Advanced RF Design and Tuning Methods of RFQ for High Intensity Proton Linacs

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Contents

1. Introduction and Theoretical Framework
2. End and Coupling Circuits Tuning
3. Stability Design, Tuning and Measurement
4. Voltage and Frequency Tuning
5. RF Power Coupling
6. Final Comments
1. Introduction and Theoretical Framework
### Our RFQ Projects for High Intensity Linacs

<table>
<thead>
<tr>
<th>Linac</th>
<th>Frequency (MHz)</th>
<th>Length (m)</th>
<th>Current (mA)</th>
<th>Mode</th>
<th>Energy (MeV)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IPHI</strong></td>
<td>352.2</td>
<td>6.0</td>
<td>100</td>
<td>CW</td>
<td>3</td>
<td>3 coupled segments of 2 brazed modules each. Status: tuned, start commissioning 2014 Q3</td>
</tr>
<tr>
<td><strong>LINAC4</strong></td>
<td>352.2</td>
<td>3.0</td>
<td>80</td>
<td>7.5%</td>
<td>3</td>
<td>1 segment of 3 brazed modules. Status: operational.</td>
</tr>
<tr>
<td><strong>SPIRAL2</strong></td>
<td>88.05</td>
<td>5.0</td>
<td>5</td>
<td>CW</td>
<td>3</td>
<td>1 segment of 5 bolted modules. Status: start tuning 2014 Q3.</td>
</tr>
<tr>
<td><strong>ESS</strong></td>
<td>352.2</td>
<td>4.5</td>
<td>62.5</td>
<td>4%</td>
<td>3.6</td>
<td>1 segment of 5 brazed modules. Status: design completed 2014 Q3.</td>
</tr>
</tbody>
</table>
The Loaded Lossless 4-Wire Transmission Line Model (TLM)

Axial region: $H_z \approx 0$, TEM 4-wire line
- 4 capacitances between adjacent electrodes $C_1$ to $C_4$ (F/m)
- 2 capacitances between opposite electrodes $C_a$, $C_b$ (F/m)
- Fundamental TEM relation: $v^2 L_s C = I$

Quadrants are $\lambda/4$ resonators
- Complement with 4 inductances $L_1$ to $L_4$ (H.m)

Transmission line equation (dim. 3, since three cuts make the system of conductors simply connected):

$$-\frac{\partial}{\partial z} \left( C \frac{\partial v}{\partial z} \right) + \frac{1}{v^2} L v = \frac{\omega^2}{v^2} C v$$
The TLM Canonical Basis \{Q,S,T\}

For an ideal (quaternary-symmetric) RFQ:

- $C_Q$ & $L_Q$ are diagonal
- $Q$, $S$ & $T$ are decoupled
TLM Boundary Conditions

arbitrary reciprocal lossless circuits, defined in \{Q,S,T\} basis
by their admittance matrixes (which are assumed to exist)

circuit theory: admittance matrixes \( y_a \ y_{ci} \ y_b \)

\[
I(a) = -y_a \ U(a) \quad \begin{vmatrix} I(c_i^-) \\ -I(c_i^+) \end{vmatrix} = y_{ci} \begin{vmatrix} U(c_i^-) \\ U(c_i^+) \end{vmatrix} \quad I(b) = +y_b \ U(b)
\]

transmission line theory: s-matrixes \( s_a \ s_{ci} \ s_b \)

\[
\frac{\partial U(a)}{\partial z} = -s_a \ U(a) \quad \begin{vmatrix} \frac{\partial U(c_i^-)}{\partial z} \\ \frac{\partial U(c_i^+)}{\partial z} \end{vmatrix} = +s_{ci} \begin{vmatrix} U(c_i^-) \\ U(c_i^+) \end{vmatrix} \quad \frac{\partial U(b)}{\partial z} = +s_b \ U(b)
\]

