

Advanced RF Design and Tuning Methods of RFQ for High Intensity Proton Linacs

Alain C. France, CEA / Saclay

IPAC 2014, Dresden

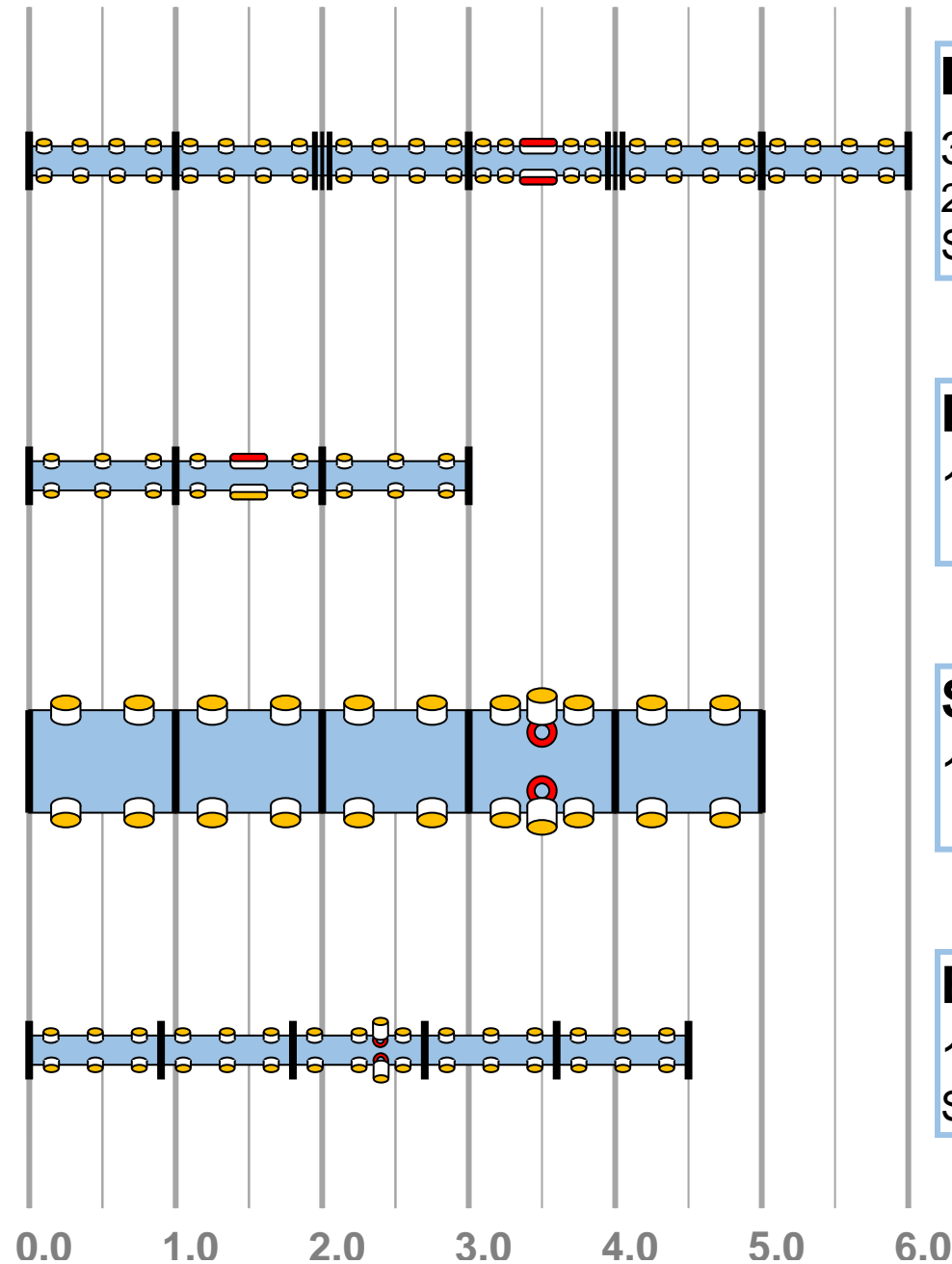
Contents

- 1. Introduction and Theoretical Framework**
- 2. End and Coupling Circuits Tuning**
- 3. Stability Design, Tuning and Measurement**
- 4. Voltage and Frequency Tuning**
- 5. RF Power Coupling**
- 6. Final Comments**



1. Introduction and Theoretical Framework

Our RFQ Projects for High Intensity Linacs



IPHI 352.2 MHz 6.0 m 100 mA CW 3 MeV

3 coupled segments of
2 brazed modules each

4 iris + qwt

96 tuners

Status: tuned, start commissioning 2014 Q3

LINAC4 352.2 MHz 3.0 m 80 mA 7.5% 3 MeV

1 segments of 3 brazed modules

1 iris + qwt

32 tuners

Status: operational

SPIRAL2 88.05 MHz 5.0 m 5 mA CW 3 MeV

1 segment of 5 bolted modules

4 loops

40 tuners

Status: start tuning 2014 Q3

ESS 352.2 MHz 4.5 m 62.5 mA 4% 3.6 MeV

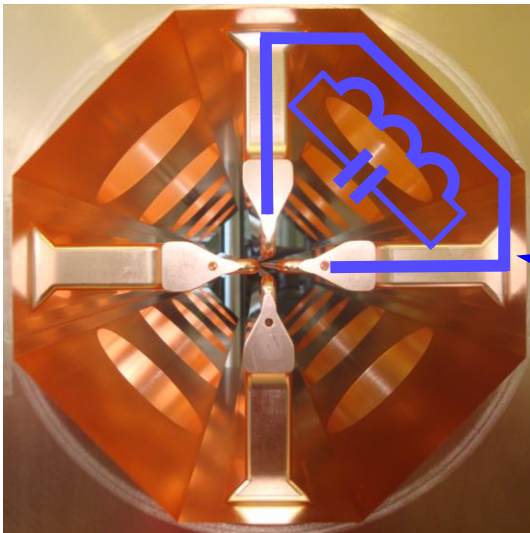
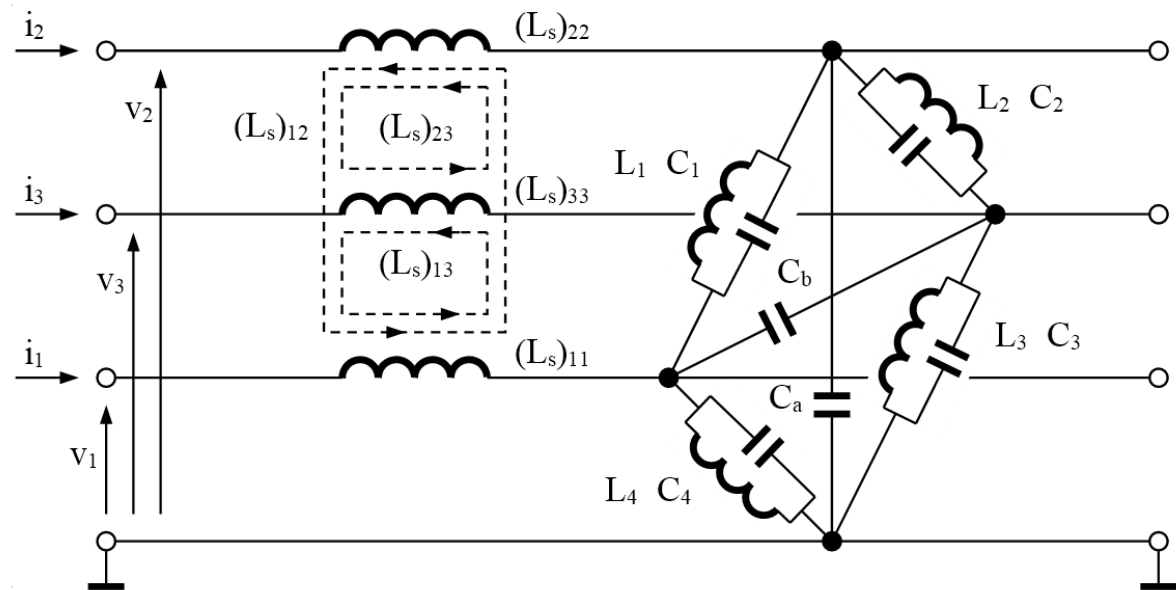
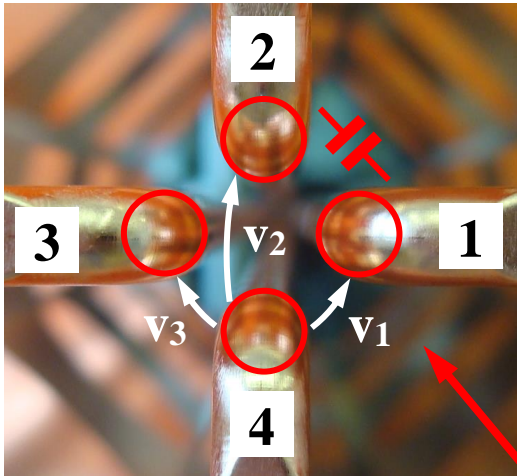
1 segment of 5 brazed modules

2 loops

60 tuners

Status: design completed 2014 Q3

The Loaded Lossless 4-Wire Transmission Line Model (TLM)



Axial region: $H_z \approx 0$, TEM 4-wire line

4 capacitances between adjacent electrodes C_1 to C_4 (F/m)

2 capacitances between opposite electrodes C_a, C_b (F/m)

fundamental TEM relation: $v^2 L_s C = I$

Quadrants are $\lambda/4$ resonators

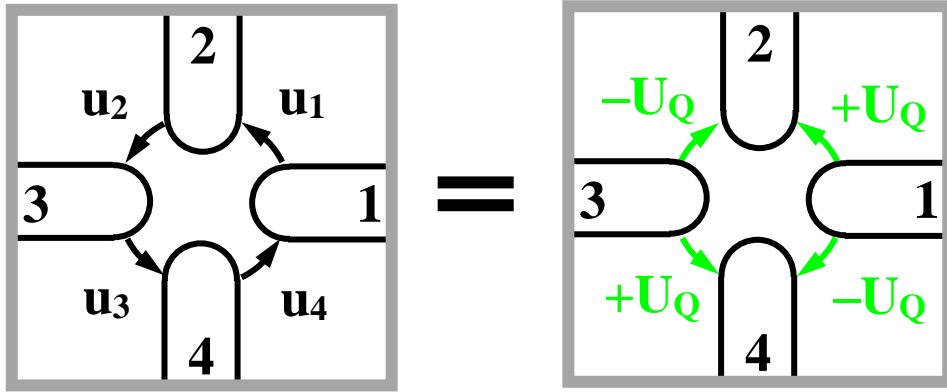
complement with 4 inductances L_1 to L_4 (H.m)

Transmission line equation (dim. 3, since three cuts make the system of conductors simply connected) :

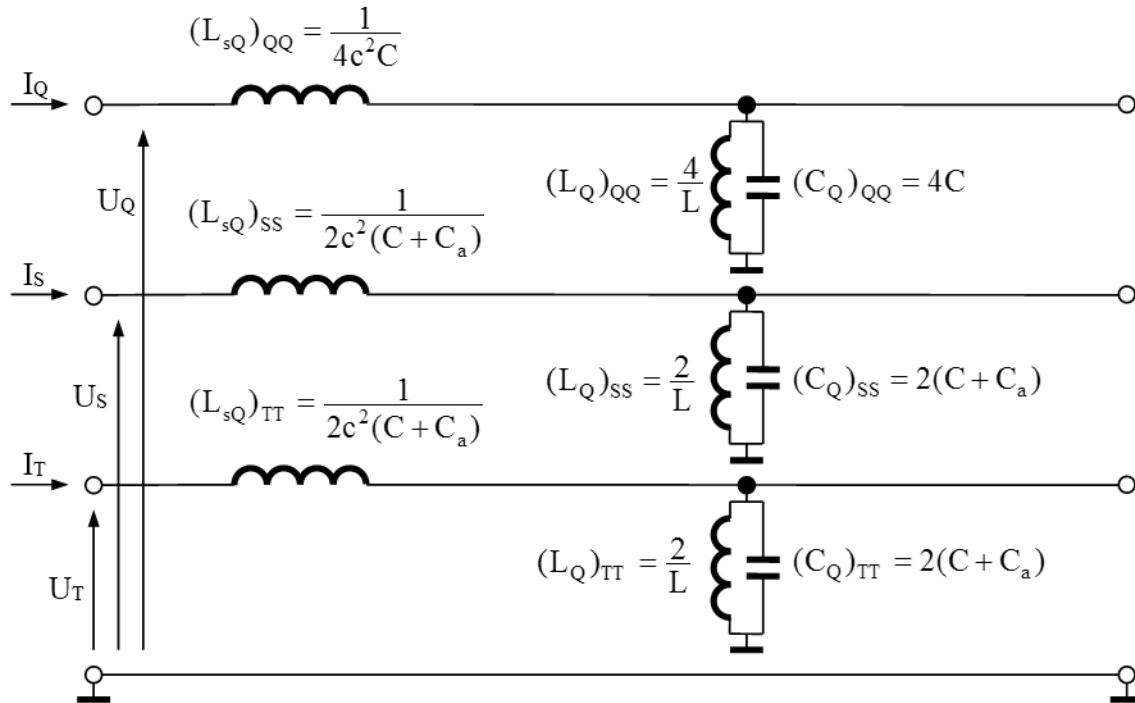
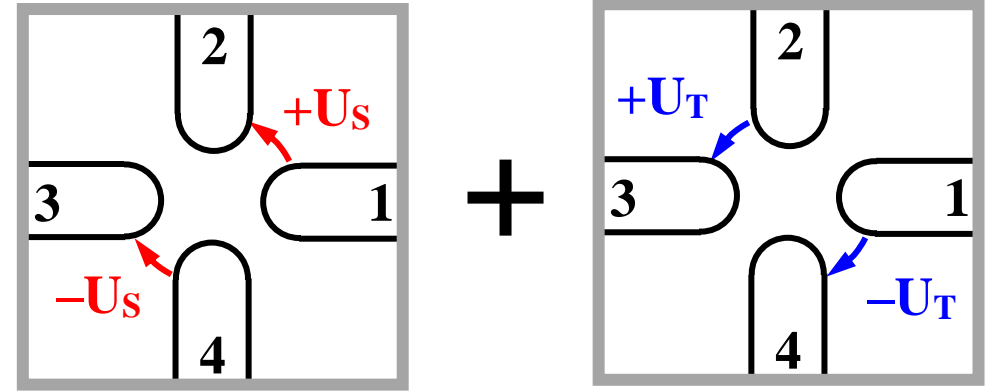
$$-\frac{\partial}{\partial z} \left(C \frac{\partial \mathbf{v}}{\partial z} \right) + \frac{1}{v^2} L \mathbf{v} = \frac{\omega^2}{v^2} C \mathbf{v}$$

