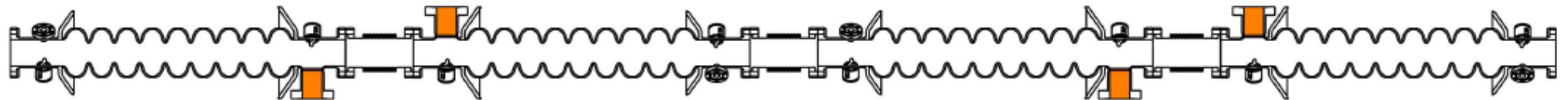
factor describes the interaction between modes and beam and vice versa (should be large for Π -mode and small for other modes)

$$V_\nu = \int_0^L \tilde{E}_{\nu,z}(0, 0, z) e^{j\tilde{\omega}_\nu z/c} dz$$

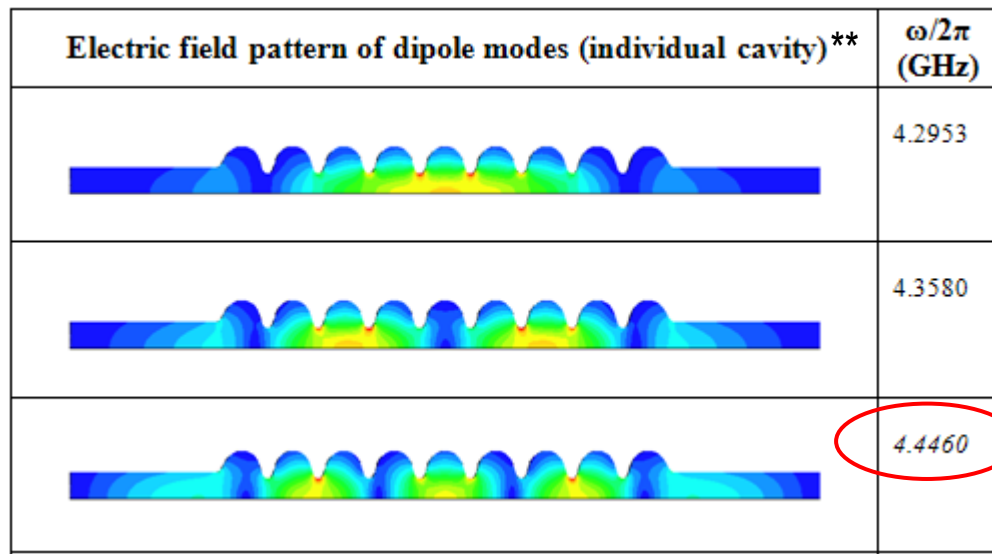
*amongst other

Challenges in Computation of Eigenmodes, R/Q Parameters and Q_{ext} and in Long Chains

String of Cavities in ACC39 @ FLASH Beamline



String of cavities in ACC39 mounted in FLASH*



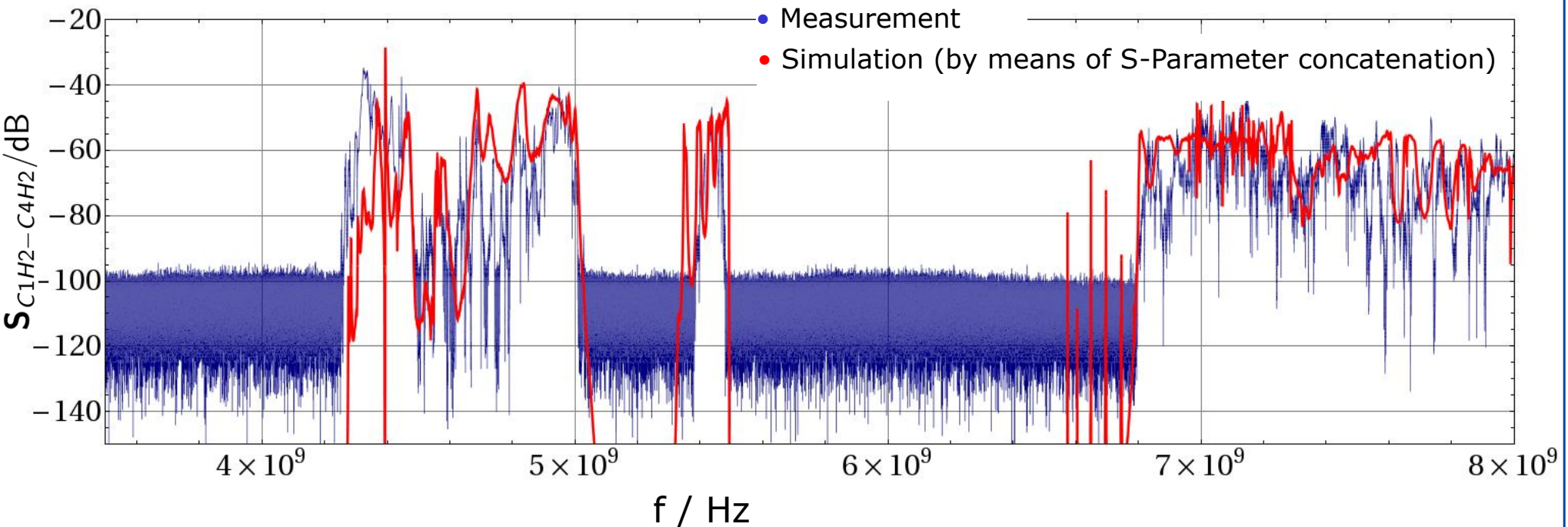
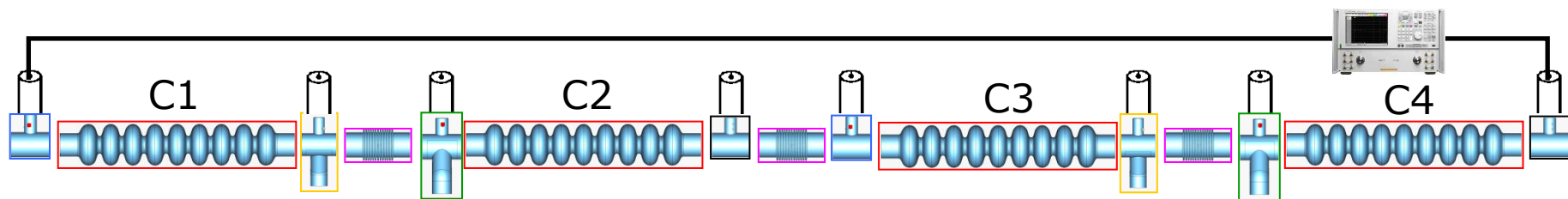
Cutoff frequencies of beam pipes:

| | | |
|----------|--------|-----------------------|
| 1. TE11 | Pol. 1 | $f_{co} = 4.3920$ GHz |
| 2. TE11 | Pol. 2 | $f_{co} = 4.3920$ GHz |
| 3. TM01 | | $f_{co} = 5.7371$ GHz |
| 4. TE21 | Pol. 1 | $f_{co} = 7.2858$ GHz |
| 5. TE21 | Pol. 2 | $f_{co} = 7.2858$ GHz |
| 6. TE01 | | $f_{co} = 9.1412$ GHz |
| 7. TM11 | Pol. 1 | $f_{co} = 9.1412$ GHz |
| 8. TM11 | Pol. 2 | $f_{co} = 9.1412$ GHz |
| 9. TE31 | Pol. 1 | $f_{co} = 10.022$ GHz |
| 10. TE31 | Pol. 2 | $f_{co} = 10.022$ GHz |

*Picture courtesy E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008.