\[
s_a = -j\omega \ L_{sQ}(a) \ y_a \quad s_{ci} = -j\omega \begin{vmatrix} L_{sQ}(c_i^-) \\ 0 \end{vmatrix} \begin{vmatrix} 0 \\ L_{sQ}(c_i^+) \end{vmatrix} y_{ci} \quad s_b = -j\omega \ L_{sQ}(b) \ y_b
\]
The TLM takes the form of a vector regular Sturm-Liouville problem

\[ \mathcal{L} U = -C_Q^{-1} \frac{\partial}{\partial z} \left( C_Q \frac{\partial U}{\partial z} \right) + \frac{1}{\nu^2} C^{-1} L_Q U \]

\[ \mathcal{L} U = \frac{\omega^2}{\nu^2} U \]

\( \mathcal{L} \) is un-bounded, with bounded compact inverse, and is self-adjoint for the inner-product

\[ \langle u, v \rangle = \int_{\Omega} v^* C_Q u \, dz \quad \text{(with given boundary conditions)} \]

three subsets Q, S and T of countable eigenpairs

\[
\begin{align*}
\omega_{Q_i}, V_{Q_i}(z) &= \begin{bmatrix} V_{Q_i,Q}(z) \\ V_{Q_i,S}(z) \\ V_{Q_i,T}(z) \end{bmatrix} \\
\omega_{S_j}, V_{S_j}(z) &= \begin{bmatrix} V_{S_j,Q}(z) \\ V_{S_j,S}(z) \\ V_{S_j,T}(z) \end{bmatrix} \\
\omega_{T_k}, V_{T_k}(z) &= \begin{bmatrix} V_{T_k,Q}(z) \\ V_{T_k,S}(z) \\ V_{T_k,T}(z) \end{bmatrix}
\end{align*}
\]
TLM Properties (2/3)

for an ideal (quaternary-symmetric) RFQ

\[
V_Q_i = \begin{bmatrix} V_{Q_1, Q} & 0 \\ 0 & 0 \end{bmatrix}, \quad V_{S_j} = \begin{bmatrix} V_{S_j, Q} \\ 0 \end{bmatrix}, \quad V_T_k = \begin{bmatrix} 0 \\ V_{T_k, Q} \end{bmatrix}
\]

the plot shows the 6 first eigenfunctions of the Q subset for IPHI

"Q_n" is the nickname for the accelerating mode; here "Q_n" is Q 0+0+0
First-order perturbation analysis reveals dual bases for parameter perturbation functions and resulting voltage perturbation functions. Example:

**Capacitance perturbations**

\[ C_1 = C_{QQ} + C_{SQ} + C_{SSTT} \]
\[ C_2 = C_{QQ} - C_{TQ} - C_{SSTT} \]
\[ C_3 = C_{QQ} - C_{SQ} + C_{SSTT} \]
\[ C_4 = C_{QQ} + C_{TQ} - C_{SSTT} \]

\[
\begin{bmatrix}
\Delta C_{QQ} \\
\Delta C_{SQ} \\
\Delta C_{TQ}
\end{bmatrix} = \sum_{\delta=0}^{\infty} p_{QQ\delta} C_{Q\delta} + \sum_{\alpha=0}^{\infty} p_{SQ\alpha} C_{S\alpha} + \sum_{\beta=0}^{\infty} p_{TQ\beta} C_{T\beta}
\]

**Eigenpair perturbation with duality relations**

\[
\Delta \lambda_{Qn}, \quad \Delta V_{Qn} = \sum_{i=0}^{\infty} c_{Qi} V_{Qi} + \sum_{j=0}^{\infty} c_{Sj} V_{Sj} + \sum_{k=0}^{\infty} c_{Tk} V_{Tk}
\]

\[
\Delta \lambda_{Qn} = p_{QQn} \quad \text{for } i \neq n
\]
\[
c_{Qi} (\lambda_{Qn} - \lambda_{Qi}) = p_{QQi}
\]
\[
c_{Sj} (\lambda_{Qn} - \lambda_{Sj}) = p_{SQj}
\]
\[
c_{Tk} (\lambda_{Qn} - \lambda_{Tk}) = p_{TQk}
\]