The TLM Canonical Basis {Q,S,T}

Quadrupole-like subspace



Dipole-like subspace (dim. 2)



$$-\frac{\partial}{\partial z} \left(C_Q \frac{\partial U}{\partial z} \right) + \frac{1}{v^2} L_Q U = \frac{\omega^2}{v^2} C_Q U$$

For an ideal (quaternary-symmetric) RFQ:

C_Q & L_Q are diagonal

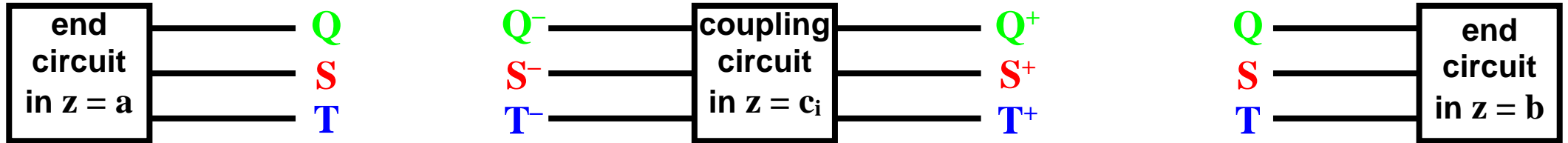
Q, S & T are decoupled

TLM Boundary Conditions

arbitrary reciprocal lossless circuits, defined in {Q,S,T} basis by their admittance matrixes (which are assumed to exist)

circuit theory: admittance matrixes y_a y_{c_i} y_b

$$I(a) = -y_a U(a) \quad \begin{vmatrix} I(c_i^-) \\ -I(c_i^+) \end{vmatrix} = y_{c_i} \begin{vmatrix} U(c_i^-) \\ U(c_i^+) \end{vmatrix} \quad I(b) = +y_b U(b)$$



transmission line theory: s-matrixes s_a s_{c_i} s_b

$$\frac{\partial U(a)}{\partial z} = -s_a U(a) \quad \begin{vmatrix} \frac{\partial U(c_i^-)}{\partial z} \\ -\frac{\partial U(c_i^+)}{\partial z} \end{vmatrix} = +s_{c_i} \begin{vmatrix} U(c_i^-) \\ U(c_i^+) \end{vmatrix} \quad \frac{\partial U(b)}{\partial z} = +s_b U(b)$$

$$s_a = -j\omega L_{sQ}(a) y_a \quad s_{c_i} = -j\omega \begin{vmatrix} L_{sQ}(c_i^-) & 0 \\ 0 & L_{sQ}(c_i^+) \end{vmatrix} y_{c_i} \quad s_b = -j\omega L_{sQ}(b) y_b$$

TLM Properties (1/3)

The TLM takes the form of a vector regular Sturm-Liouville problem

<p>Linear operator \mathcal{L} :</p> <p>U satisfies boundary conditions,</p> $\mathcal{L} U = -C_Q^{-1} \frac{\partial}{\partial z} \left(C_Q \frac{\partial U}{\partial z} \right) + \frac{1}{v^2} C^{-1} L_Q U$	<p>Eigenvalue problem</p> $\mathcal{L} U = \frac{\omega^2}{v^2} U$
--	--

\mathcal{L} is un-bounded, with bounded compact inverse, and is self-adjoint for the inner-product

$$\langle u, v \rangle = \int_{\Omega} v^* C_Q u \, dz \quad (\text{with given boundary conditions})$$

three subsets Q , S and T of countable eigenpairs

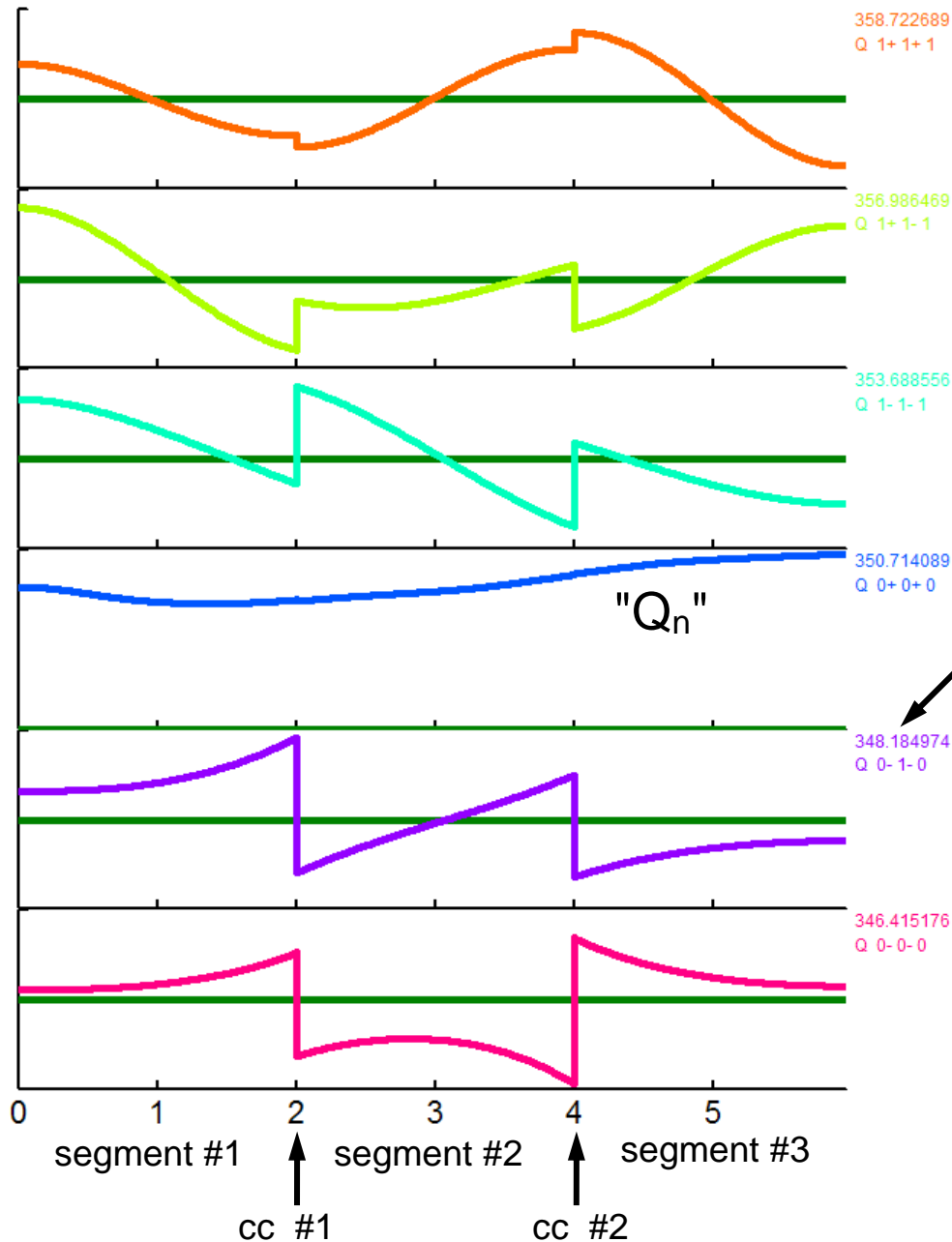
$$\left\{ \omega_{Q_i}, V_{Q_i}(z) = \begin{vmatrix} V_{Q_i,Q}(z) \\ V_{Q_i,S}(z) \\ V_{Q_i,T}(z) \end{vmatrix} \right\} \quad \left\{ \omega_{S_j}, V_{S_j}(z) = \begin{vmatrix} V_{S_j,Q}(z) \\ V_{S_j,S}(z) \\ V_{S_j,T}(z) \end{vmatrix} \right\} \quad \left\{ \omega_{T_k}, V_{T_k}(z) = \begin{vmatrix} V_{T_k,Q}(z) \\ V_{T_k,S}(z) \\ V_{T_k,T}(z) \end{vmatrix} \right\}$$

TLM Properties (2/3)

for an ideal (quaternary-symmetric) RFQ

$$V_{Q_i} = \begin{vmatrix} V_{Q_i,Q} \\ 0 \\ 0 \end{vmatrix} \quad V_{S_j} = \begin{vmatrix} 0 \\ V_{S_j,S} \\ 0 \end{vmatrix} \quad V_{T_k} = \begin{vmatrix} 0 \\ 0 \\ V_{T_k,T} \end{vmatrix}$$

the plot shows the 6 first eigenfunctions of the Q subset for IPHI



Mode designators

348.184974 ← eigenfrequency

Q 0-1-0

- ↑ # of voltage nodes in segment #3
- ↑ state of coupling-circuit #2 ("-" = inverting)
- ↑ # of voltage nodes in segment #2
- ↑ state of coupling-circuit #1 ("-" = inverting)
- ↑ # of voltage nodes in segment #1

"Q_n" is the nickname for the accelerating mode; here "Q_n" is Q 0+0+0

TLM Properties (3/3)

First-order perturbation analysis reveals **dual bases** for parameter perturbation functions and resulting voltage perturbation functions. Example:

capacitance perturbations

$$C_1 = C_{QQ} + C_{SQ} + C_{SSTT}$$

$$C_2 = C_{QQ} - C_{TQ} - C_{SSTT}$$

$$C_3 = C_{QQ} - C_{SQ} + C_{SSTT}$$

$$C_4 = C_{QQ} + C_{TQ} - C_{SSTT}$$

$$\begin{pmatrix} \Delta C_{QQ} \\ \Delta C_{SQ} \\ \Delta C_{TQ} \end{pmatrix} = \sum_{\delta=0}^{\infty} p_{QQ\delta} C_{Q\delta} + \sum_{\alpha=0}^{\infty} p_{SQ\alpha} C_{S\alpha} + \sum_{\beta=0}^{\infty} p_{TQ\beta} C_{T\beta}$$

eigenpair perturbation with duality relations

$$\Delta \lambda_{Q_n}, \Delta V_{Q_n} = \sum_{\substack{i=0 \\ i \neq n}}^{\infty} c_{Q_i} V_{Q_i} + \sum_{j=0}^{\infty} c_{S_j} V_{S_j} + \sum_{k=0}^{\infty} c_{T_k} V_{T_k}$$

$$\begin{aligned} \Delta \lambda_{Q_n} &= p_{QQ_n} \\ c_{Q_i} (\lambda_{Q_n} - \lambda_{Q_i}) &= p_{QQ_i} \end{aligned}$$

$$c_{S_j} (\lambda_{Q_n} - \lambda_{S_j}) = p_{SQ_j}$$

$$c_{T_k} (\lambda_{Q_n} - \lambda_{T_k}) = p_{TQ_k}$$

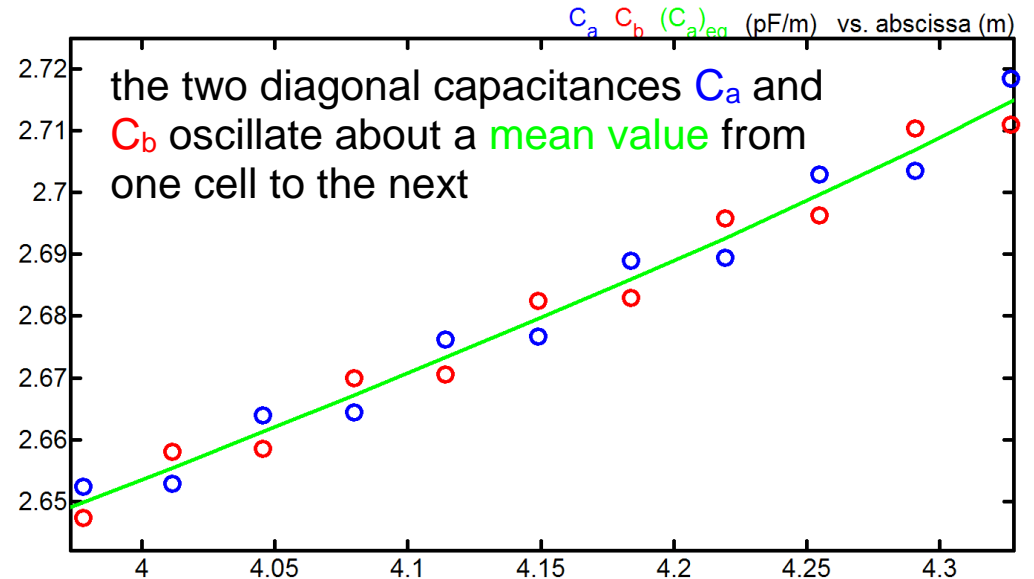
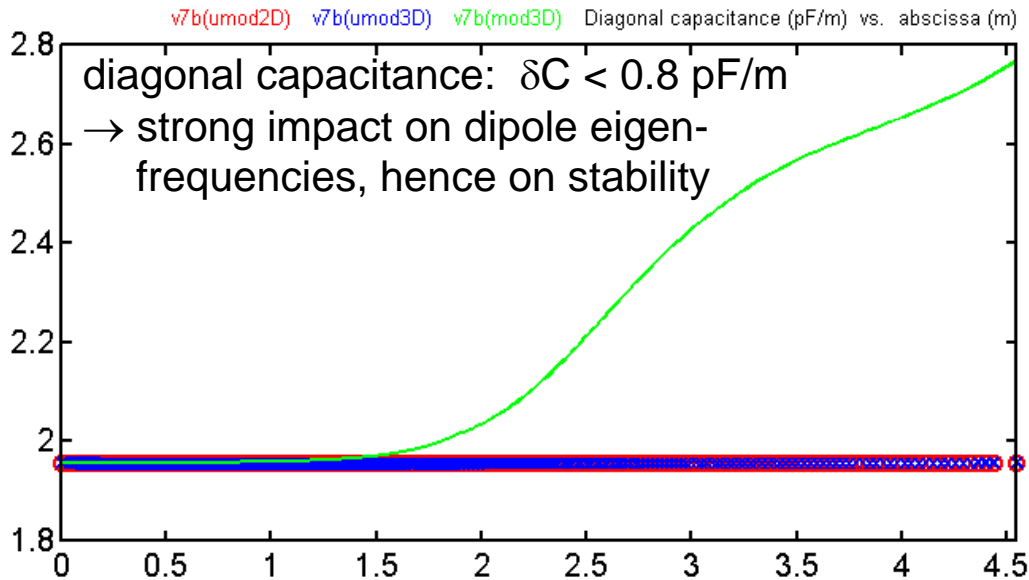
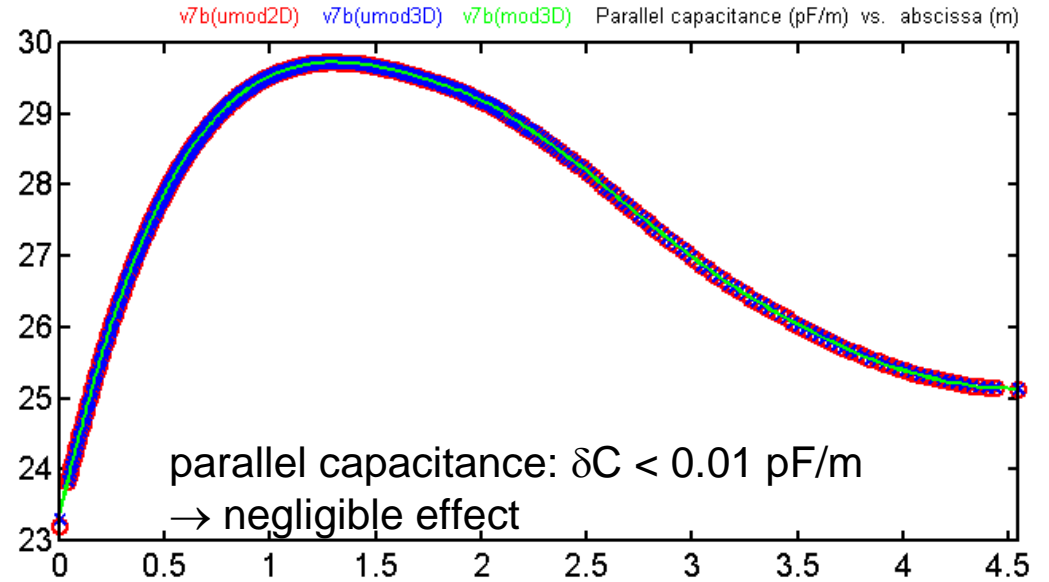
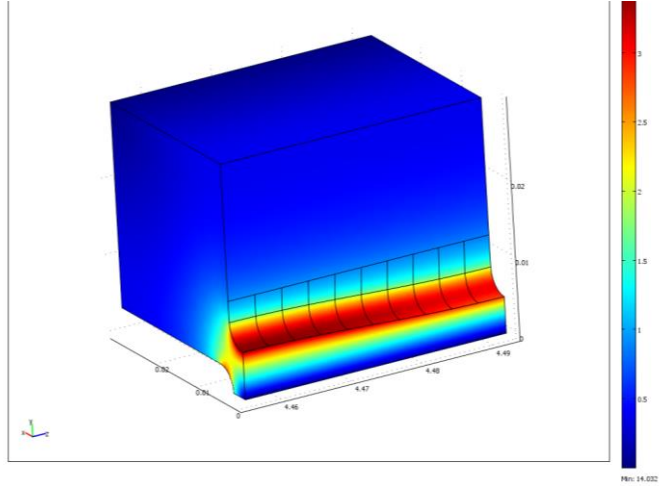
$\{ c_{Q_i}, c_{S_j}, c_{T_k} \}_{0 \leq i, j, k \leq \infty}$ is the spectral analysis of ΔV_{Q_n}