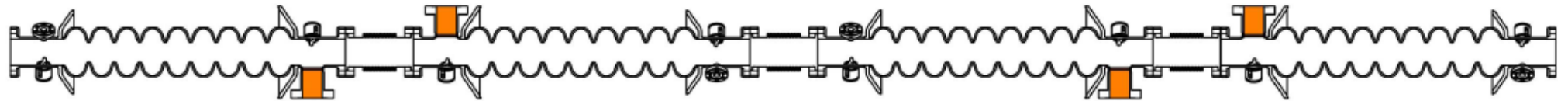
**I. R. R. Shinton, N. Juntong, R. M. Jones: "Modal Dictionary of Cavity Modes for the Third Harmonic XFEL/FLASH Cavities", DESY note: DESY 12-053.

Previous Results: Transmission via ACC39 String

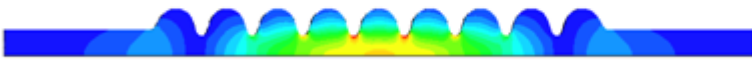
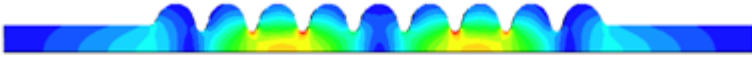
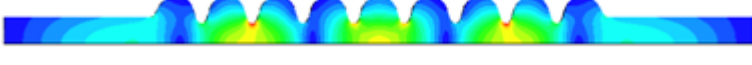


T. Flisgen, H.-W. Glock, P. Zhang, I. R. R. Shinton, N. Baboi, R. M. Jones, and U. van Rienen: "Scattering parameters of the 3.9 GHz accelerating module in a free-electron laser linac: A rigorous comparison between simulations and measurements", Phys. Rev. ST Accel. Beams, 17:022003, February 2014

Eigenmodes of Cavities String in ACC39



String of cavities in ACC39 mounted in FLASH*

| Electric field pattern of dipole modes (individual cavity)** | $\omega/2\pi$ (GHz) |
|---|------------------------|
|  | 4.2953 |
|  | 4.3580 |
|  | 4.4460 |

Cutoff frequencies of beam pipes:

| | | |
|----------|--------|-----------------------|
| 1. TE11 | Pol. 1 | $f_{co} = 4.3920$ GHz |
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- Eigenmodes are determined by entire string
- Computation of eigenmodes is expensive

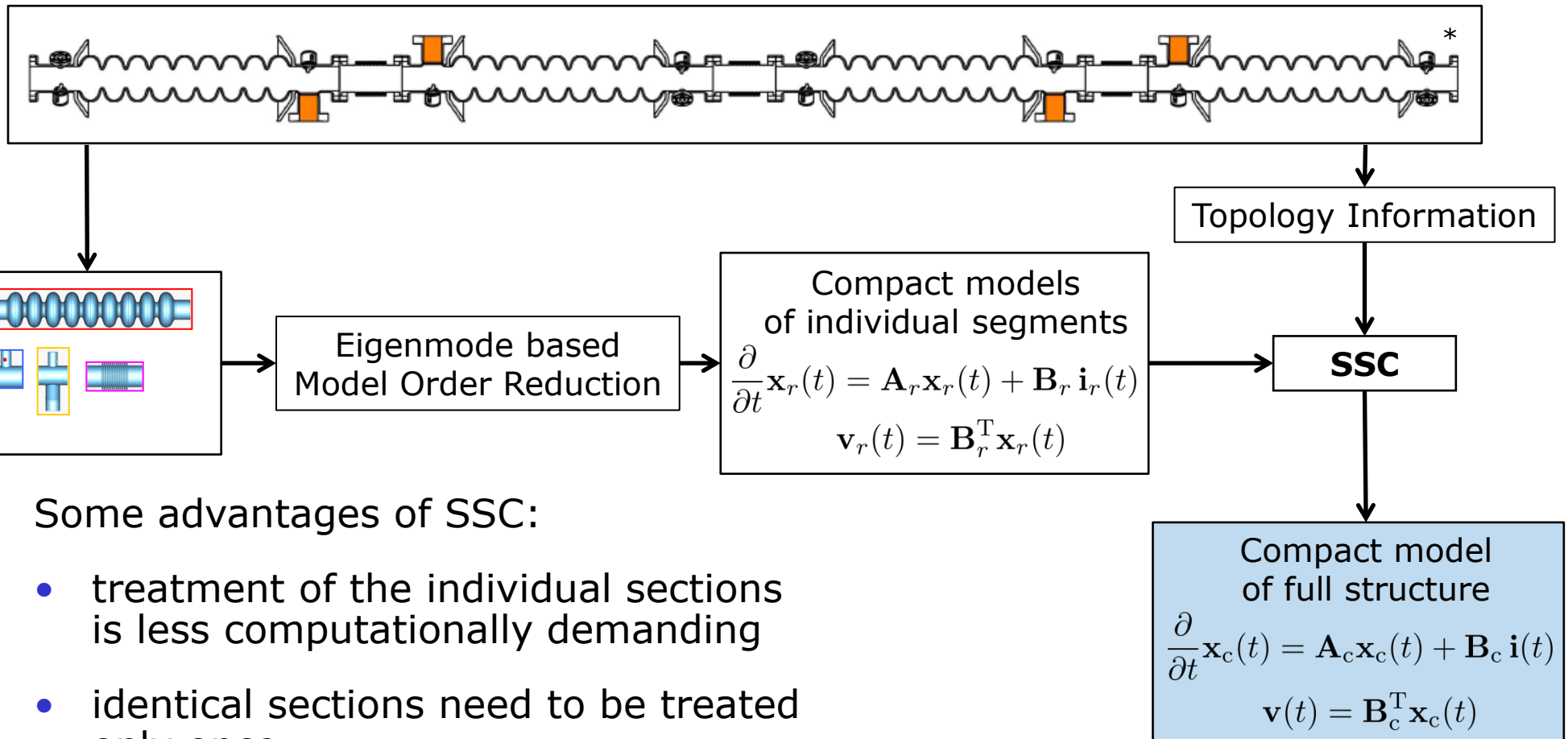
*Picture courtesy E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008.

**I. R. R. Shinton, N. Juntong, R. M. Jones: "Modal Dictionary of Cavity Modes for the Third Harmonic XFEL/FLASH Cavities", DESY note: DESY 12-053.

Concatenation Approach with Field Distributions: State-Space Concatenations*

*T. Flisgen, H.-W. Glock, and U. van Rienen: "Compact Time-Domain Models of Complex RF Structures Based on the Real Eigenmodes of Segments", IEEE Transactions on Microwave Theory and Techniques, 61(6), June 2013.