\[
\{ c_{Qi}, c_{Sj}, c_{Tk} \}_{0 \leq i, j, k \leq \infty} \quad \text{is the spectral analysis of } \Delta V_{Qn}\]
Effects of Modulations on Line Parameters
ESS RFQ 2D/3D simulations
un-modulated 2D / un-modulated 2D / modulated 3D

one simulation cell = one half-period

parallel capacitance: \( \delta C < 0.01 \text{ pF/m} \) → negligible effect

the two diagonal capacitances \( C_a \) and \( C_b \) oscillate about a mean value from one cell to the next

diagonal capacitance: \( \delta C < 0.8 \text{ pF/m} \) → strong impact on dipole eigen-frequencies, hence on stability
Effects of Modulation Style
LINAC4 RFQ 2D/3D simulations (Comsol)

→ un-modulated profile of LINAC4 electrodes is constant

→ 3 simulations:
  – in green : un-modulated electrodes
  – in red : sine modulation
  – in blue : 2-term potential modulation

→ the sine modulation induces too much detuning for reasonable slug dimensions. RFQ cross-section could not be kept constant.
2. End and Coupling Circuits Tuning
End and Coupling Circuits Tuning

End circuit s matrix (ex. in \( z = a \))

\[
\begin{bmatrix}
\partial U_Q / \partial z \\
\partial U_S / \partial z \\
\partial U_T / \partial z
\end{bmatrix}
= -
\begin{bmatrix}
s_{QQ} & s_{QS} & s_{QT} \\
s_{SQ} & s_{SS} & s_{ST} \\
s_{TQ} & s_{TS} & s_{TT}
\end{bmatrix}
\begin{bmatrix}
U_Q \\
U_S \\
U_T
\end{bmatrix}
\]

\[
= 0
\]

(since \( U_T(z) = U_S(z) = 0 \) \( \forall z \) in the tuned RFQ)

\[ s_{QQ} = -\frac{1}{V(a)} \frac{\partial V(a)}{\partial z} \]

\( V(z) = \) specified voltage

Coupling circuit s matrix (in \( z = c \))

\[
\begin{bmatrix}
\partial U_Q^- / \partial z \\
- \partial U_Q^+ / \partial z
\end{bmatrix}
= \begin{bmatrix}
s_c^- & s_c^- & -s_c^- \\
- s_c^- & s_c^+ & s_c^+ + s_c^-)
\end{bmatrix}
\]

\[
= 0
\]

coupling coefficient

\[ s_c = \frac{\omega^2 C_c}{v^2 4C} \]

\( U_Q^- = U_Q^+ \) in the tuned RFQ

\[ s_c^- = -s_c^+ = \frac{1}{V(c)} \frac{\partial V(c)}{\partial z} \]

tuning

\[ s_{\Sigma\Sigma} = \frac{1}{2} (s_c^- - s_c^+) = \frac{1}{V(c)} \frac{\partial V(c)}{\partial z} \]

\[ s_{\Delta\Sigma} = -\frac{1}{2} (s_c^- + s_c^+) = 0 \]

matching

tuning : adequate voltage slope across boundary

matching : continuous voltage across boundary
Tunable Devices for End and Coupling Circuits

- Adjustable thickness
  - IPHI input end-plate
  - IPHI coupling-plates #1 and #2

- Adjustable "quadrupole" rods
  - IPHI output end-plate
  - LINAC4 input and output plates
  - SPIRAL2 input and output end-plates
  - ESS input and output end-plates
Use $M$ linearly independent pairs $\{\partial U/\partial z, U\}$ to estimate unknown coefficients of $s$ matrixes. Excitations are obtained with $M$ preset tuner positioning at some distance from boundary. $M = 5$ for end circuits; $M = 11$ for coupling circuits (number of bead-pulls is $M$).