Effects of Modulations on Line Parameters

ESS RFQ 2D/3D simulations

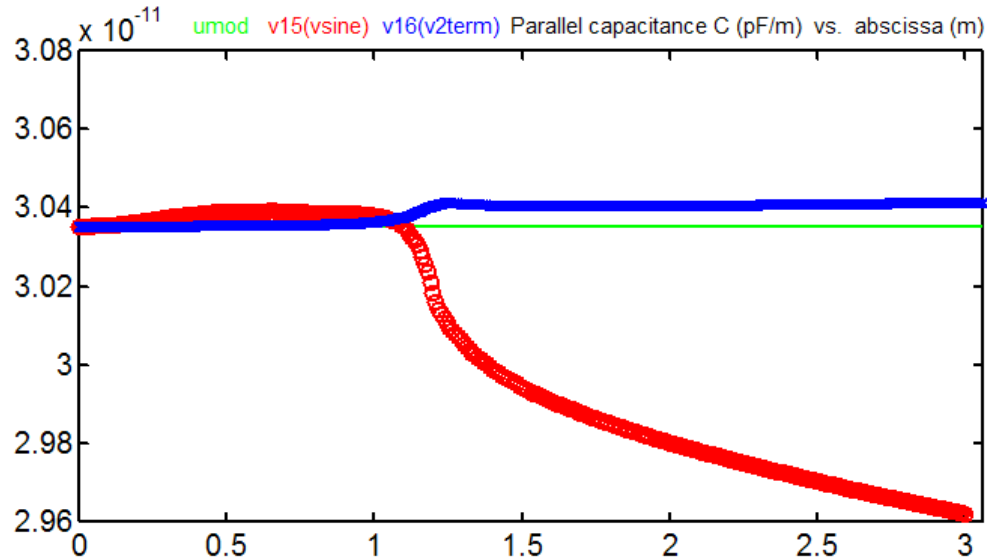
un-modulated 2D / un-modulated 2D / modulated 3D

one simulation cell = one half-period



Effects of Modulation Style

LINAC4 RFQ 2D/3D simulations (Comsol)



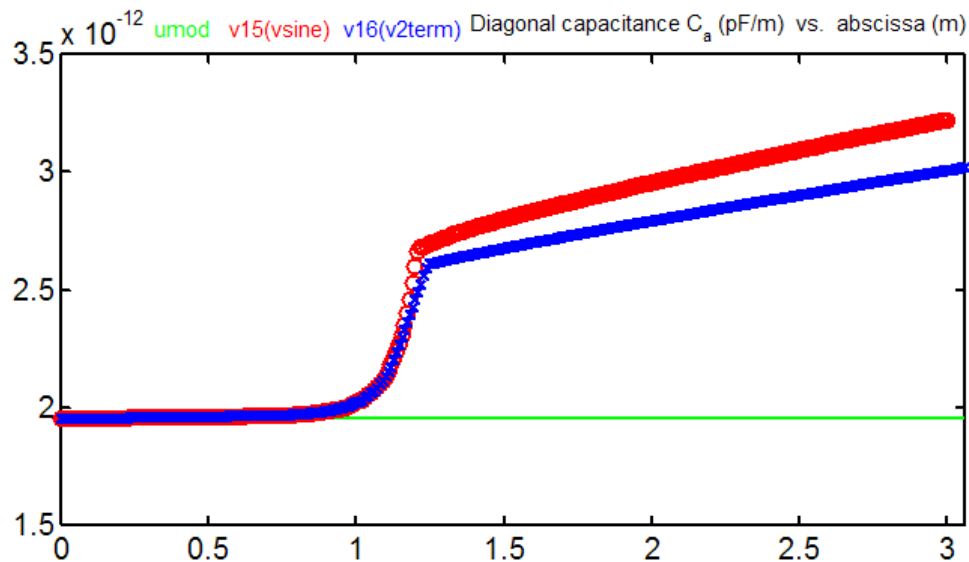
→ un-modulated profile of LINAC4 electrodes is constant

→ 3 simulations:

– in green : un-modulated electrodes

– in red : sine modulation

– in blue : 2-term potential modulation



→ the sine modulation induces too much detuning for reasonable slug dimensions. RFQ cross-section could not be kept constant.

2. End and Coupling Circuits Tuning

End and Coupling Circuits Tuning

End circuit s matrix (ex. in $z = a$)

$$\begin{pmatrix} \partial U_Q / \partial z \\ \partial U_S / \partial z \\ \partial U_T / \partial z \end{pmatrix} = - \begin{pmatrix} s_{QQ} & s_{QS} & s_{QT} \\ s_{SQ} & s_{SS} & s_{ST} \\ s_{TQ} & s_{TS} & s_{TT} \end{pmatrix} \begin{pmatrix} U_Q \\ U_S \\ U_T \end{pmatrix}$$

$= 0$ $= 0$
 (since $U_T(z) = U_S(z) = 0 \quad \forall z$ in the tuned RFQ)

$$s_{QQ} = - \frac{1}{V(a)} \frac{\partial V(a)}{\partial z}$$

$V(z)$ = specified voltage

Coupling circuit s matrix (in $z = c$)

$$\begin{pmatrix} \partial U_Q^- / \partial z \\ - \partial U_Q^+ / \partial z \end{pmatrix} = \begin{pmatrix} s_e^- + s_c & -s_c \\ -s_c & s_e^+ + s_c \end{pmatrix} \begin{pmatrix} U_Q^- \\ U_Q^+ \end{pmatrix}$$

coupling coefficient $s_c = \frac{\omega^2}{v^2} \frac{C_c}{4C}$

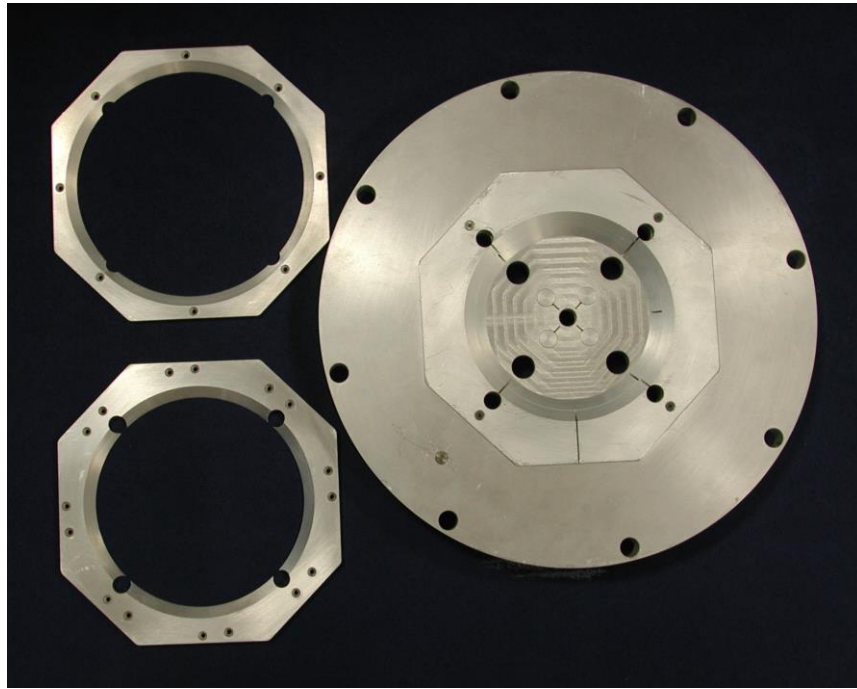
$U_Q^- = U_Q^+$ in the tuned RFQ $s_e^- = -s_e^+ = \frac{1}{V(c)} \frac{\partial V(c)}{\partial z}$

$$s_{\Sigma\Sigma} = \frac{1}{2} (s_e^- - s_e^+) = \frac{1}{V(c)} \frac{\partial V(c)}{\partial z} \quad \text{tuning}$$

$$s_{\Delta\Sigma} = -\frac{1}{2} (s_e^- + s_e^+) = 0 \quad \text{matching}$$

tuning : adequate voltage slope across boundary
matching : continuous voltage across boundary

Tunable Devices for End and Coupling Circuits



adjustable thickness

IPHI input end-plate
IPHI coupling-plates #1 and #2



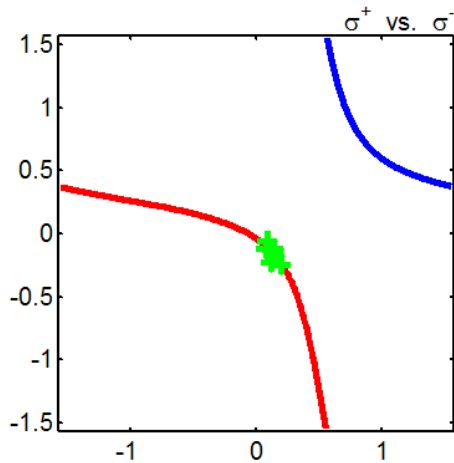
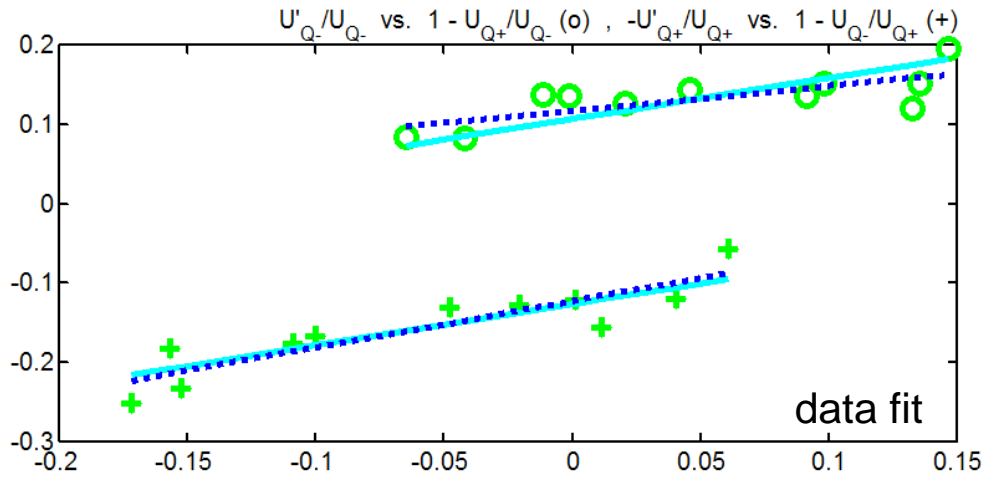
adjustable "quadrupole" rods

IPHI output end-plate
LINAC4 input and output plates
SPIRAL2 input and output end-plates
ESS input and output end-plates