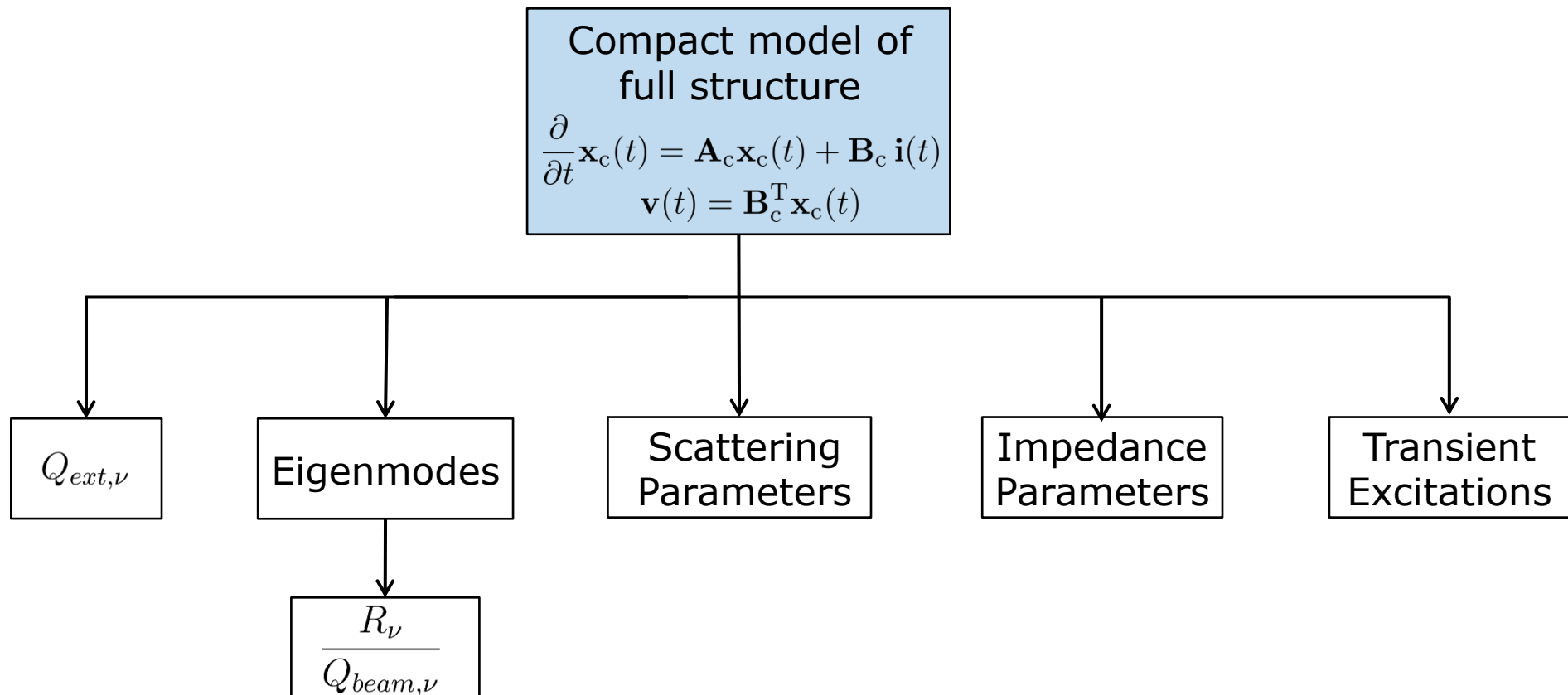
Workflow State Space Concatenations



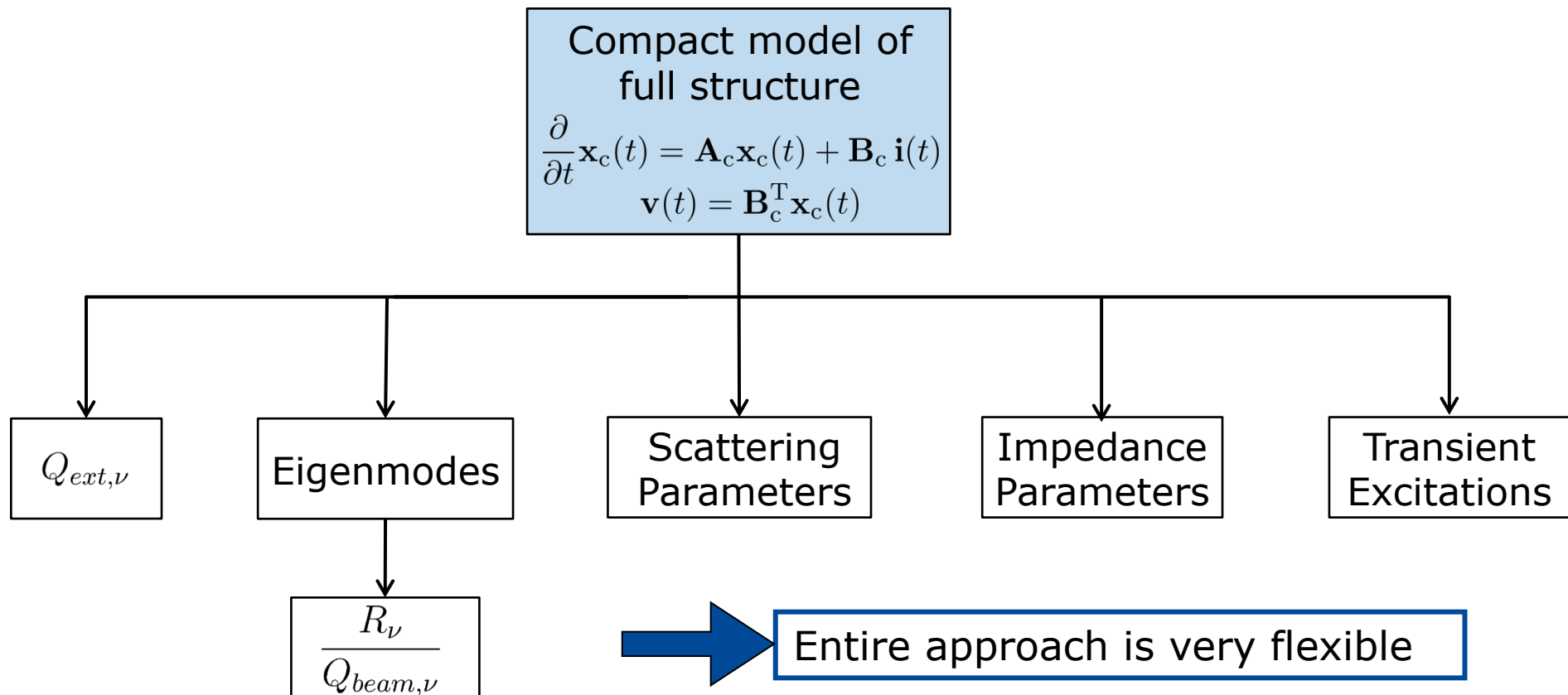
Some advantages of SSC:

- treatment of the individual sections is less computationally demanding
- identical sections need to be treated only once

Secondary Quantities of Full Structure Readily Available by Compact (Reduced Order) Model



Secondary Quantities of Full Structure Readily Available by Compact (Reduced Order) Model



SSC in Comparison with other Coupling Methods

| | SSC[1] | CSC[2] | GSM[3] | Mode Matching[4] | Spice[5] | HPC[6] | ... |
|-------------------------------------|--------|--------|--------|------------------|----------|--------|-----|
| Time Domain | ✓ | ✗ | ✗ | ✗ | ✓ | ✓ | |
| Frequency Domain | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | |
| Model Order Reduction | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | |
| Arbitrary Structures and Topologies | ✓ | ✓ | ✗ | ○ | ✓ | ✓ | |
| 3D Field Information | ✓ | ○ | ○ | ✓ | ✗ | ✓ | |

[1] T. Flisgen et al., "Compact Time-Domain Models of Complex RF Structures Based on the Real Eigenmodes of Segments", IEEE MTT, 61(6), June 2013.

[2] H.-W. Glock et al., "CSC - A procedure for coupled S-parameter calculations", IEEE Trans. Magn., 38(2), March 2002.

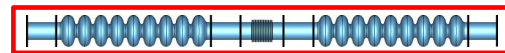
[3] I. Shinton et al., "Large Scale Linac Simulations using a Globalised Scattering Matrix Approach ", Proc. EPAC08, Italy, 2008.

[4] W. Wessel et al., "Mode-matching analysis of general waveguide multiport junctions", IEEE MTT-S Int. Microw. Symp. Dig., June 1999, vol. 3.

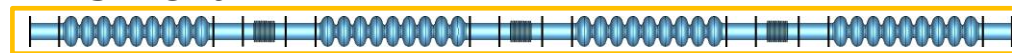
[5] T. Wittig, et al. "Model order reduction for large systems in computational electromagnetics", LinAlgApp, vol. 415, no. 2-3, June 2006.