Example: IPHI coupling circuit #2

- State equation: non-inverting branch in red
- Matching and tuning conditions
### IPHI and LINAC4 Realized Boundary Conditions

<table>
<thead>
<tr>
<th></th>
<th>legend: good / not good, don't know why / fair, know why. s parameters in m⁻¹ (&quot;V/m/V&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IPHI</strong></td>
<td>expected</td>
</tr>
<tr>
<td>input end-circuit</td>
<td>$s_{QQ}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(s_{QQ})$</td>
</tr>
<tr>
<td>coupling-circuit #1</td>
<td>$s_{\Sigma \Sigma}$</td>
</tr>
<tr>
<td></td>
<td>$s_{\Delta \Sigma}$</td>
</tr>
<tr>
<td></td>
<td>$C_{c}$</td>
</tr>
<tr>
<td>coupling-circuit #2</td>
<td>$s_{\Sigma \Sigma}$</td>
</tr>
<tr>
<td></td>
<td>$s_{\Delta \Sigma}$</td>
</tr>
<tr>
<td></td>
<td>$C_{c}$</td>
</tr>
<tr>
<td>output end-circuit</td>
<td>$s_{QQ}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(s_{QQ})$</td>
</tr>
<tr>
<td><strong>LINAC4</strong></td>
<td>expected</td>
</tr>
<tr>
<td>input end-circuit</td>
<td>$s_{QQ}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(s_{QQ})$</td>
</tr>
<tr>
<td>output end-circuit</td>
<td>$s_{QQ}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(s_{QQ})$</td>
</tr>
</tbody>
</table>
3. Stability Design, Tuning and Measurement
Stability Analysis
"stability" w.r.t. undesired perturbations under operation

impulse error function

the voltage relative perturbation vs. $z$

$$\frac{\Delta V_{Qn,Q}(z)}{V_{Qn,Q}(z)} = h_{Qn,Q}(z, z_0) \frac{\Delta c_{QQ}}{C(z_0)}$$

a Dirac-like QQ perturbation with mass $\Delta c_{QQ}/C$ located in $z = z_0$

idem for SQ and TQ functions:

$$\frac{\Delta V_{Qn,S}(z)}{V_{Qn,Q}(z)} = h_{Qn,S}(z, z_0) \frac{\Delta c_{SQ}}{C(z_0)}$$

$$\frac{\Delta V_{Qn,T}(z)}{V_{Qn,Q}(z)} = h_{Qn,T}(z, z_0) \frac{\Delta c_{TQ}}{C(z_0)}$$

compare RFQ designs with the norms

$$\|h_{Qn,Q}\| := \sup_{z_0 \in \Omega} \sup_{z \in \Omega} \left|h_{Qn,Q}(z, z_0)\right|, \quad \|h_{Qn,S}\|, \quad \|h_{Qn,T}\|$$

these functions depend on

$$\frac{1}{\lambda_{Qn} - \lambda_i}, \quad \frac{1}{\lambda_{Qn} - \lambda_{S_j}}, \quad \frac{1}{\lambda_{Qn} - \lambda_{T_k}},$$

i.e. on quadratic differences

$$f_{Qn}^2 - f_i^2, \quad f_{Qn}^2 - f_{S_j}^2, \quad f_{Qn}^2 - f_{T_k}^2,$$

and are infinite when an eigenmode coincides with $Q_n$. 
**IPHI Stability**

legend: unsegmented / segmented, specification / segmented, realized

Eigenfrequencies and quadratic frequency separations (QFS) in MHz.

<table>
<thead>
<tr>
<th>mode des.</th>
<th>specification</th>
<th>prior to slug tuning</th>
<th>after slug tuning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q 0–1–0</td>
<td>348.18 [-42.0]</td>
<td>349.55 [-25.1]</td>
<td>351.25 [-24.5]</td>
</tr>
<tr>
<td>Q 0+0+0  &quot;Qₙ&quot;</td>
<td>350.71 [ 0.00]</td>
<td>350.45 [ 0.0]</td>
<td>352.10 [ 0.0]</td>
</tr>
<tr>
<td>Q 1–1–1</td>
<td>353.69 [+47.8]</td>
<td>354.65 [+54.4]</td>
<td>356.40 [+55.2]</td>
</tr>
<tr>
<td>D 1+1+1</td>
<td>347.67 [-46.1]</td>
<td>348.10 [-40.5]</td>
<td>349.30 [-44.3]</td>
</tr>
<tr>
<td>Q 0+0+0  &quot;Qₙ&quot;</td>
<td>350.71 [ 0.0]</td>
<td>350.45 [ 0.0]</td>
<td>352.10 [ 0.0]</td>
</tr>
<tr>
<td>D 2–2–2</td>
<td>363.16 [+94.3]</td>
<td>362.60 [+93.1]</td>
<td>364.30 [+93.5]</td>
</tr>
</tbody>
</table>