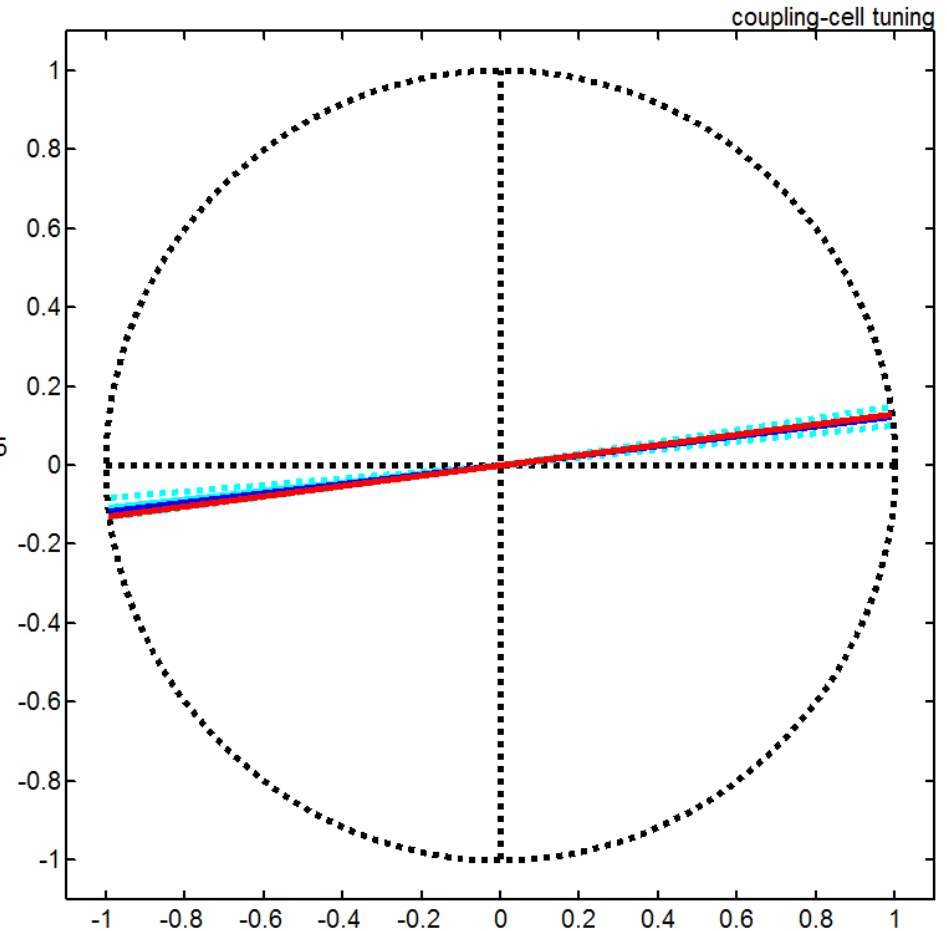
The Excitation Set Method

Use M linearly independent pairs $\{\partial U/\partial z, U\}$ to estimate unknown coefficients of s matrixes. Excitations are obtained with M preset tuner positioning at some distance from boundary. $M = 5$ for end circuits; $M = 11$ for coupling circuits (number of bead-pulls is M).

Example: IPHI coupling circuit #2



state equation: non-inverting branch in red



matching and tuning conditions

IPHI and LINAC4 Realized Boundary Conditions

legend: **good** / **not good, don't know why** / **fair, know why**. s parameters in m^{-1} ("V/m/V")

		expected	tuning (aluminum)	tuned (copper)
IPHI				
input end-circuit	s_{QQ}	0.0	$-5.46 \cdot 10^{-3}$	$-3.04 \cdot 10^{-2}$
	$\sigma(s_{QQ})$		$1.55 \cdot 10^{-2}$	$3.01 \cdot 10^{-2}$
coupling-circuit #1	$s_{\Sigma\Sigma}$	$+7.91 \cdot 10^{-2}$	$+7.33 \cdot 10^{-2}$	$+9.45 \cdot 10^{-2}$
	$s_{\Delta\Sigma}$	0.0	$+9.29 \cdot 10^{-3}$	$-9.59 \cdot 10^{-2}$
	C_c	1.1 pF	0.71 pF	0.53 pF
coupling-circuit #2	$s_{\Sigma\Sigma}$	$1.30 \cdot 10^{-1}$	$+1.07 \cdot 10^{-1}$	$+1.20 \cdot 10^{-1}$
	$s_{\Delta\Sigma}$	0.0	$+1.82 \cdot 10^{-2}$	$+2.39 \cdot 10^{-3}$
	C_c	1.1 pF	0.93 pF	0.95 pF
output end-circuit	s_{QQ}	$2.11 \cdot 10^{-2}$	n/a	$+2.85 \cdot 10^{-2}$
	$\sigma(s_{QQ})$		n/a	$1.67 \cdot 10^{-2}$
LINAC4				
input end-circuit	s_{QQ}	0.0	$+2.78 \cdot 10^{-2}$	$+6.26 \cdot 10^{-2}$
	$\sigma(s_{QQ})$		$7.00 \cdot 10^{-2}$	$2.41 \cdot 10^{-2}$
output end-circuit	s_{QQ}	0.0	n/a	$-7.67 \cdot 10^{-2}$
	$\sigma(s_{QQ})$		n/a	$2.97 \cdot 10^{-2}$

3. Stability Design, Tuning and Measurement

Stability Analysis

"stability" w.r.t. undesired perturbations under operation

impulse error function

QQ impulse error function

the voltage relative perturbation vs. z

$$\frac{\Delta V_{Qn,Q}(z)}{V_{Qn,Q}(z)} = h_{Qn,Q}(z, z_0) \frac{\Delta c_{QQ}}{C(z_0)}$$

a Dirac-like QQ perturbation with mass $\Delta c_{QQ}/C$ located in $z = z_0$

idem for SQ and TQ functions:

$$\frac{\Delta V_{Qn,S}(z)}{V_{Qn,Q}(z)} = h_{Qn,S}(z, z_0) \frac{\Delta c_{SQ}}{C(z_0)}$$

$$\frac{\Delta V_{Qn,T}(z)}{V_{Qn,Q}(z)} = h_{Qn,T}(z, z_0) \frac{\Delta c_{TQ}}{C(z_0)}$$

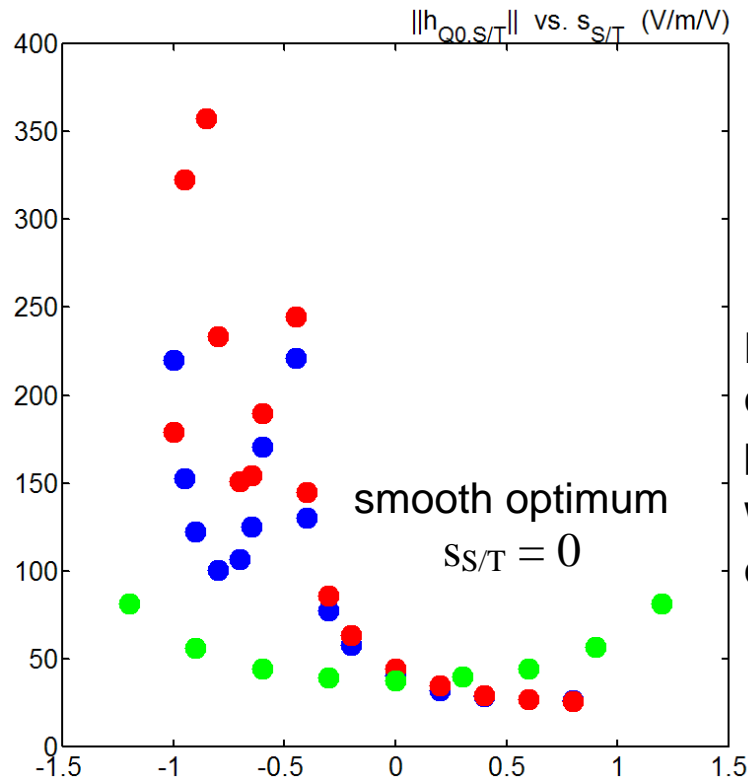
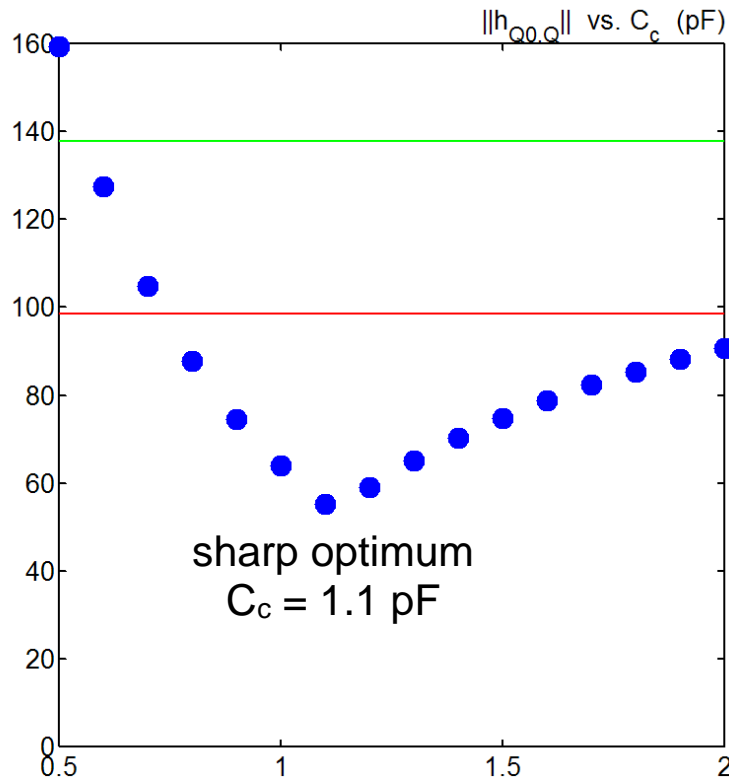
compare RFQ designs with the noms $\|h_{Qn,Q}\| := \sup_{z_0 \in \Omega} \sup_{z \in \Omega} |h_{Qn,Q}(z, z_0)|$, $\|h_{Qn,S}\|$, $\|h_{Qn,T}\|$

these functions depend on $\frac{1}{\lambda_{Qn} - \lambda_{Q_i}}$, $\frac{1}{\lambda_{Qn} - \lambda_{S_j}}$, $\frac{1}{\lambda_{Qn} - \lambda_{T_k}}$ i.e. on quadratic differences

$f_{Qn}^2 - f_{Q_i}^2$, $f_{Qn}^2 - f_{S_j}^2$, $f_{Qn}^2 - f_{T_k}^2$, and are infinite when an eigenmode coincides with Q_n .

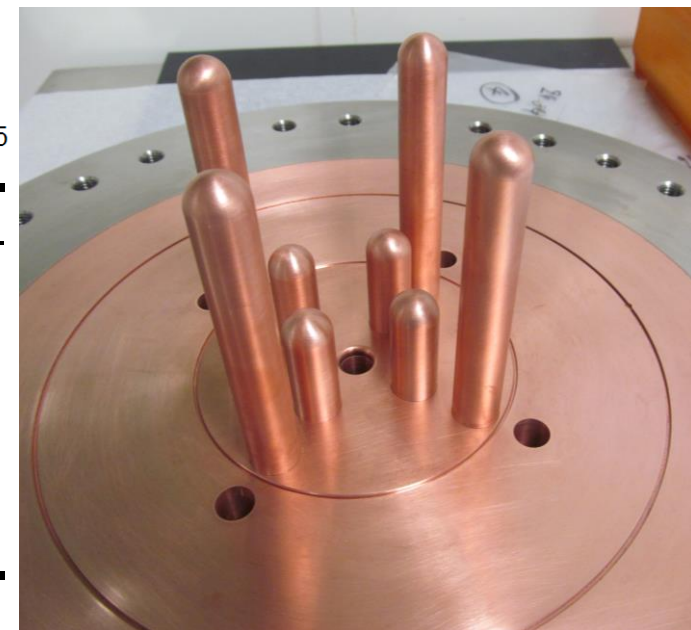
IPHI Stability

legend: unsegmented / segmented, specification / segmented, realized



$S_{SS}, S_{TT}, S_{Se}^-, S_{Se}^+, S_{Te}^-, S_{Te}^+ \approx -1$
without dipole rods, and ≈ 0
with dipole rods