[6] e.g. F. Yaman et al., "Comparison of Eigenvalue Solvers for Large Sparse Matrix Pencils", Proc. 11th Int. Comput. Accelerator Phys. Conf., Germany, August 2012.

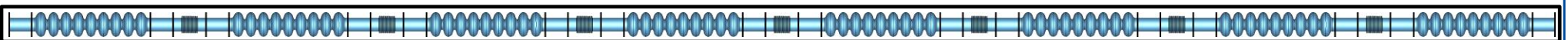
Proof of Principle: Analysis of Multi-Cavity TM₀₁ Modes in a Concatenated Arrangement of Third Harmonic Cavities with Bellows



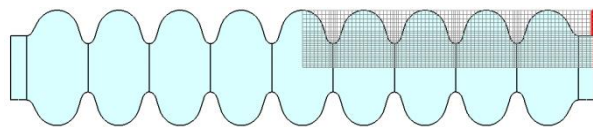
FLASH Chain



X-FEL Chain



Segments of 3rd Harmonic Cavity Chain



meshcells = 44,436
(all symmetry planes used)

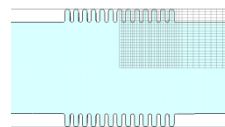
Eigenmode based
Model Order Reduction
(T ≈ 51 min)

$$\frac{\partial^2}{\partial t^2} \mathbf{x}_{\text{cav}}(t) = \mathbf{A}_{\text{cav}} \mathbf{x}_{\text{cav}}(t) + \mathbf{B}_{\text{cav}} \frac{\partial}{\partial t} \mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_{\text{cav}}^T \mathbf{x}_{\text{cav}}(t)$$

$$\mathbf{E}_{\text{cav}}(\mathbf{r}, t) = \sum_{\nu=1}^{33} \tilde{\mathbf{E}}_{\text{cav},\nu}(\mathbf{r}) x_{\text{cav},\nu}(t)$$

$$\mathbf{x}_{\text{cav}}(t) \in \mathbb{R}^{33}$$



meshcells = 16,758
(all symmetry planes used)

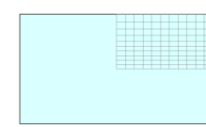
Eigenmode based
Model Order Reduction
(T ≈ 5 min)

$$\frac{\partial^2}{\partial t^2} \mathbf{x}_{\text{bel}}(t) = \mathbf{A}_{\text{bel}} \mathbf{x}_{\text{bel}}(t) + \mathbf{B}_{\text{bel}} \frac{\partial}{\partial t} \mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_{\text{bel}}^T \mathbf{x}_{\text{bel}}(t)$$

$$\mathbf{E}_{\text{bel}}(\mathbf{r}, t) = \sum_{\nu=1}^{15} \tilde{\mathbf{E}}_{\text{bel},\nu}(\mathbf{r}) x_{\text{bel},\nu}(t)$$

$$\mathbf{x}_{\text{bel}}(t) \in \mathbb{R}^{15}$$



meshcells = 1,100
(all symmetry planes used)

Eigenmode based
Model Order Reduction
(T < 1 min)

$$\frac{\partial^2}{\partial t^2} \mathbf{x}_{\text{pip}}(t) = \mathbf{A}_{\text{pip}} \mathbf{x}_{\text{pip}}(t) + \mathbf{B}_{\text{pip}} \frac{\partial}{\partial t} \mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_{\text{pip}}^T \mathbf{x}_{\text{pip}}(t)$$

$$\mathbf{E}_{\text{pip}}(\mathbf{r}, t) = \sum_{\nu=1}^8 \tilde{\mathbf{E}}_{\text{pip},\nu}(\mathbf{r}) x_{\text{pip},\nu}(t)$$

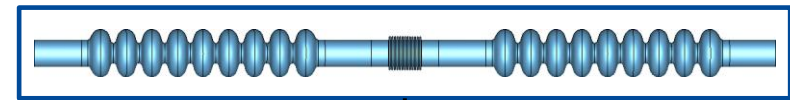
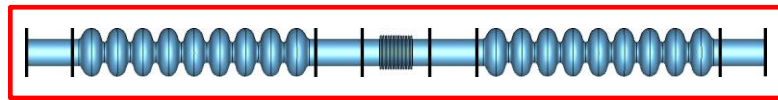
$$\mathbf{x}_{\text{pip}}(t) \in \mathbb{R}^8$$

Computations performed on an Intel Core i5-2400 CPU @ 3.10 GHz machine equipped with 8 GB RAM



Validation of State-Space Concatenations

Validation using R/Q Parameter



$$\frac{\partial^2}{\partial t^2} \mathbf{x}_c(t) = \mathbf{A}_c \mathbf{x}_c(t) + \mathbf{B}_c \frac{\partial}{\partial t} \mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_c^T \mathbf{x}_c(t)$$

(T ≈ 1 h)

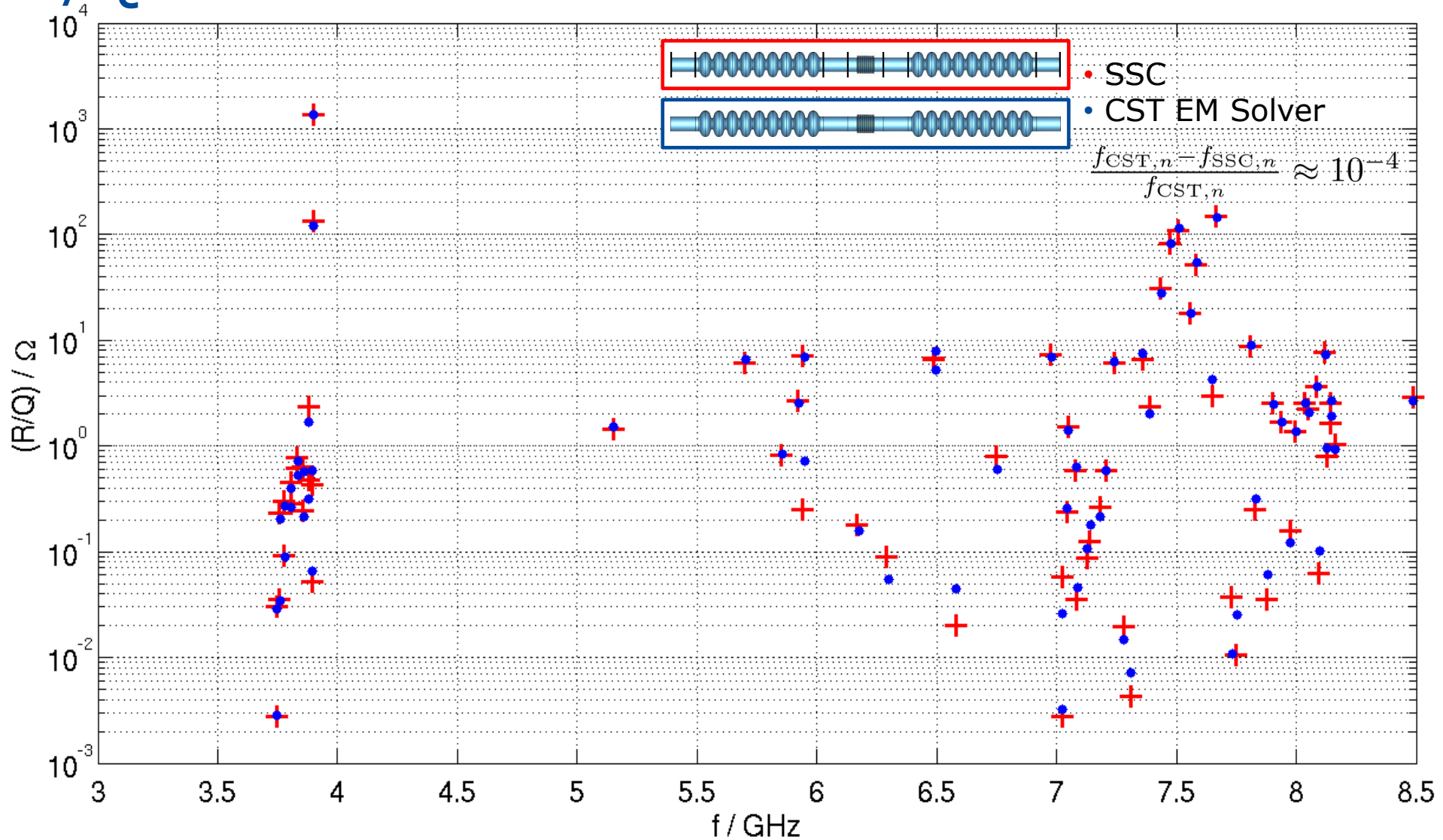
Computation of 136 eigenmodes of full structure with CST MWS® EM JDM Solver employing all three symmetry planes (T = 29 h)