Note that a short rod (ℓ < λ₀/2) is capacitive, hence its admittance is positive, and it may only increase s. When ℓ ≈ λ₀/2, s = ∞, and the RFQ end is a short-circuit.
IPHI Impulse Error Functions

Q perturbation, specified

Q perturbation, achieved

D perturbation, specified

D perturbation, achieved
LINAC4 Stability

- measured s parameter in dipole subspace not in agreement with calculated value, but in agreement with measured spectra
- rod length is chosen smaller than optimum for $s_{QQ} = 0$ to save dipole stability

<table>
<thead>
<tr>
<th>Rods (mm)</th>
<th>specification</th>
<th>prior to slug tuning</th>
<th>after slug tuning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Comsol + TLM</td>
<td>measured</td>
<td>measured</td>
</tr>
<tr>
<td>Q 0</td>
<td>&quot;Q_n&quot;</td>
<td>345.32 [ 0.00]</td>
<td>345.50 [ 0.0]</td>
</tr>
<tr>
<td>Q 1</td>
<td></td>
<td>348.82 [+49.3]</td>
<td>348.69 [+47.0]</td>
</tr>
<tr>
<td>D 1</td>
<td></td>
<td>338.45 [−68.5]</td>
<td>338.50 [−69.2]</td>
</tr>
<tr>
<td>Q 0</td>
<td>&quot;Q_n&quot;</td>
<td>345.32 [ 0.0]</td>
<td>345.50 [ 0.0]</td>
</tr>
<tr>
<td>D 2</td>
<td></td>
<td>348.42 [+46.4]</td>
<td>347.88 [+40.6]</td>
</tr>
</tbody>
</table>

(very) smooth optimum $s_{SS/T} \leq −0.3$
ESS Stability Design

$\| h_{Qn,S/T} \| \text{ vs. end boundary condition parameter } s_{S/T} \text{ and RFQ length } \ell$

optimal RFQ length $\ell^* = \sqrt{k^2 + \kappa + \frac{1}{2} \sqrt{(1 + r)/r}} \frac{\pi \nu}{\omega_{Q0}} \quad s_{S/T} = 0, \quad \kappa \in \mathbb{N}$

usual values w/o dipole rods
Sensitivity to Perturbations under Operation

CW linacs: deformations due to RF heating / water cooling combination
low duty cycle linacs: thermal expansions due to water temperature variations

→ spectral contents of perturbation is important
→ in general alternating water flow direction from one module to the next is better

apply perturbation — capacitance basis function with adequate spectral index
— peak value of relative perturbation = 0.001 arb.

calculate peak value of resulting voltage perturbation

<table>
<thead>
<tr>
<th>number of modules</th>
<th>IPHI</th>
<th>LINAC4</th>
<th>SPIRAL2</th>
<th>ESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>sup $\Delta V_{Qn,Q} / V_{Qn,Q}$</td>
<td>$5.34 \times 10^{-3}$</td>
<td>$5.53 \times 10^{-3}$</td>
<td>$3.36 \times 10^{-4}$</td>
<td>$4.48 \times 10^{-3}$</td>
</tr>
<tr>
<td>sup $\Delta V_{Qn,S/T} / V_{Qn,Q}$</td>
<td>$5.22 \times 10^{-4}$</td>
<td>$7.88 \times 10^{-3}$</td>
<td>$3.18 \times 10^{-4}$</td>
<td>$5.84 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Voltage monitoring:
- pickup loops inserted in 16 slug tuners (4 quadrants in 4 cross-sections)
- calibration: low RF power, nominal water temperatures, reference = bead-pull values
- voltage reconstruction: TLM and sampling theory