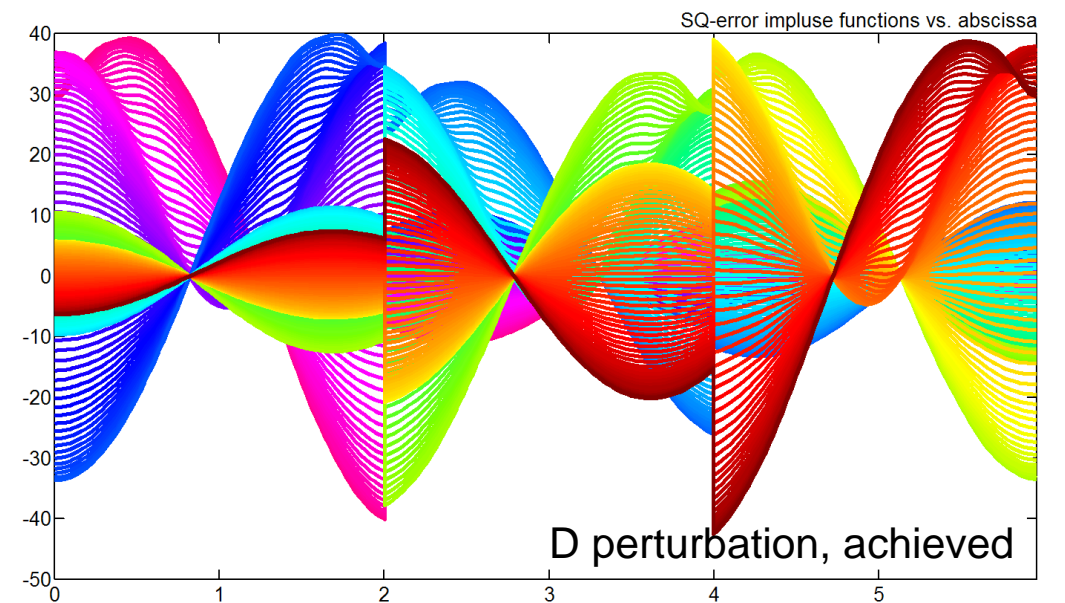
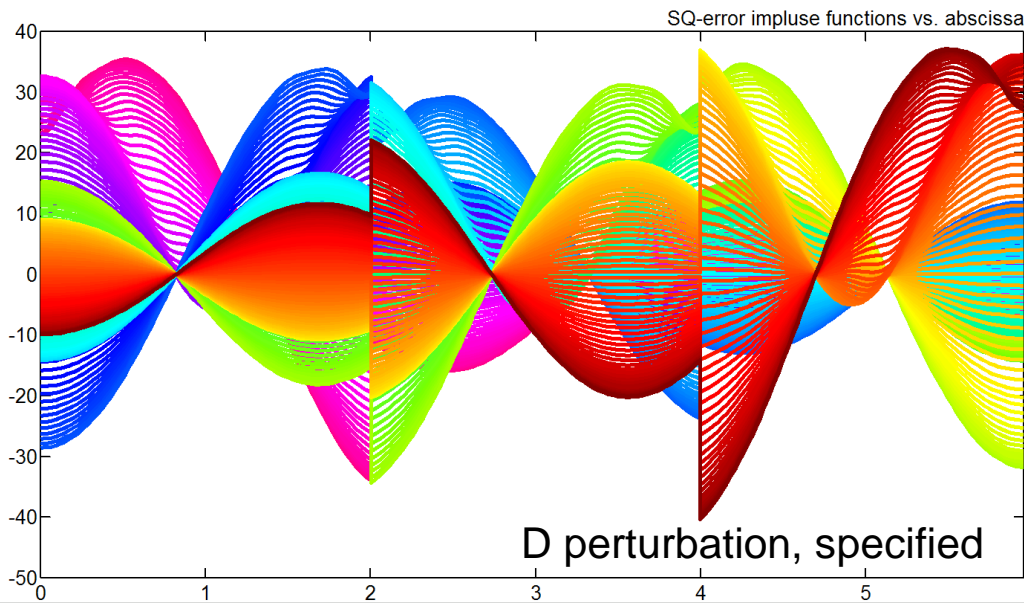
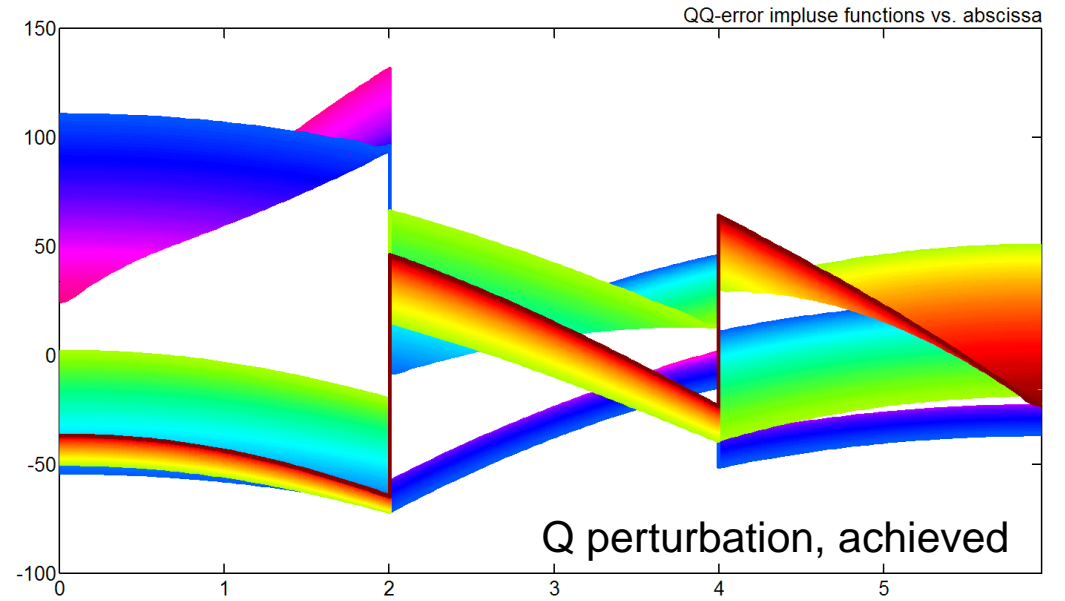
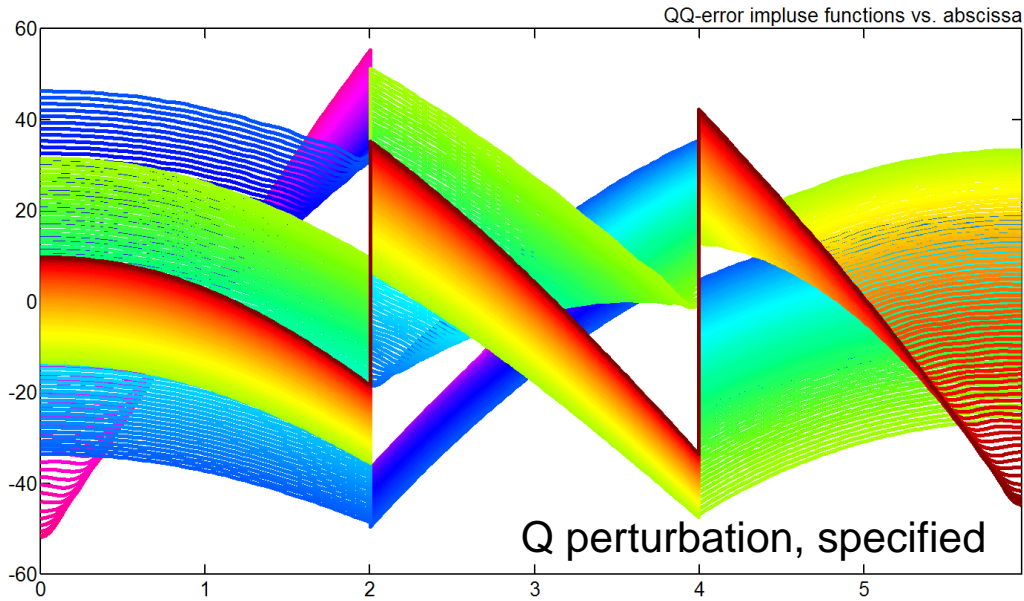
Note that a short rod ($\ell < \lambda_0/2$) is capacitive, hence its admittance is positive, and it may only increase s .
When $\ell \approx \lambda_0/2$, $s = \infty$, and the RFQ end is a short-circuit.



mode des.	specification	prior to slug tuning	after slug tuning
Q 0-1-0	348.18 [-42.0]	349.55 [-25.1]	351.25 [-24.5]
Q 0+0+0 "Q _n "	350.71 [0.00]	350.45 [0.0]	352.10 [0.0]
Q 1-1-1	353.69 [+47.8]	354.65 [+54.4]	356.40 [+55.2]
D 1+1+1	347.67 [-46.1]	348.10 [-40.5]	349.30 [-44.3]
Q 0+0+0 "Q _n "	350.71 [0.0]	350.45 [0.0]	352.10 [0.0]
D 2-2-2	363.16 [+94.3]	362.60 [+93.1]	364.30 [+93.5]

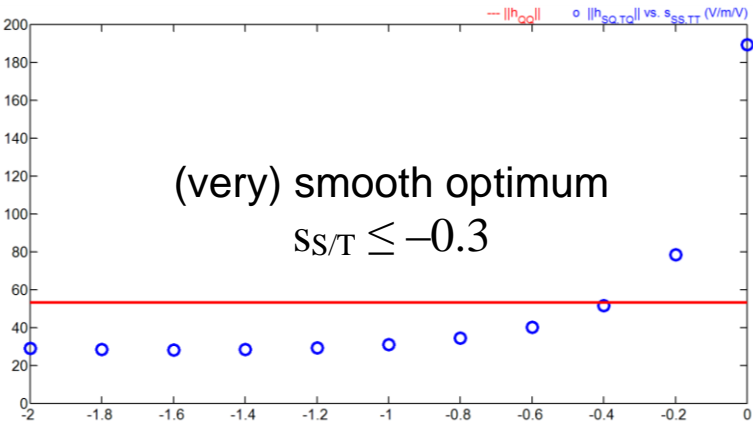
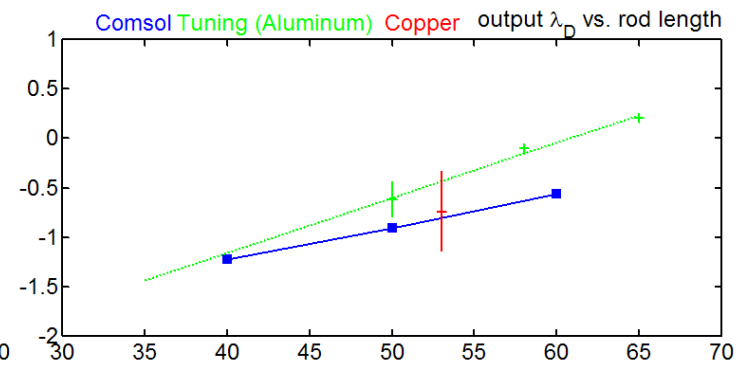
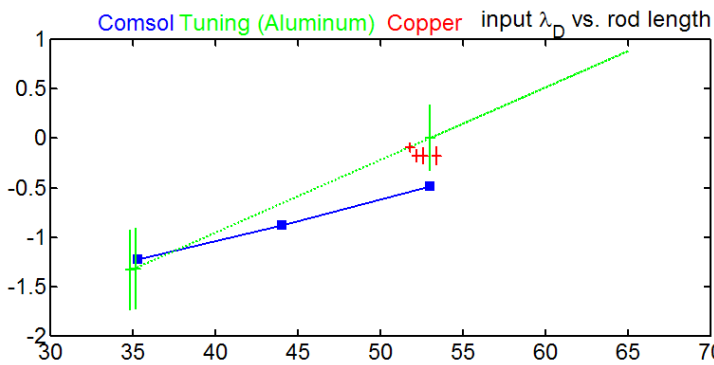
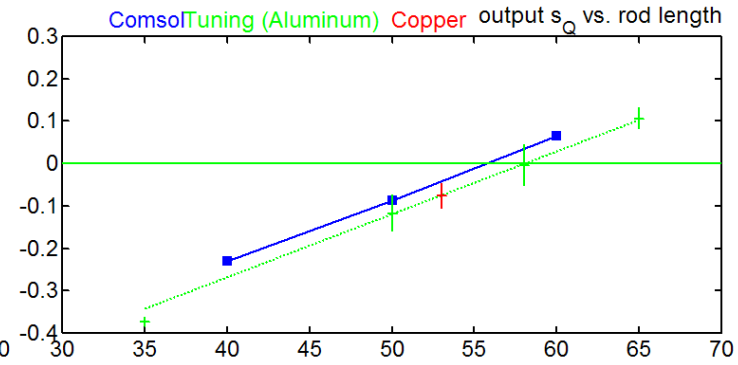
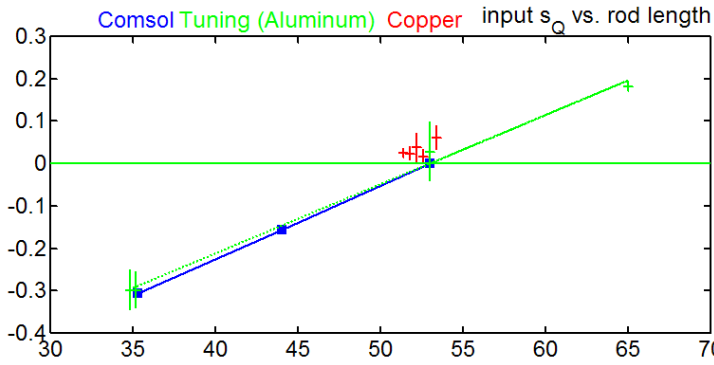
Eigenfrequencies and quadratic frequency separations (QFS) in MHz.

IPHI Impulse Error Functions



LINAC4 Stability

- measured s parameter in dipole subspace not in agreement with calculated value, but in agreement with measured spectra
- rod length is chosen smaller than optimum for $s_{QQ} = 0$ to save dipole stability

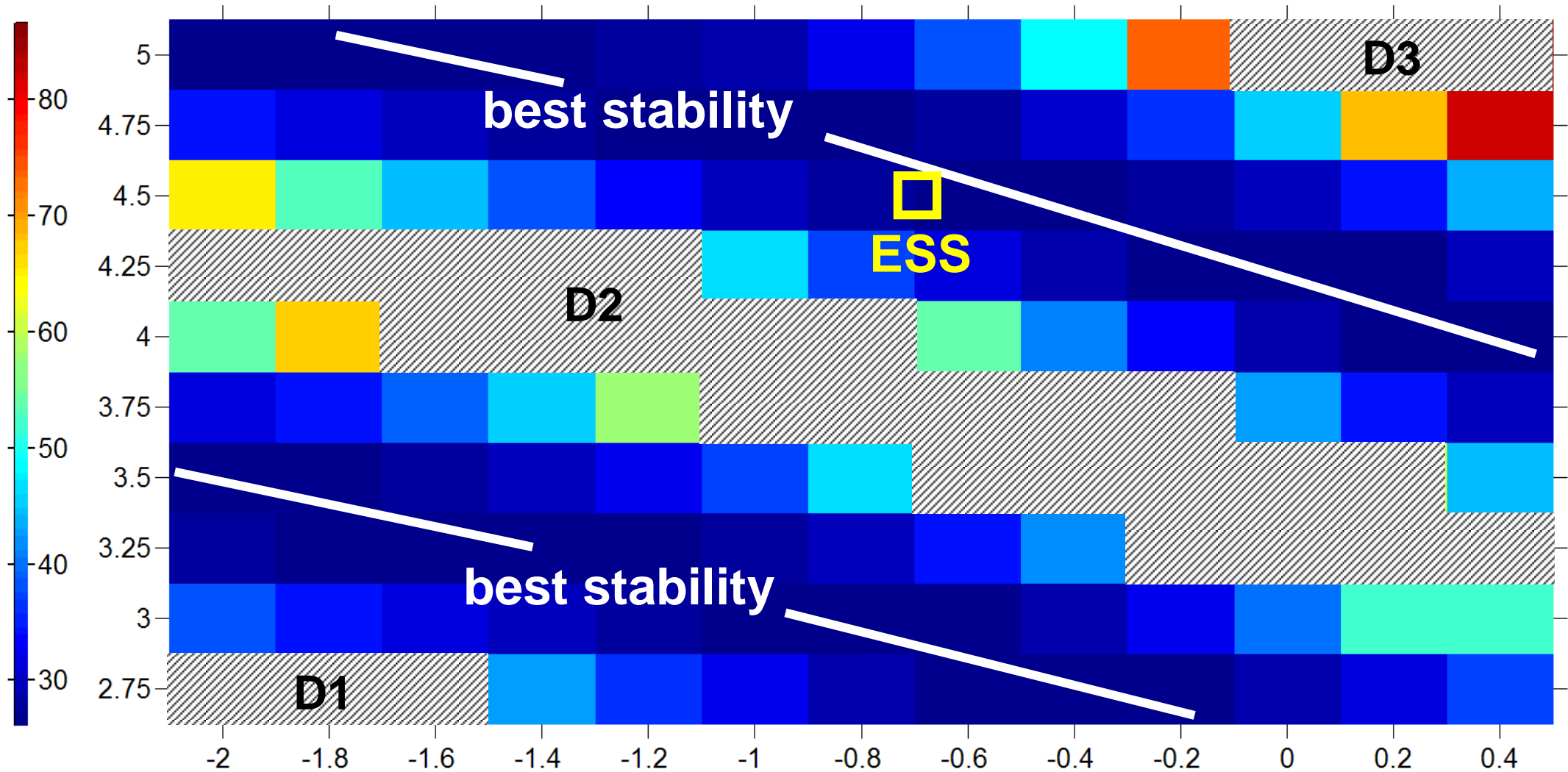


Rods (mm)	53.0 / 55.7	53.0 / 53.0		53.0 / 53.0
	specification	prior to slug tuning		after slug tuning
	Comsol + TLM	measured	TLM (measured s)	measured
Q 0 "Q _n "	345.32 [0.00]	345.50 [0.0]	345.33 [0.0]	352.13 [0.0]
Q 1	348.82 [+49.3]	348.69 [+47.0]	348.84 [+49.3]	355.31 [+47.5]
D 1	338.45 [-68.5]	338.50 [-69.2]	338.06 [-70.5]	344.63 [-72.3]
Q 0 "Q _n "	345.32 [0.0]	345.50 [0.0]	345.33 [0.0]	352.13 [0.0]
D 2	348.42 [+46.4]	347.88 [+40.6]	347.96 [+42.7]	353.50 [+31.2]