$\text{eig}(\mathbf{A}_c) = \{\mathbf{v}_1, \dots\}$
Constructing field distributions based on segment's eigenmodes and weighting factors (T << 1s)

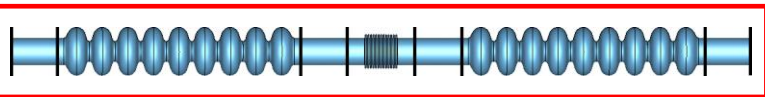
CST Template Based Post Processing

Comparison of R/Q

R/Q Parameter Validation of SSC



Validation of External Q Factor



$$\frac{\partial^2}{\partial t^2} \mathbf{x}_c(t) = \mathbf{A}_c \mathbf{x}_c(t) + \mathbf{B}_c \frac{\partial}{\partial t} \mathbf{i}(t)$$

$$\mathbf{v}(t) = \mathbf{B}_c^T \mathbf{x}_c(t)$$

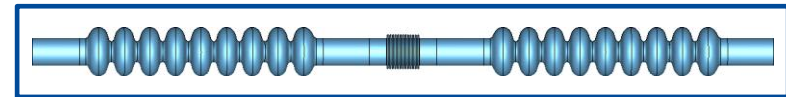
(T ≈ 1 h)

Concatenation of
Matching Models*

$$Q_{ext,\nu} = \frac{\omega_\nu W_{stored,\nu}}{P_{ports,\nu}} = \frac{\text{Im}(\lambda_\nu)}{2\text{Re}(\lambda_\nu)}$$

$$\text{eig}(\mathbf{A}_{sc}) = \{\lambda_1, \dots, \lambda_\nu, \dots\}$$

(T < 1 s)



S-Parameter computation of Full
structure using CST MWS® FR Solver
with two symmetry planes (T = 22 min)

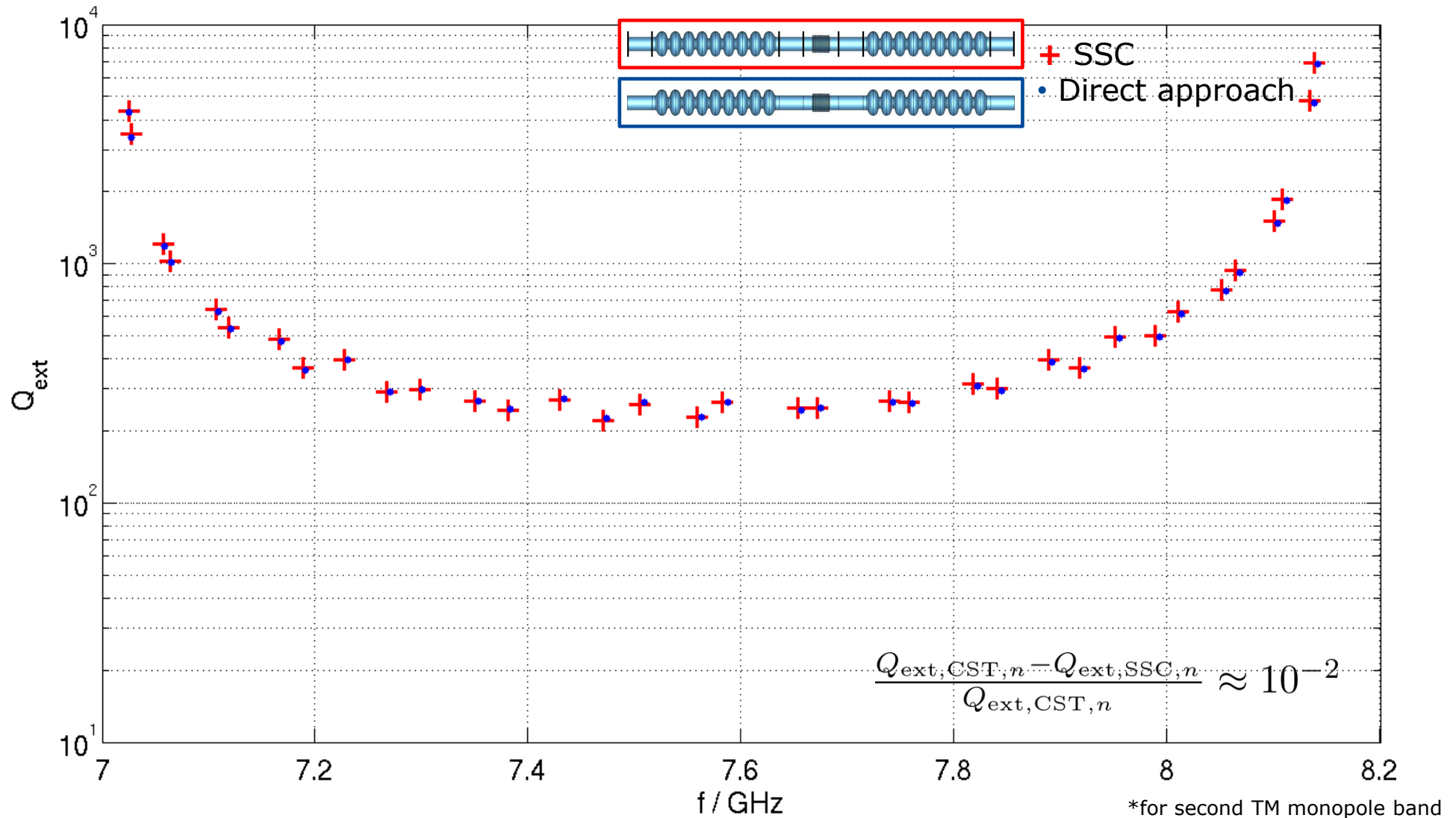
Pole Fitter** on S-Parameters (T = 2 min)

Comparison of
external Q's

*T. Flisgen, J. Heller, and U. van Rienen: "Time-Domain Absorbing Boundary Terminations for Waveguide Ports Based on State-Space Models", IEEE Transactions on Magnetics, 50(2), February 2014.

**B. Gustavsen et al.: "Rational approximation of frequency domain responses by vector fitting", IEEE Trans. Power Delivery, vol. 14, no. 3, July 1999.