Temperature variations:
- water temperatures in the 3 RFQ modules are controlled independently
- 5 temperature distributions:
  - O 26.0 – 26.0 – 26.0 (nominal)
  - A 25.5 – 26.0 – 26.5
  - B 26.5 – 26.0 – 25.5
  - C 26.5 – 26.0 – 26.5
  - D 25.5 – 26.0 – 25.5
- 3 RF powers 38 kW, 100 kW, 430 kW (PD = 250 μs, PRI = 1.2 s)
Measured Voltage Stability of LINAC4 (2/2)

- Measured, 38 kW
- Measured, 100 kW
- Measured, 430 kW
- Expected
4. Voltage and Frequency Tuning
The Voltage & Frequency Tuning Loop (1/3)

Idea: apply 1\textsuperscript{st}-order perturbation theory to TLM to build dual bases:
- a discrete basis of tuner command functions
  (tuner position or equivalently inductance perturbation)
- a truncated basis of voltage eigenfunctions
\rightarrow both are calculated with given boundary conditions, which should be tuned first

IPHI's 6 first tuner command functions in $Q$ subset.
dim. = 25 (tuning devices in 25 cross-sections)

"$Q_n$" : frequency tuning \rightarrow

spectral coefficients of voltage perturbation resulting from each command function
The Voltage & Frequency Tuning Loop (2/3)

- **tuner command functions**
  - $\xi_{TQ_{tk}}$
  - $\xi_{SQ_{tj}}$
  - $\xi_{QQ_{ti}}$
  - $\xi_{QQ_{tn}}$

- **loop gain**
  - $K_{Q_n}$
  - $K_{Q_i}$
  - $K_{S_j}$
  - $K_{T_k}$

- **inverse transfer function of ideal RFQ**
  - $H^{-1}_{Q_n}$
  - $H^{-1}_{Q_i}$
  - $H^{-1}_{S_j}$
  - $H^{-1}_{T_k}$

- **transfer function of true RFQ**
  - $G$

- **tuner positions**
  - $\lambda_{Q_n}$

- **voltage vector function**
  - $U_T = \sum c_{T_k} V_{T_k,T}$
  - $U_S = \sum c_{S_j} V_{S_j,S}$
  - $U_Q = \sum c_{Q_i} V_{Q_i,Q}$

- **measured spectral coefficients**
  - $\tilde{c}_{Q_n}$
  - $\tilde{c}_{Q_i}$
  - $\tilde{c}_{S_j}$
  - $\tilde{c}_{T_k}$

- **linear filter banks**
- **noise reduction FIR filters**

- **required spectral coefficients**
  - $\tilde{c}_{Q_n}^0$
  - $\tilde{c}_{Q_i}^0$
  - $0$
  - $0$

- **specification frequency** $\omega_{Q_n}$

- **V(z) specified voltage function**

- **abscissa sampling**
  - $*\delta(z-Z_s)$
Voltage & Frequency Tuning Loop (3/3)

**Working conditions**

**Voltage sampling:**
- Magnetic field samples (bead-pull) should reside far enough from local perturbations.
- Tune vacuum ports in electrically neutral position prior to braze if possible (Linac4).
- Full-rank sampling.
- Output of filter banks free from aliasing.

**Inductance sampling:**
- Full-rank sampling (include RF ports in tuning devices set).

**Tuner efficiency:**
- Use 3D simulation to determine individual tuner slope $\partial L/\partial h$ for TLM.
- Derive capacitance vs. intervane gap function from simulations.
- Transform mechanical tolerance into capacitance error polyhedron.
- Use TLM + linear programming (Danzig) to determine worst tuning case.