ESS Stability Design

$\|h_{Qn,S/T}\|$ vs. end boundary condition parameter $s_{S/T}$ and RFQ length ℓ
 optimal RFQ length $\ell^* = \sqrt{\kappa^2 + \kappa + \frac{1}{2}} \sqrt{(1+r)/r} \frac{\pi V}{\omega_{Q0}} \quad s_{S/T} = 0, \quad \kappa \in \mathbb{N}$

usual values w/o dipole rods



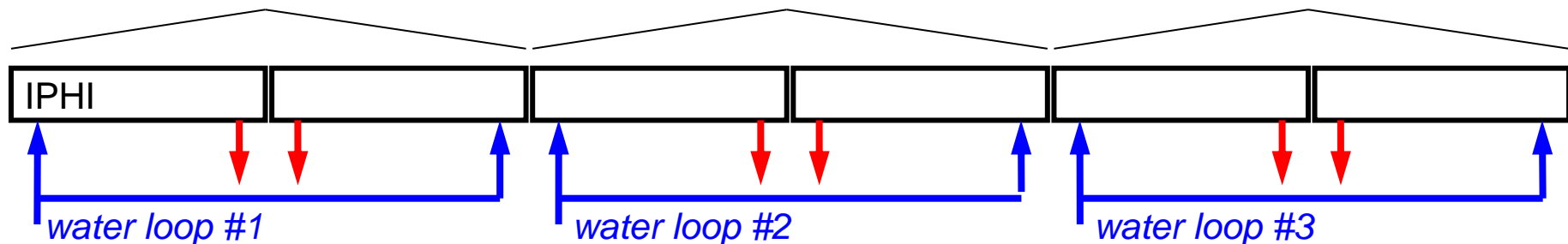
Sensitivity to Perturbations under Operation

CW linacs : deformations due to RF heating / water cooling combination

low duty cycle linacs : thermal expansions due to water temperature variations

→ spectral contents of perturbation is important

→ in general alternating water flow direction from one module to the next is better



apply perturbation – capacitance basis function with adequate spectral index
– peak value of relative perturbation = 0.001 arb.

calculate peak value of resulting voltage perturbation

	IPHI	LINAC4	SPIRAL2	ESS
number of modules	6	3	5	5
sup $\Delta V_{Qn,Q} / V_{Qn,Q}$	$5.34 \cdot 10^{-3}$	$5.53 \cdot 10^{-3}$	$3.36 \cdot 10^{-4}$	$4.48 \cdot 10^{-3}$
sup $\Delta V_{Qn,S/T} / V_{Qn,Q}$	$5.22 \cdot 10^{-4}$	$7.88 \cdot 10^{-3}$	$3.18 \cdot 10^{-4}$	$5.84 \cdot 10^{-3}$

Measured Voltage Stability of LINAC4 (1/2)

Voltage monitoring:

pickup loops inserted in 16 slug tuners (4 quadrants in 4 cross-sections)

calibration: low RF power, nominal water temperatures, reference = bead-pull values

voltage reconstruction: TLM and sampling theory

Temperature variations:

water temperatures in the 3 RFQ modules are controlled independently

5 temperature distributions: O 26.0 – 26.0 – 26.0 (nominal)

A 25.5 – 26.0 – 26.5

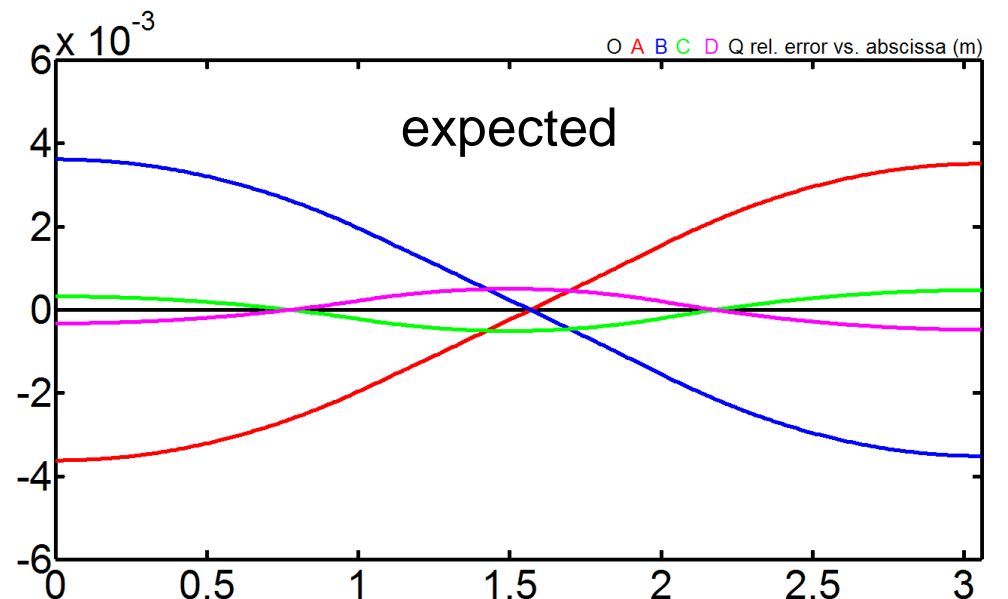
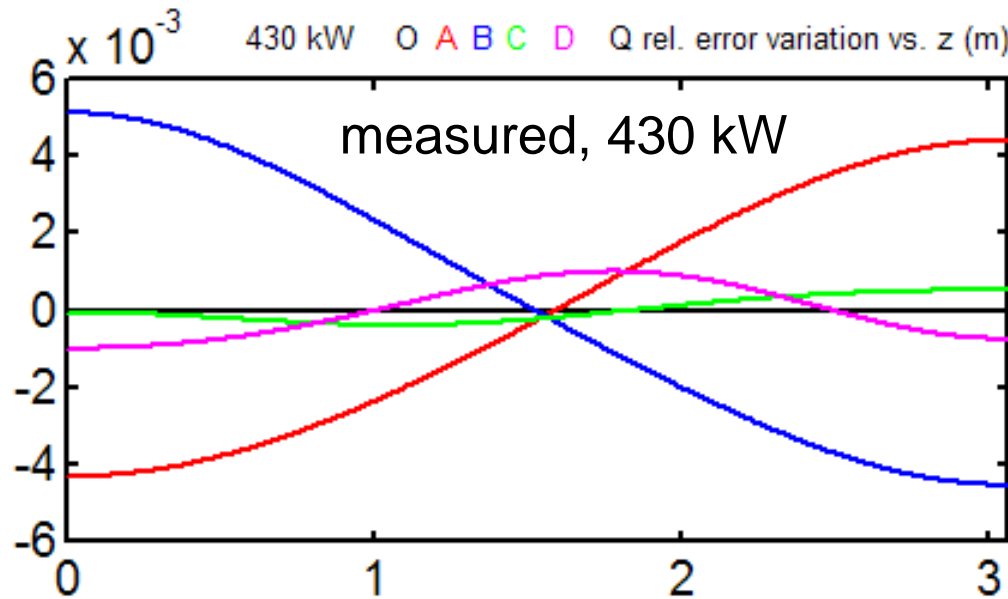
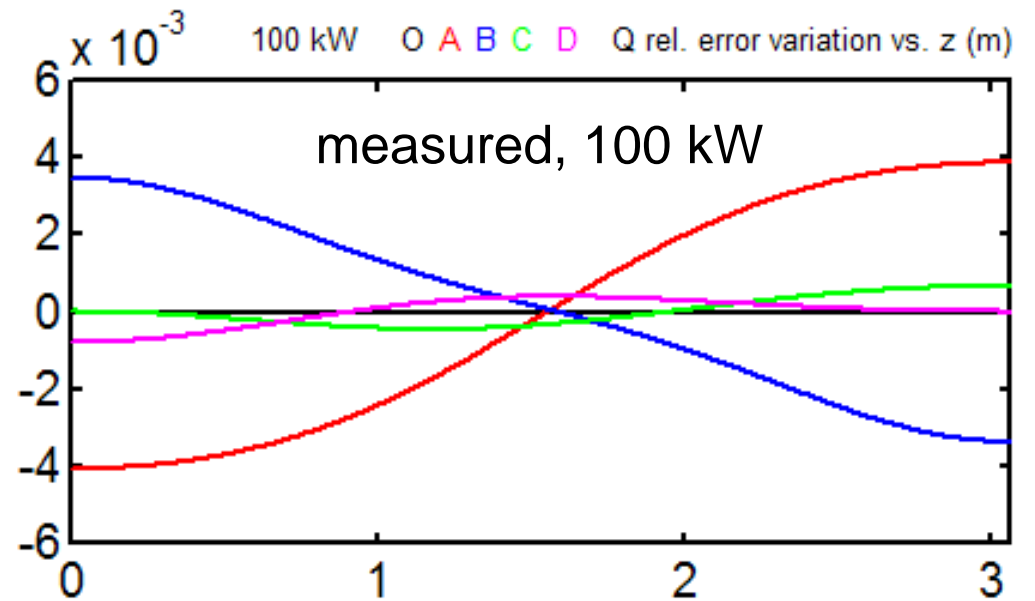
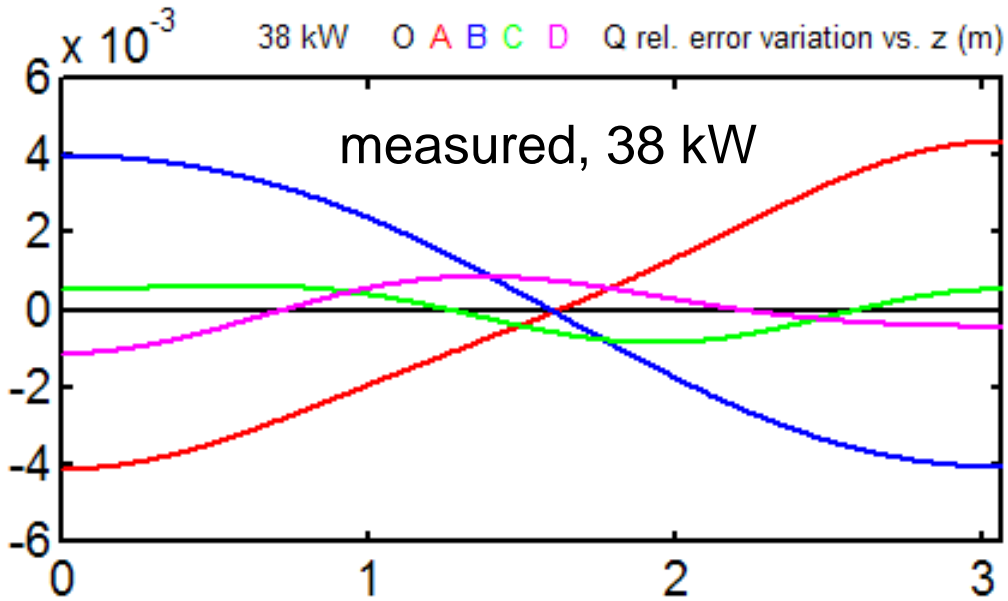
B 26.5 – 26.0 – 25.5

C 26.5 – 26.0 – 26.5

D 25.5 – 26.0 – 25.5

3 RF powers 38 kW, 100 kW, 430 kW (PD = 250 μ s, PRI = 1.2 s)

Measured Voltage Stability of LINAC4 (2/2)



4. Voltage and Frequency Tuning

The Voltage & Frequency Tuning Loop (1/3)

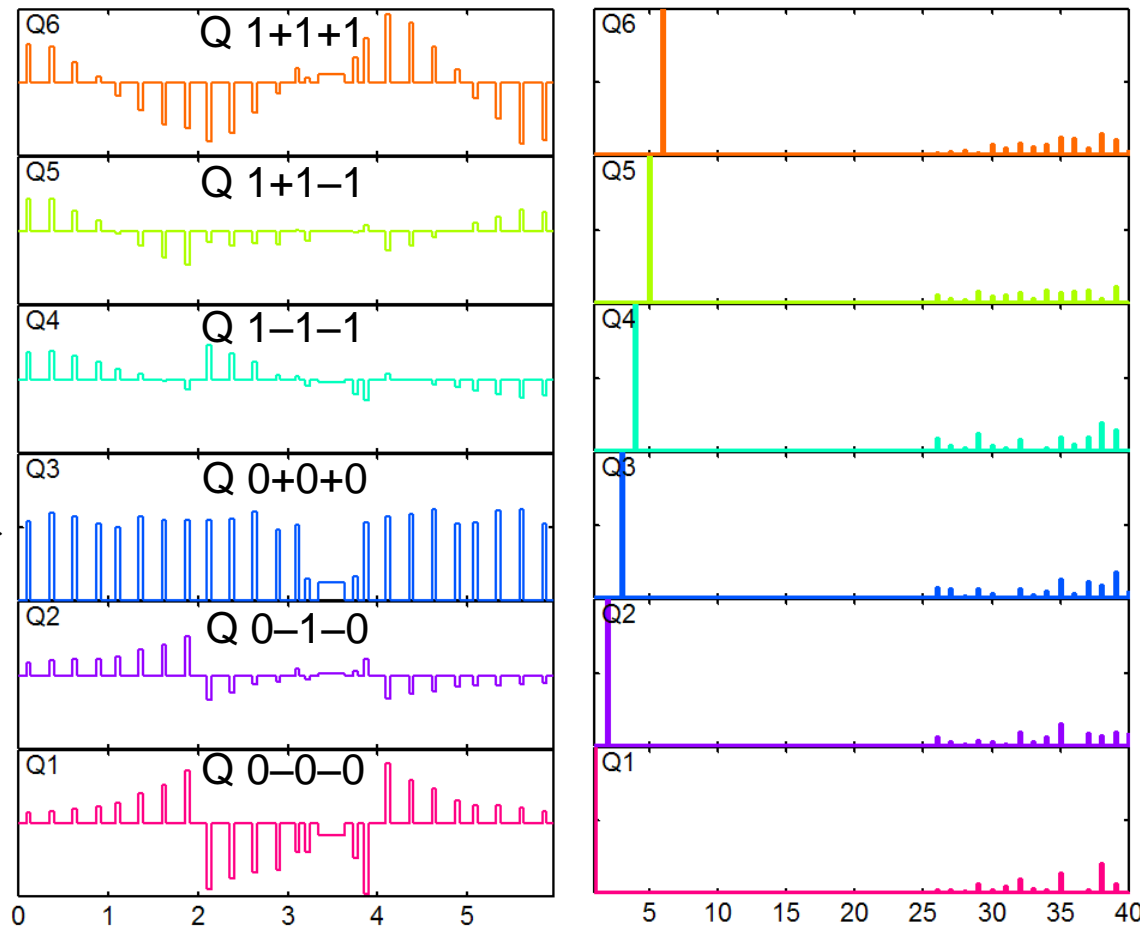
Idea: apply 1st-order perturbation theory to TLM to build dual bases:

- a discrete basis of tuner command functions
(tuner position or equivalently inductance perturbation)
- a truncated basis of voltage eigenfunctions
→ both are calculated with given boundary conditions, which should be tuned first

IPHI's 6 first tuner command functions in Q subset.