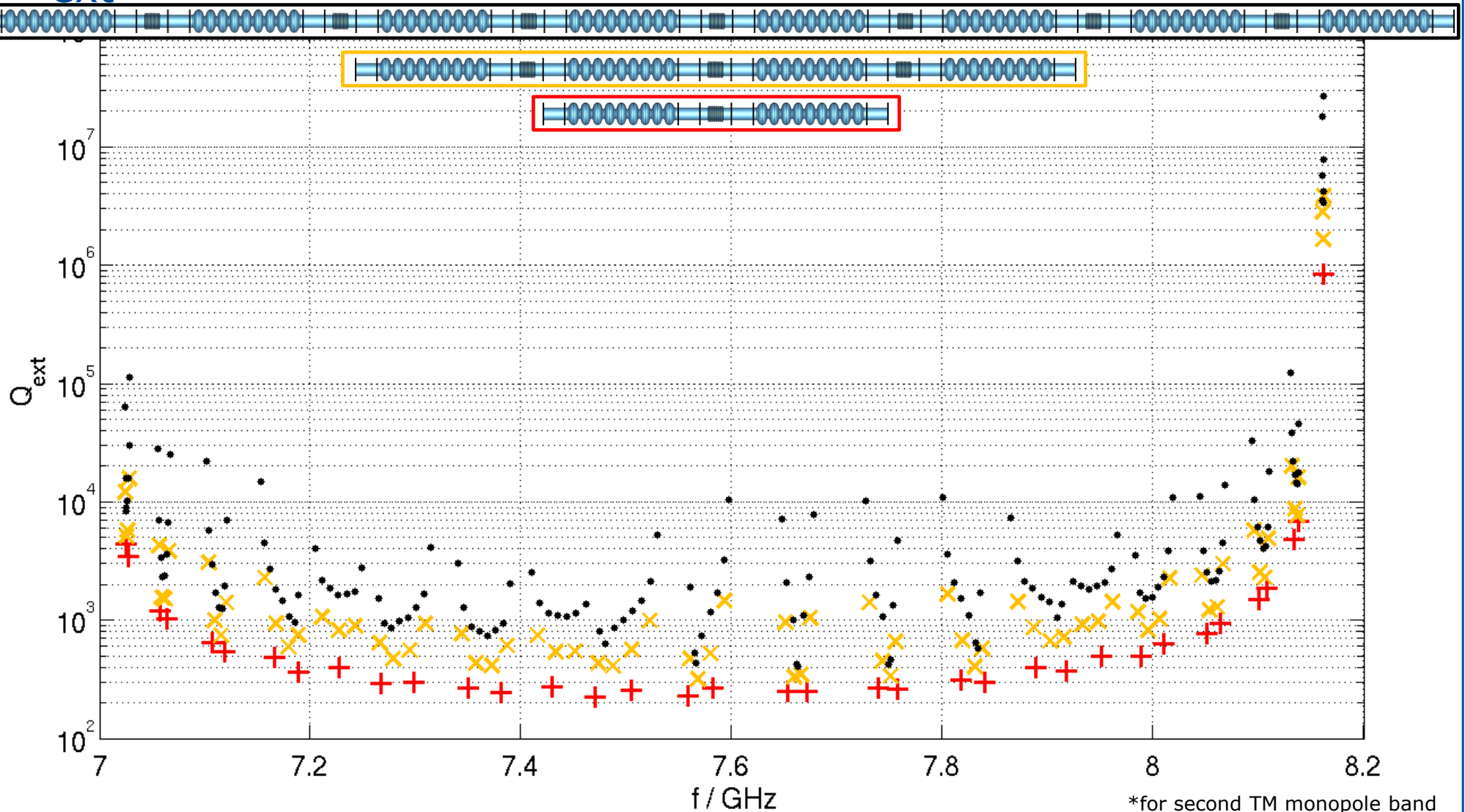
Q_{ext} Validation of SSC





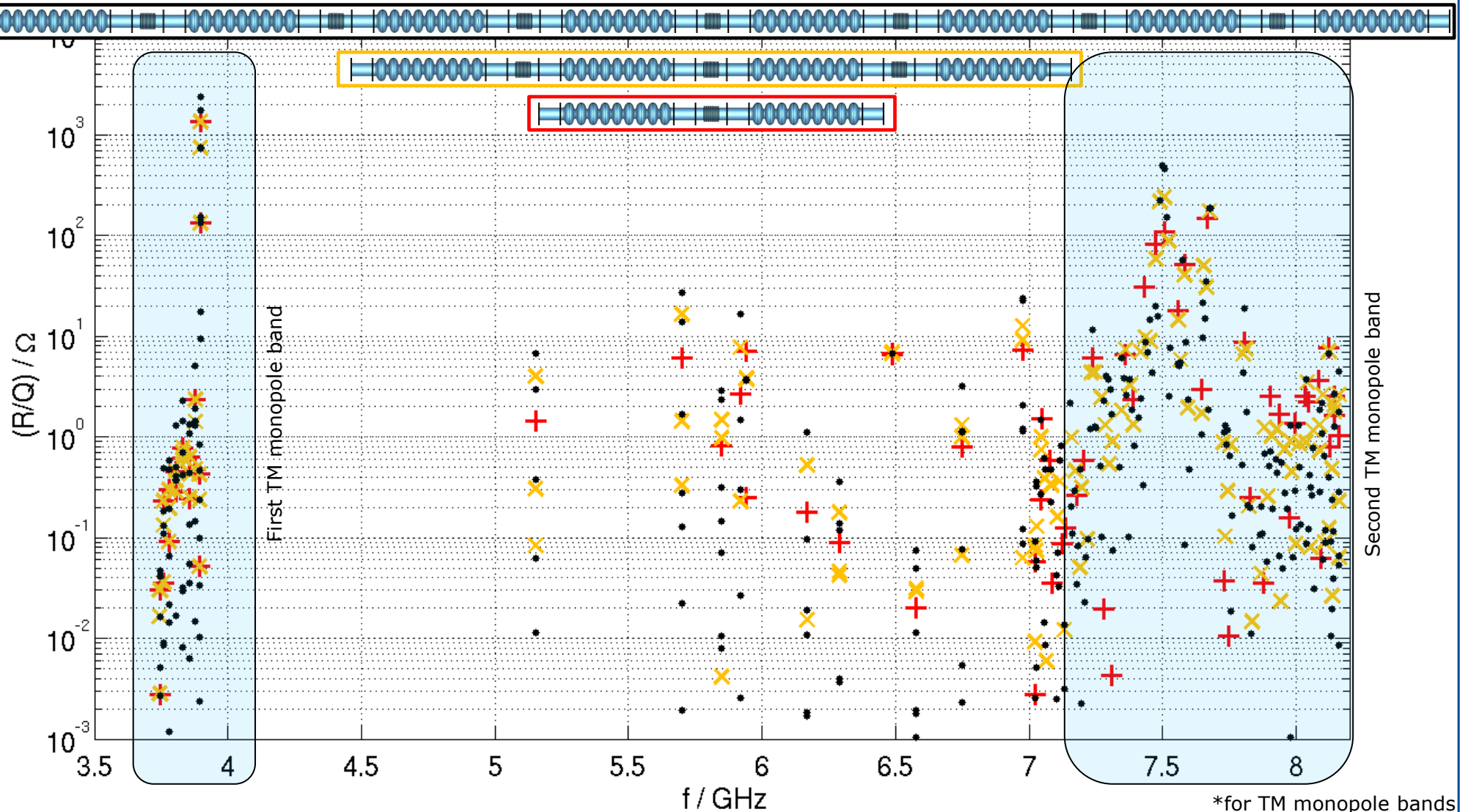
Comparison of Modal Properties in Three Different Arrangements employing SSC