**Tuning loop:**
- Unbiased.
- Equivalent to fixed-point iteration of the operator $A = I - GKH^{-1}$ (with all the convergence properties of fixed-point iterations!)
- Converges iff $A$ is a contraction, here satisfied iff eigenmodes are identically sorted for the ideal and the true RFQs according to eigenvalue order.
- Convergence is monotonic if $A$ is diagonal, but may be non-monotonic otherwise.

\[ \Delta = \ell/(T + \tau) \]

\[ \tau \approx -0.7 \sim -0.5 \]

\[ \zeta \approx 0.15 \sim 0.25 \]
## IPHI and LINAC4 Tuning

### IPHI

<table>
<thead>
<tr>
<th>Voltage Peak Relative Errors (%)</th>
<th>Q</th>
<th>S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dummy RF Ports, Un-tuned</td>
<td>90</td>
<td>17.6</td>
<td>14.5</td>
</tr>
<tr>
<td>Adjustable Slugs, RF Ports, Tuned</td>
<td>0.78</td>
<td>0.28</td>
<td>0.63</td>
</tr>
<tr>
<td>Copper Slugs</td>
<td>3.97</td>
<td>1.32</td>
<td>2.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tuner Positions (mm)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Specified</td>
<td>+1.0</td>
<td>+19.0</td>
<td></td>
</tr>
<tr>
<td>Specified, with safety margin</td>
<td>−5.0</td>
<td>+25.0</td>
<td></td>
</tr>
<tr>
<td>Tuned RFQ</td>
<td>−1.7</td>
<td>+12.5</td>
<td></td>
</tr>
</tbody>
</table>

### LINAC4

<table>
<thead>
<tr>
<th>Voltage Peak Relative Errors (%)</th>
<th>Q</th>
<th>S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dummy RF Port, Un-tuned</td>
<td>5.55</td>
<td>5.53</td>
<td>7.19</td>
</tr>
<tr>
<td>Adjustable Slugs, RF Port, Tuned</td>
<td>0.70</td>
<td>1.48</td>
<td>3.07</td>
</tr>
<tr>
<td>Copper Slugs</td>
<td>0.63</td>
<td>3.45</td>
<td>2.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tuner Positions (mm)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Specified</td>
<td>−4.0</td>
<td>+30.0</td>
<td></td>
</tr>
<tr>
<td>Tuned RFQ</td>
<td>+9.0</td>
<td>+12.1</td>
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</tbody>
</table>
IPHI Voltage Tuning
spectral coefficients vs. tuning step index in initial pre-tuning sequence
5. RF Power Coupling
The Structure of the $4 \times 4$ Scattering Matrix

in the case of ideal, quaternary-symmetric RFQs

$$S = \begin{bmatrix}
S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\
S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\
S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\
S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4}
\end{bmatrix}$$

$$s_{i,j}(\omega) \approx \frac{\sqrt{\beta_i \beta_j}}{1 + \beta} \left(1 + e^{j\alpha(\omega)}\right) \zeta_{i,j}(\omega) \quad (\beta_i \sim 1/K_i^2)$$

$$s_{i,j}(\omega) \approx -\left(\frac{\sqrt{\beta_i \beta_j}}{1 + \beta} \left(1 + e^{j\alpha(\omega)}\right) + j\beta_i \beta_j H(\omega)\right) \zeta_{i,j}(\omega)$$

$$s_{i,j}(\omega) \approx \left(1 - \frac{\beta_i}{1 + \beta} \left(1 + e^{j\alpha(\omega)}\right) + j\beta_i H(\omega)\right) \zeta_{i,i}(\omega)$$

$\text{VNA-measured}$
$\text{de-embedded}$
$\text{VNA-measured}$

\(50\,\Omega\text{ coax}\)
\(\ell_2\text{ WR2300}\)
\(\ell_1\text{ WR2300}\)
\(\ell_3\text{ WR2300}\)
\(\ell_4\text{ WR2300}\)

\(\text{VNA-}\text{measured}\)
\(\text{de-}\text{embedded}\)
\(\text{VNA-}\text{measured}\)
A Few Essential S-matrix Properties

1. Quarter-wave transformers may be represented by K-inverters with excellent accuracy in the complex plane (10^-8 in simulations).