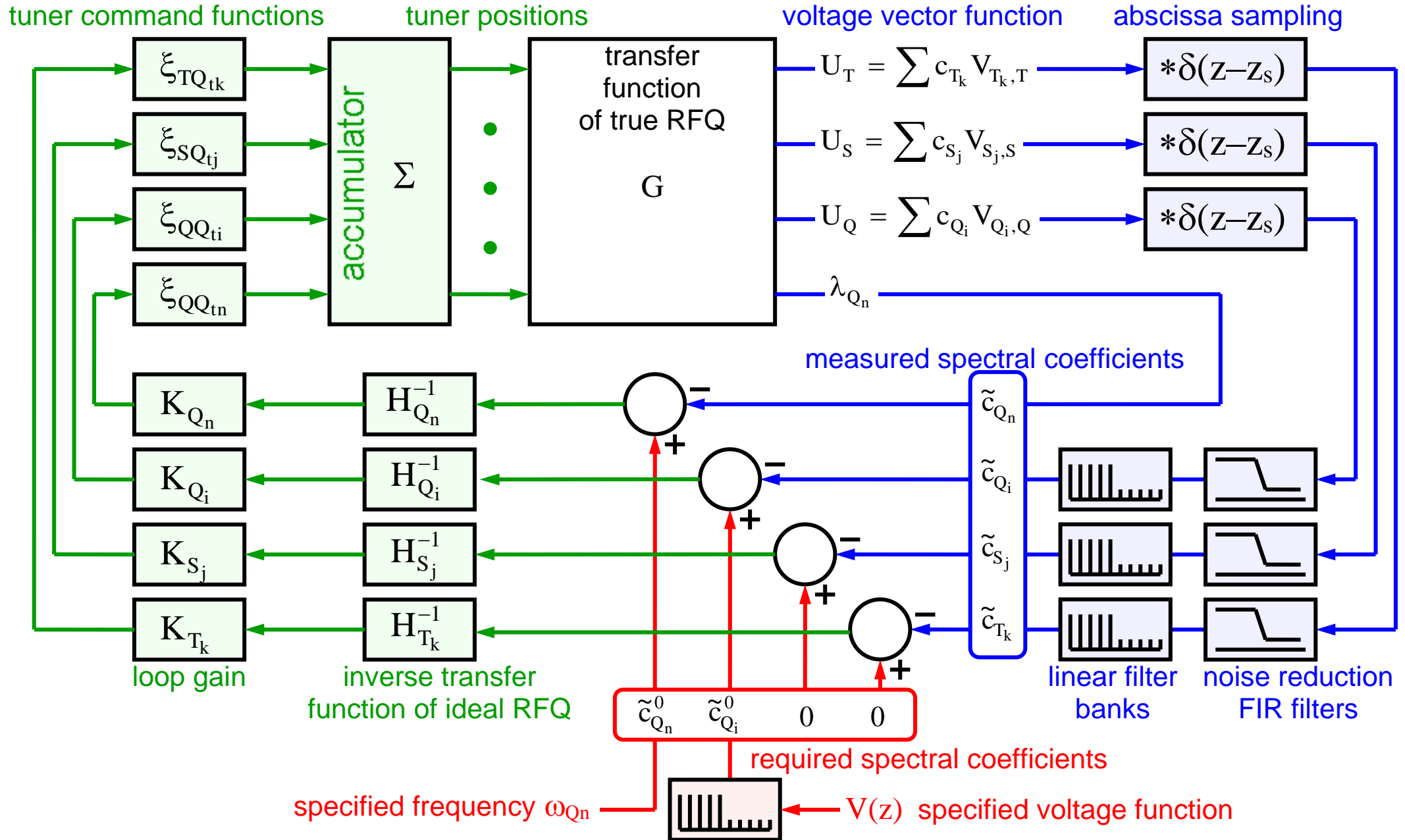
dim. = 25 (tuning devices in 25 cross-sections)

"Q_n" : frequency tuning →



spectral coefficients of voltage perturbation resulting from each command function

The Voltage & Frequency Tuning Loop (2/3)

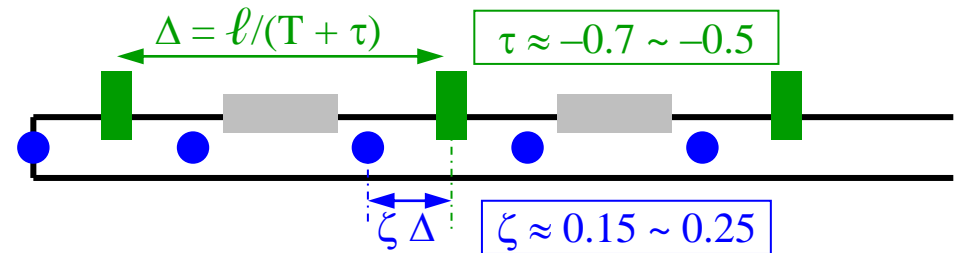


The Voltage & Frequency Tuning Loop (3/3)

working conditions

Voltage sampling:

- magnetic field samples (bead-pull) should reside far enough from local perturbations
- tune vacuum ports in electrically neutral position prior to braze if possible (Linac4)
- full-rank sampling
- output of filter banks free from aliasing

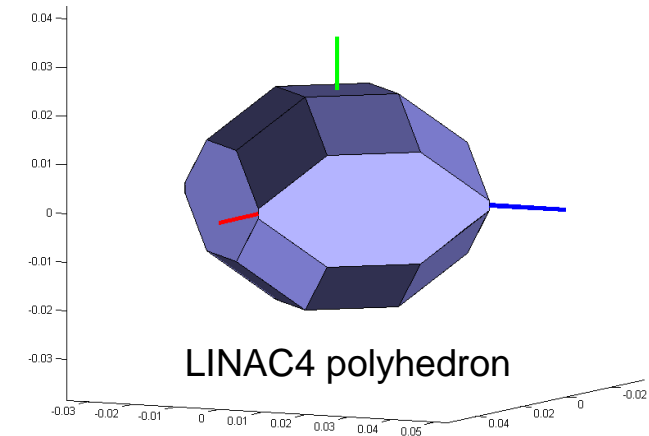


Inductance sampling:

- full-rank sampling (include RF ports in tuning devices set)

Tuner efficiency:

- use 3D simulation to determine individual tuner slope $\partial L / \partial h$ for TLM
- derive capacitance vs. intervane gap fcn. from simulations
- transform mechanical tolerance into capacitance error polyhedron
- use TLM + linear programming (Danzig) to determine worst tuning case



Tuning loop:

- unbiased
- equivalent to fixed-point iteration of the operator $A = I - G K H^{-1}$ (with all the convergence properties of fixed-point iterations!)
- converges iff A is a contraction, here satisfied iff eigenmodes are identically sorted for the ideal and the true RFQs according to eigenvalue order
- convergence is monotonic if A is diagonal, but may be non-monotonic otherwise

IPHI and LINAC4 Tuning

IPHI

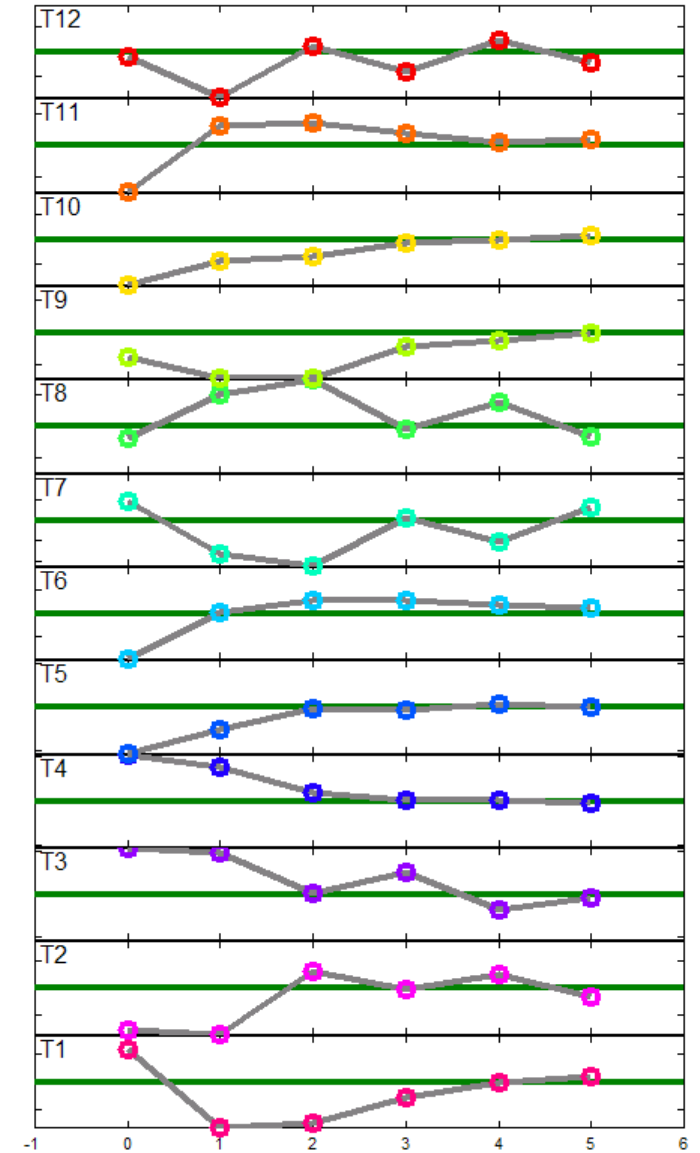
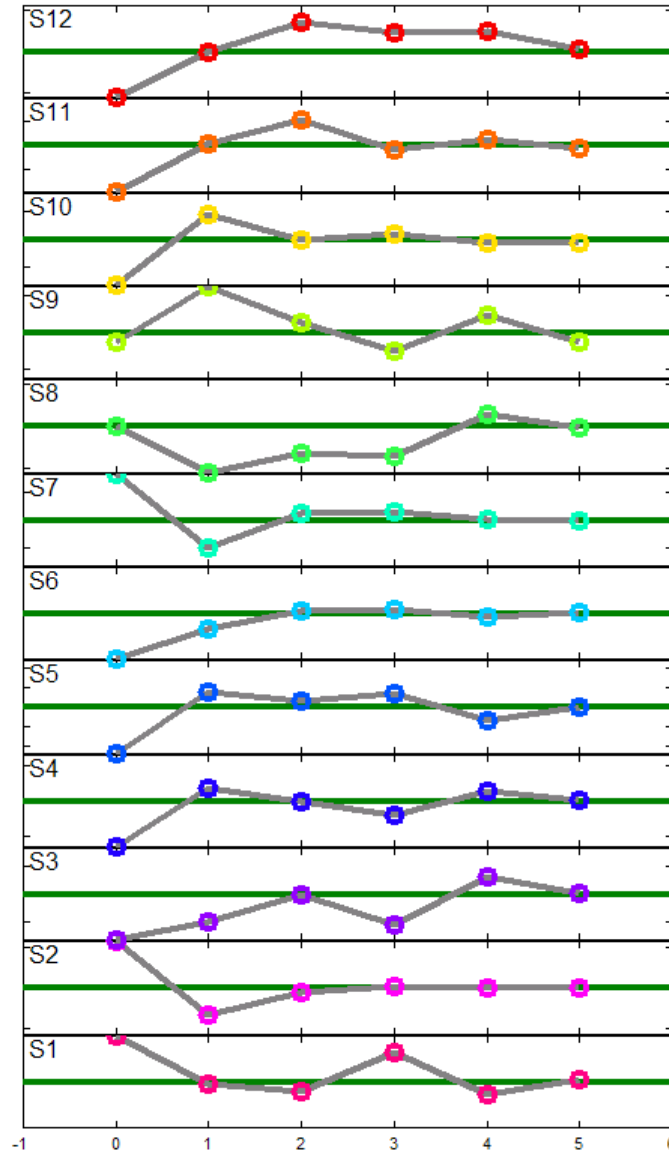
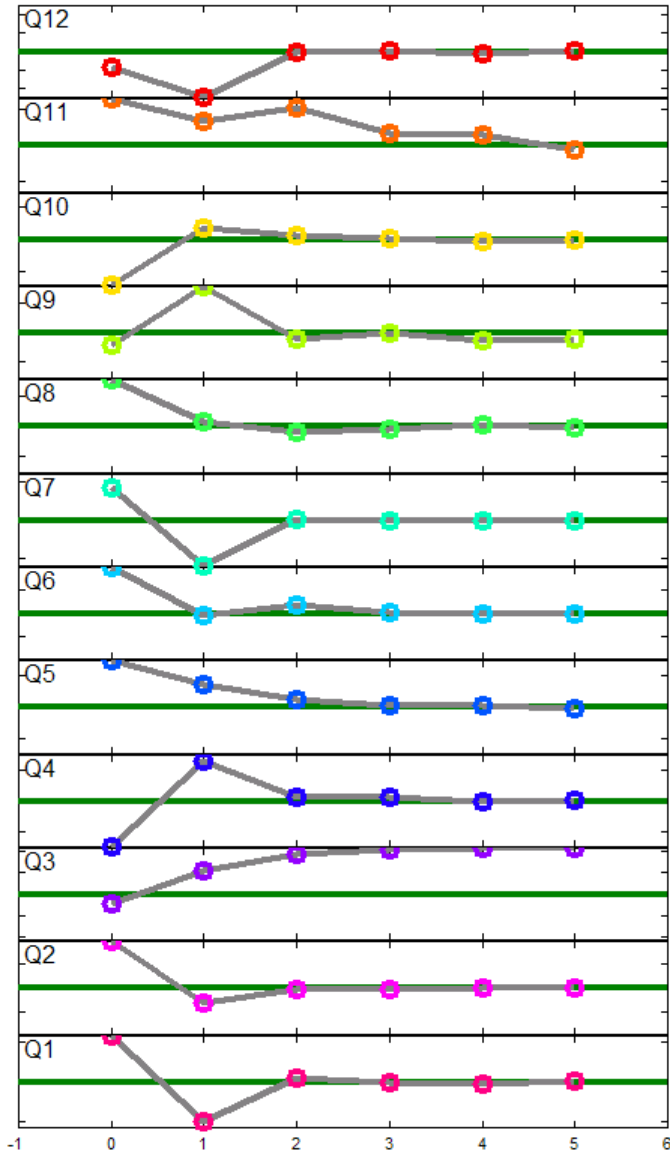
voltage peak relative errors (%):	Q	S	T
dummy RF ports, un-tuned	90	17.6	14.5
adjustable slugs, RF ports, tuned	0.78	0.28	0.63
copper slugs	3.97	1.32	2.07
tuner positions (mm):			
specified		+1.0 / +19.0	
specified, with safety margin		-5.0 / +25.0	
tuned RFQ		-1.7 / +12.5	