Q_{ext} Factor* Comparison

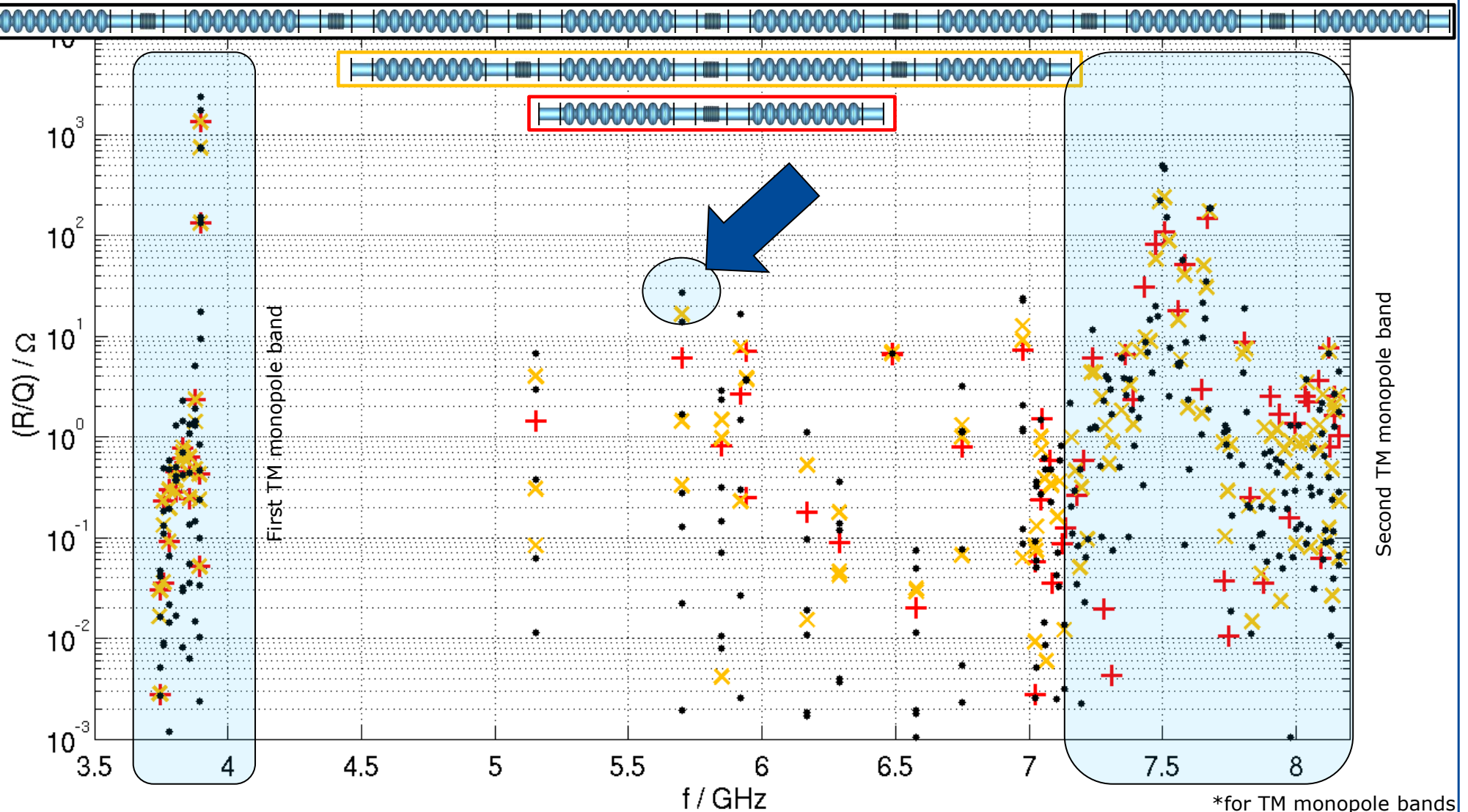


*for second TM monopole band

R/Q Parameter* Comparison

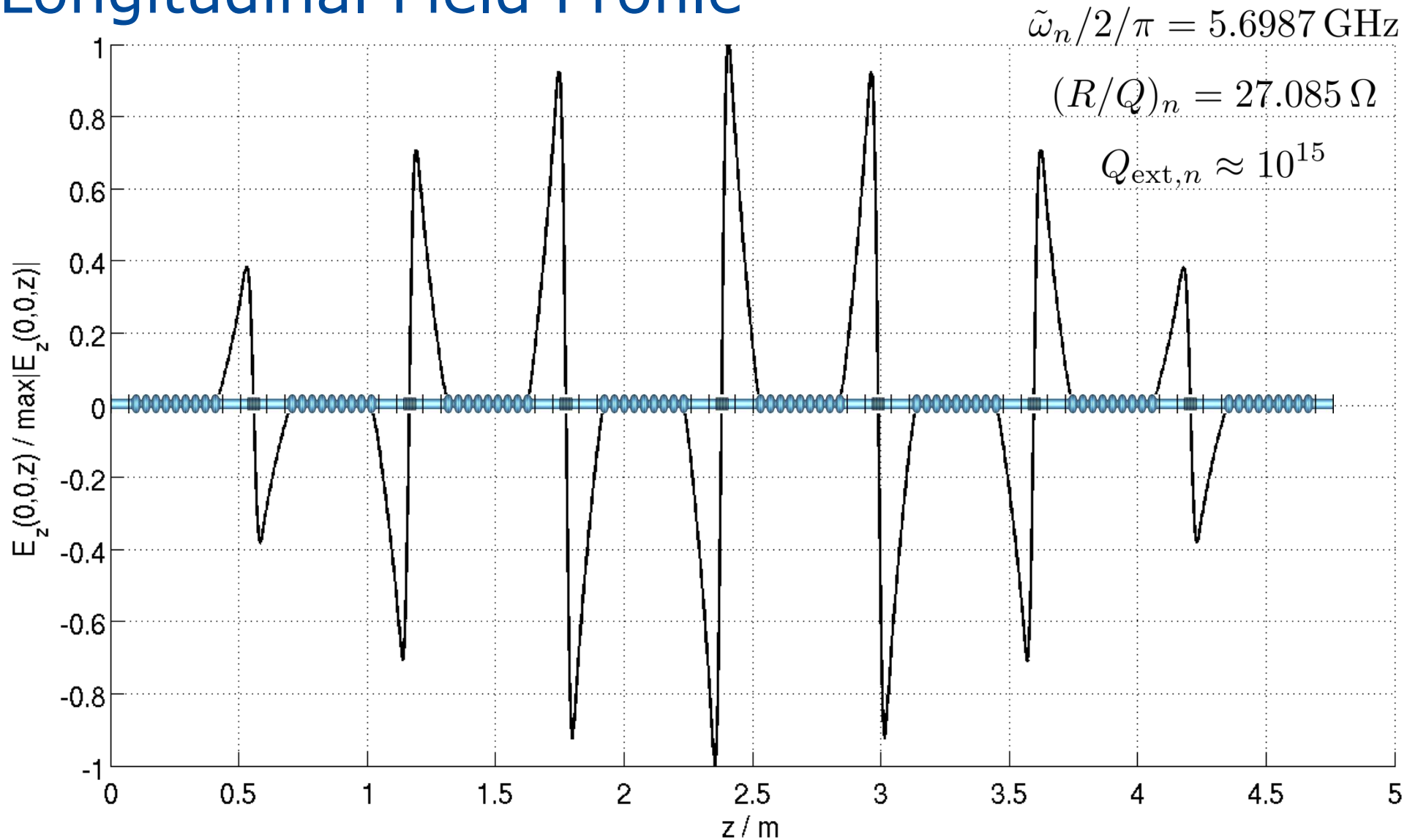


R/Q Parameter* Comparison

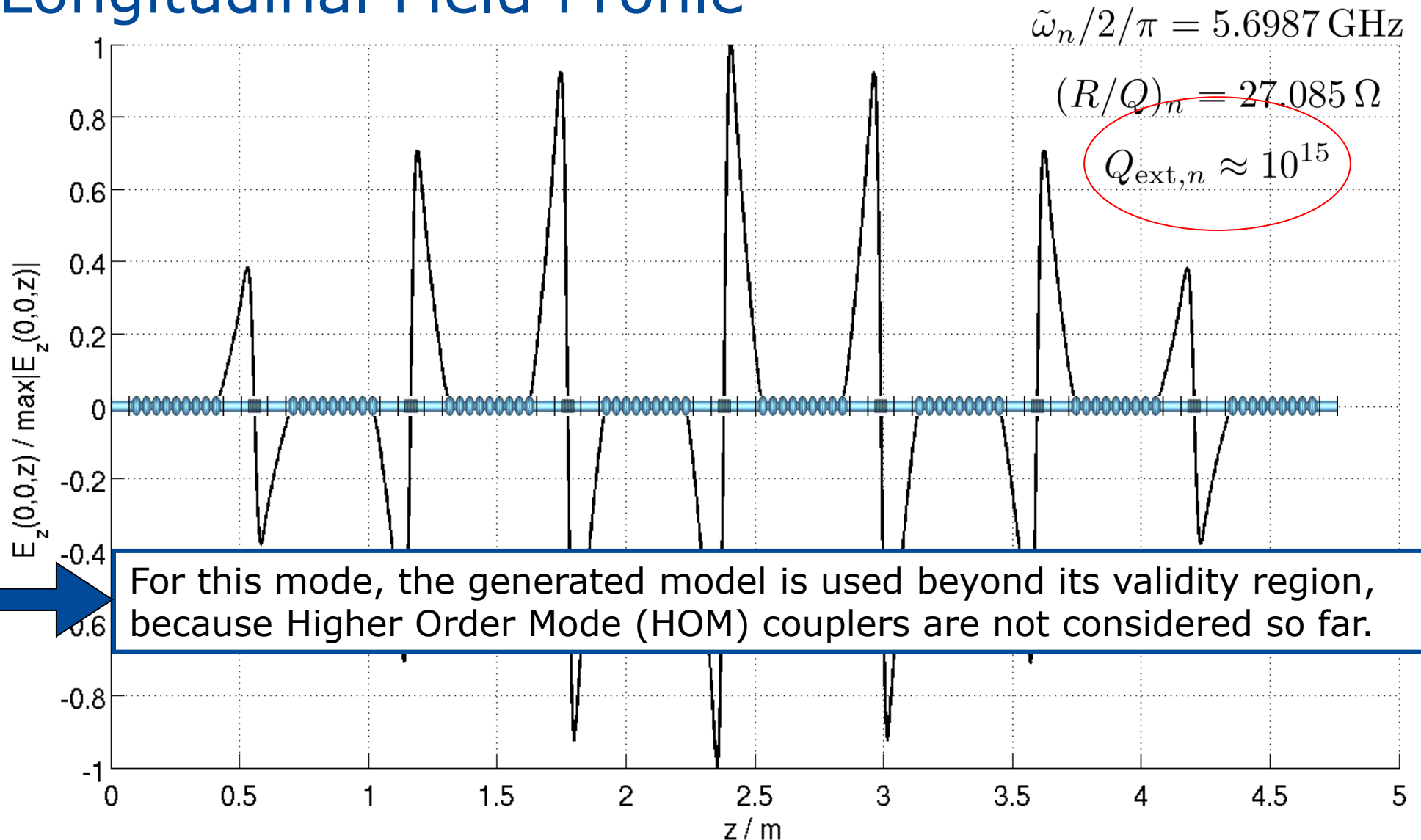


*for TM monopole bands

Longitudinal Field Profile

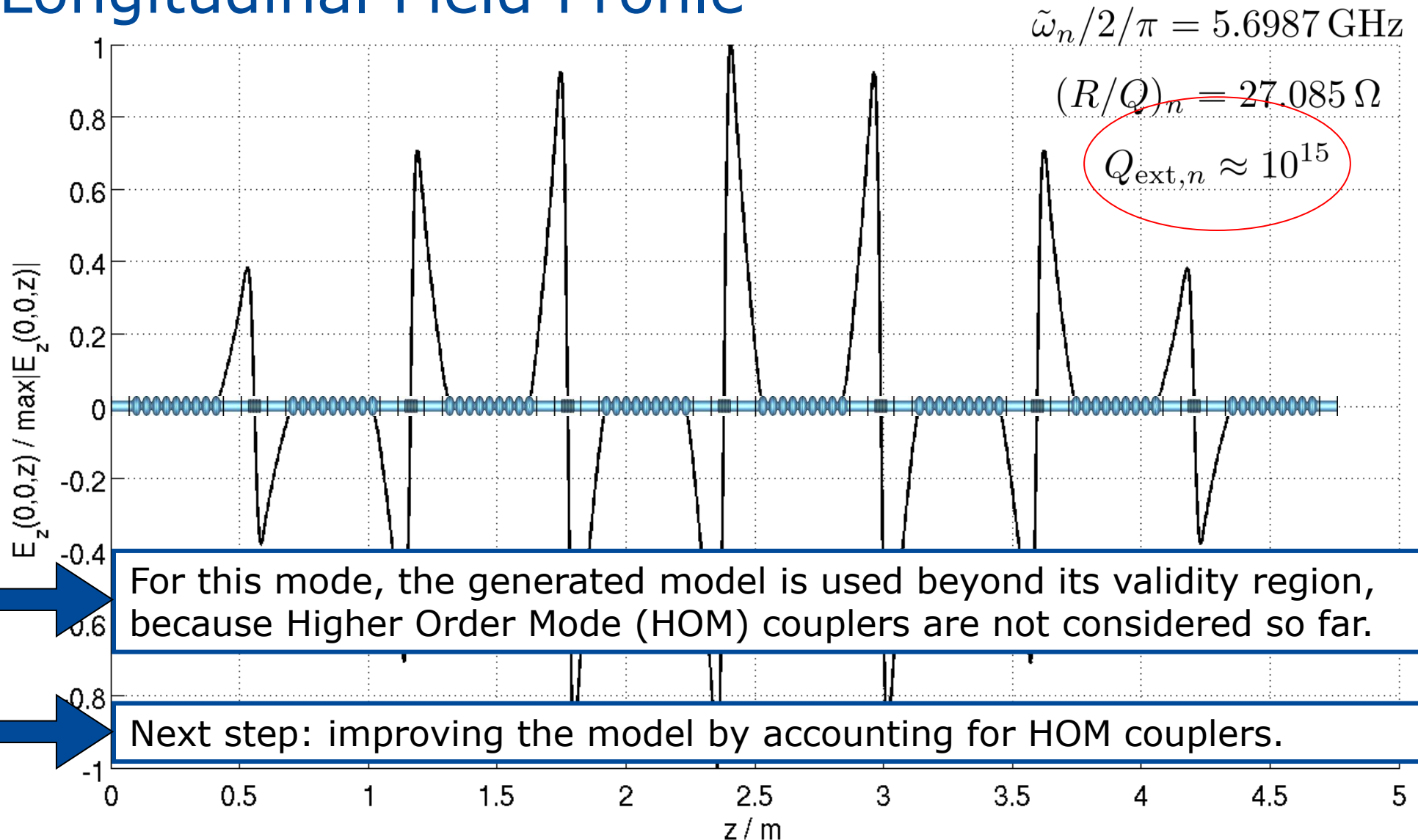


Longitudinal Field Profile



For this mode, the generated model is used beyond its validity region, because Higher Order Mode (HOM) couplers are not considered so far.

Longitudinal Field Profile



For this mode, the generated model is used beyond its validity region, because Higher Order Mode (HOM) couplers are not considered so far.

Next step: improving the model by accounting for HOM couplers.



Conclusions and Outlook

Summary, Conclusions, and Outlook

- The State-Space Concatenation approach allows for eigenmode computation of long cavity chains
- Scheme delivers (amongst others) eigenmodes, Q_{ext} 's and R/Q's
- Scheme is successfully validated by means of straightforward computations
- Further studies based on SSC are in preparation which account for rotational symmetry breaking HOM couplers

Acknowledgement



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Grant Agreement 312453



Relative Error in R/Q Parameter