2. S-parameters of asymmetric RFQ may be represented by S-parameters of quaternary-symmetric RFQ with very small errors in the complex plane (10^-4 ~ 10^-3 in measurements, even smaller in simulations) → electrical asymmetries are non-observable in standard VNA measurements.

3. $Q_0$, $\omega_0$ and total coupling coefficient are correctly estimated, but partial coupling coefficients have to be corrected for voltage asymmetries (derived from bead-pull measurements):

\[ \beta_i \rightarrow \beta_i \left( \frac{u_i^2}{\beta} \sum_{j=1}^{4} \beta_j u_j^2 \right)^{-1} \]

4. Multiport matching: total power reflection coefficient is $\Gamma^2 = (a^*S^*Sa)/(a^*a)$: the 4-port circuit is matched when the excitation vector $a$ is an eigen-vector $a_1$ corresponding to the smallest eigenvalue $\lambda_1$ of $S^*S$. → $a_1$ and $\lambda_1(\omega)$ also give estimates of $Q_0$, $\omega_0$, $\beta$ and $\beta_i$'s, without reference to the matrix structure.
# The IPHI 4-Port Scattering Matrix
under vacuum

<table>
<thead>
<tr>
<th></th>
<th>$\omega_0$</th>
<th>$Q_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta$</th>
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<tbody>
<tr>
<td>matrix reconstruction</td>
<td>352.1421</td>
<td>6875</td>
<td>0.2679</td>
<td>0.2795</td>
<td>0.3123</td>
<td>0.2782</td>
<td>1.1379</td>
</tr>
<tr>
<td><em>with correction</em></td>
<td></td>
<td></td>
<td>0.2693</td>
<td>0.2837</td>
<td>0.3107</td>
<td>0.2741</td>
<td></td>
</tr>
<tr>
<td>multiport matching</td>
<td>352.1422</td>
<td>6786</td>
<td>0.2797</td>
<td>0.2979</td>
<td>0.3218</td>
<td>0.2988</td>
<td>1.1982</td>
</tr>
</tbody>
</table>

- **S-matrix reconstruction errors**
  - Transmission to adjacent quadrants $(i,j) \in \{(1,2) (1,4) (2,3) (3,4)\}$:
    - $S$-parameter reconstruction error: $|\text{Re} \Delta s_{i,j}|, |\text{Re} \Delta s_{i,j}| < 2 \times 10^{-3}$
    - Polar angle error: $|\alpha_{i,j} - \alpha_0| < 0.8^\circ$
    - Phasor closure error: $|\arg(\zeta_{1,2}\zeta_{2,3}\zeta_{3,4})| < 0.8^\circ$
  - Transmission to opposite quadrant $(i,j) \in \{(1,3) (2,4)\}$:
    - $S$-parameter reconstruction error: $|\text{Re} \Delta s_{i,j}|, |\text{Re} \Delta s_{i,j}| < 2 \times 10^{-3}$
    - Offset function: $|H_{i,j} - H_0| < 2 \times 10^{-3}$
    - Phasor closure error: $|\arg(\zeta_{1,2}\zeta_{2,3}\zeta_{3,4})| < 3.2^\circ$

---

## Estimated power budget

- $\text{RFQ} = 1.17$ to $1.18$ MW
- $\text{Beam} = 0.3$ MW
- Ideal $\beta = 1.254$ to $1.257$
- Estimated $\Gamma = -26$ dB (reconstructed matrix)
- Estimated $\Gamma = -32$ dB (multiport matching)
Final Comments

– the TLM creates accurate and invertible bridges between 3D simulations, electromagnetic specifications and measurable/observable quantities

– IPHI and Linac4 are accurately tuned

– thermal stability of Linac4 is experimentally demonstrated to be in agreement with design

– the sophistication of the electromagnetic perturbation analysis deserves an improvement of the way mechanical tolerances are specified
Thank you for attention!