LINAC4

voltage peak relative errors (%):	Q	S	T
dummy RF port, un-tuned	5.55	5.53	7.19
adjustable slugs, RF port, tuned	0.70	1.48	3.07
copper slugs	0.63	3.45	2.29
tuner positions (mm):			
specified		-4.0 / +30.0	
tuned RFQ		+9.0 / +12.1	

IPHI Voltage Tuning

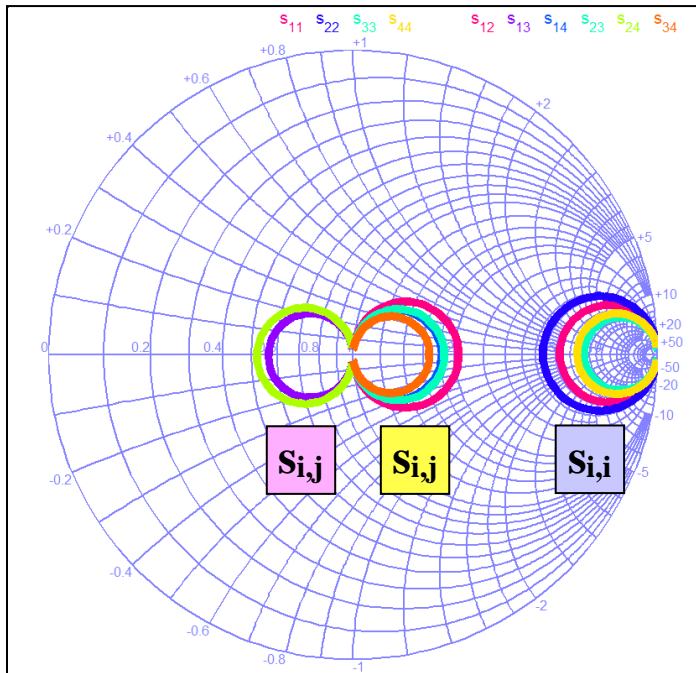
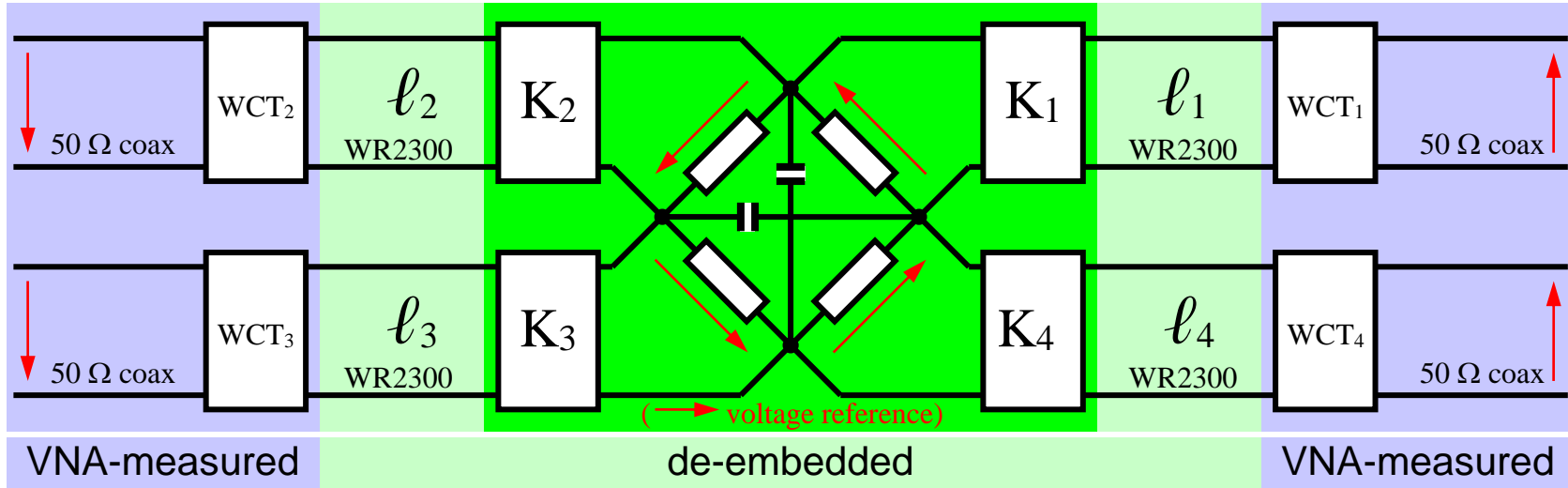
spectral coefficients vs. tuning step index in initial pre-tuning sequence



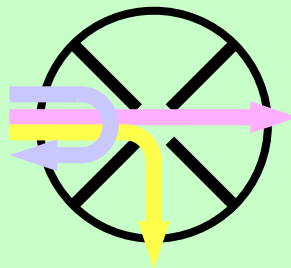
5. RF Power Coupling

The Structure of the 4×4 Scattering Matrix

in the case of ideal, quaternary-symmetric RFQ



$$S = \begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \end{bmatrix}$$



$$S_{i,j}(\omega) \approx \frac{\sqrt{\beta_i \beta_j}}{1 + \beta} (1 + e^{j\alpha(\omega)}) \zeta_{i,j}(\omega) \quad (\beta_i \sim 1/K_i^2)$$

$$S_{i,j}(\omega) \approx - \left(\frac{\sqrt{\beta_i \beta_j}}{1 + \beta} (1 + e^{j\alpha(\omega)}) + j\sqrt{\beta_i \beta_j} H(\omega) \right) \zeta_{i,j}(\omega)$$

$$S_{i,j}(\omega) \approx \left(1 - \frac{\beta_i}{1 + \beta} (1 + e^{j\alpha(\omega)}) + j\beta_i H(\omega) \right) \zeta_{i,i}(\omega)$$

circle ↑
 imaginary offset ↑
 propagation phasor (circular projection) ↑

A Few Essential S-matrix Properties

1. quarter-wave transformers may be represented by K-inverters with excellent accuracy in the complex plane (10^{-8} in simulations)
2. S-parameters of asymmetric RFQ may be represented by S-parameters of quaternary-symmetric RFQ with very small errors in the complex plane ($10^{-4} \sim 10^{-3}$ in measurements, even smaller in simulations)
→ electrical asymmetries are non-observable in standard VNA measurements
3. Q_0 , ω_0 and total coupling coefficient are correctly estimated, but partial coupling coefficients have to be corrected for voltage asymmetries (derived from bead-pull measurements)

$$\beta_i \rightarrow \beta_i \left(\frac{u_i^2}{\beta} \sum_{j=1}^4 \frac{\beta_j}{u_j^2} \right)^{-1}$$

4. multiport matching: total power reflection coefficient is $\Gamma^2 = (a^* S^* S a) / (a^* a)$: the 4-port circuit is matched when the excitation vector a is an eigen-vector a_1 corresponding to the smallest eigenvalue λ_1 of $S^* S$.
→ a_1 and $\lambda_1(\omega)$ also give estimates of Q_0 , ω_0 , β and β_i 's, without reference to the matrix structure

The IPHI 4-Port Scattering Matrix

under vacuum

	ω_0	Q_0	β_1	β_2	β_3	β_4	β
matrix reconstruction	352.1421	6875	0.2679	0.2795	0.3123	0.2782	1.1379
<i>with correction</i>			0.2693	0.2837	0.3107	0.2741	
multiport matching	352.1422	6786	0.2797	0.2979	0.3218	0.2988	1.1982

S-matrix reconstruction errors

Transmission to adjacent quadrants $(i,j) \in \{ (1,2) (1,4) (2,3) (3,4) \}$:

S-parameter reconstruction error $|\text{Re } \Delta s_{i,j}|, |\text{Im } \Delta s_{i,j}| < 2 \cdot 10^{-3}$

polar angle error $|\alpha_{i,j} - \alpha_0| < 0.8^\circ$

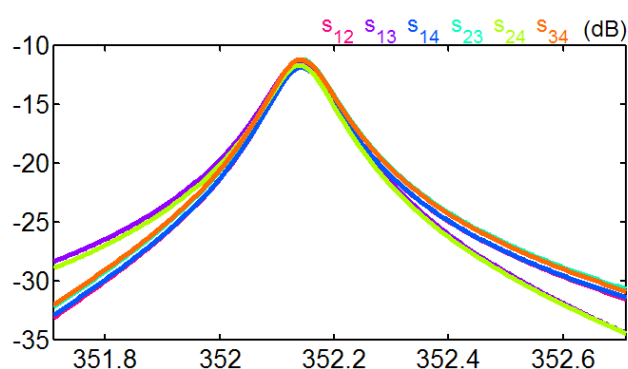
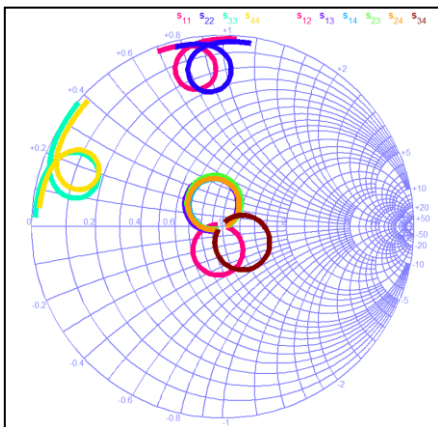
phasor closure error $|\arg(\zeta_{1,4}\zeta_{2,3}/\zeta_{1,2}\zeta_{3,4})| < 0.8^\circ$

Transmission to opposite quadrant $(i,j) \in \{ (1,3) (2,4) \}$:

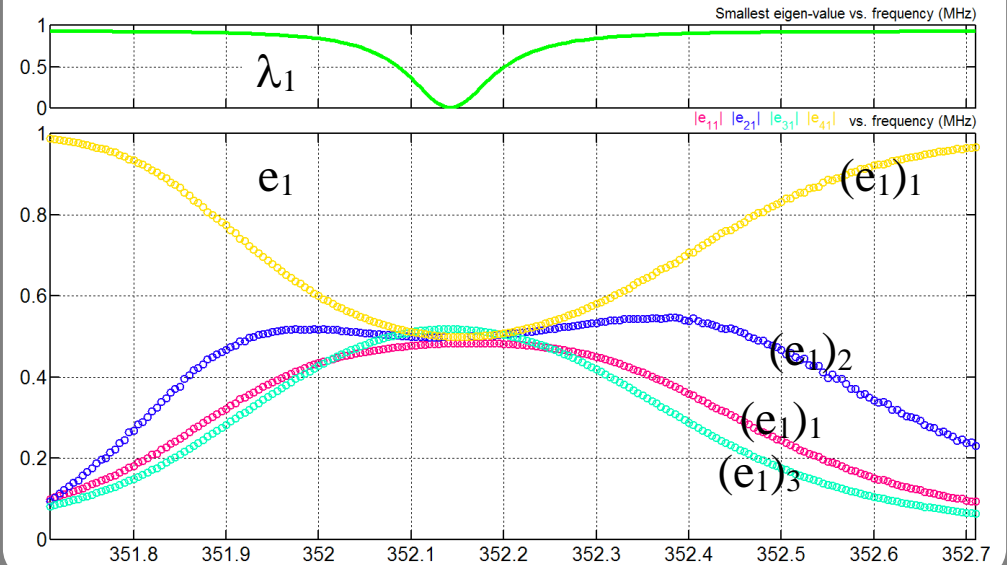
S-parameter reconstruction error $|\text{Re } \Delta s_{i,j}|, |\text{Im } \Delta s_{i,j}| < 2 \cdot 10^{-3}$

offset function $|H_{i,j} - H_0| < 2 \cdot 10^{-3}$

phasor closure error $|\arg(\zeta_{1,3}\zeta_{2,4}/\zeta_{1,2}\zeta_{3,4})| < 3.2^\circ$



multiport matching



Estimated power budget

RFQ = 1.17 to 1.18 MW Beam = 0.3 MW

ideal $\beta = 1.254$ to 1.257

estimated $\Gamma = -26$ dB (reconstructed matrix)

= -32 dB (multiport matching)

Final Comments

- the TLM creates accurate and invertible bridges between 3D simulations, electromagnetic specifications and measurable/observable quantities
- IPHI and Linac4 are accurately tuned
- thermal stability of Linac4 is experimentally demonstrated to be in agreement with design
- the sophistication of the electromagnetic perturbation analysis deserves an improvement of the way mechanical tolerances are specified



Thank you for attention !