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UNDULATOR RADIATION SPECTRAL BROADENING DUE TO RADIATION ENERGY LOSS*

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Abstract

A relativistic electron passing through an undulator generates electromagnetic radiation at the expenses of its own kinetic energy. This effect is usually not taken into account if the number of periods of the undulator is relatively small (100 - 200). However, at FEL facilities, long installations have been built, planned or are under construction, where many undulators are installed one after another for a total of several thousand undulator periods. For instance, the SASE1 and SASE2 lines at the European XFEL will consist of 35 undulators with 124 periods each. In this case, because of the electron energy decrease along its trajectory, the radiation from different undulators will drop out of synchronism. As a result, the radiation spectral line will be much wider. In the presented report, this effect was analyzed analytically and numerically for the case of spontaneous undulator radiation. An expression for the critical number of undulator periods, when the effect of electron energy loss should be properly taken into account, is derived. It is found that, for the case of the European XFEL, this number is about 1400 periods.

INTRODUCTION

Travelling down the undulator, the electron transfers part of its energy to the light wave and consequently decreases its kinetic energy. This effect is of crucial importance in free electron lasers. In this case, the beam energy decreases with the undulator distance z, and the undulator deflection parameter K should be tapered accordingly to maintain the resonant condition in order to both maintain minimal SASE bandwidth and not degrade the gain. The idea of tapering the undulator period and/or field amplitude along its axis was initially suggested in [1], and now is widely covered in the literature (see, for example, [2] and references therein). The spontaneous radiation from such devices has been analyzed analytically as well as numerically in [3 - 9].

Let us consider a non-tapered undulator with number of periods N. An electron kinetic energy loss increases proportionally with increasing number of undulator periods N, and the electron moves out of the resonant condition, broadening in such a manner spectral width of undulator spontaneous radiation harmonic. On the other hand, as larger is the number of undulator periods, as narrower is the harmonic spectral width because it is inversely proportional to N. It is apparent that at some sufficiently large number of undulator periods $N = N_{loss}$ the radiation energy loss will have a pronounced effect on radiation harmonic spectral width.

In this contribution we analyze the undulator spontaneous radiation spectral broadening due to radiation energy loss. It has been shown that $N_{loss} \cong \frac{1}{2\pi K}\sqrt{\frac{3\lambda_u}{\gamma r_e}}$.

Here $r_e \cong 2.818\cdot 10^{-13}\ cm$ is the classical electron radius, λ_u is the undulator period, K is the undulator deflection parameter, γ is the electron reduced energy.

RADIATION ENERGY LOSS

For simplicity, we consider ultra-relativistic electron propagating in a planar undulator with vertical sinusoidal magnetic field (see Fig.1) $B_y(z) = B_0 \sin(\frac{2\pi}{\lambda_u} z)$, where λ_u is the undulator period length.

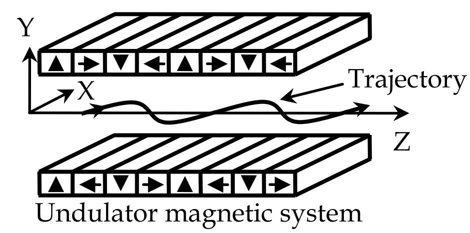

Figure 1: Sketch of a permanent magnet undulator.

The instantaneous radiation energy loss of the electron is given by

$$\frac{dE_r}{dt} = \frac{2e^2}{3c}\gamma^4\beta^2 = \frac{2}{3}r_e^2 c\gamma^2 B_0^2 \sin^2\left(\frac{2\pi}{\lambda_u}z(t)\right). \quad (1)$$

Here e is the electron charge, c is the speed of light, γ and β are the electron reduced energy and acceleration.

Integrating over the period, we get the following expression for the energy loss per one undulator period:

$$\Delta E_r(period) = \frac{\pi}{3}\alpha\varepsilon_1 K^2\left(1+\frac{K^2}{2}\right) \quad (2)$$

Here α is the fine structure constant, $\varepsilon_1 = 2\pi c / \lambda_1$ is photon energy of the fundamental, $\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$ is wavelength of the fundamental.

It can be easily derived from Eq. (2) that the relative energy loss δ per one period is equal to:

$$\delta = \frac{\Delta\gamma(period)}{\gamma} = \frac{(2\pi)^2}{3}\gamma K^2 \frac{r_e}{\lambda_u} \quad (3)$$

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RADIATION SPECTRAL DISTRIBUTIONS

Let us consider the radiation field of a moving electron, which is seen by the observer at time τ and at the observation point $X^* = \{x^*, y^*, z^*\}$ in the far-field zone, so z^* tends to infinity. The far-field radiation component is given by the following expression:

$$E(\tau) = \frac{e}{c|X^*|} \cdot \frac{[n \times [(n - \beta(t)) \times \dot{\beta}(t)]]}{(1 - (n \cdot \beta(t)))^3}, \tag{4}$$

Here, $n = X^* / |X^*|$, $r(t)$ and $\beta(t) = \frac{dr(t)}{cdt}$ are the electron trajectory and reduced velocity respectively. The quantities $\beta(t)$ and $\dot{\beta}(t)$ are to be evaluated at the retarded time t which must obey the equation:

$$c\tau = ct + |X^* - r(t)|. \tag{5}$$

It can be shown by direct calculations that:

$$\tau(t) = \tau(0) + \int_0^t (1 - (n \cdot \beta(t'))dt' . \tag{6}$$

The number of photons $dN_{x,y}$ with horizontal (x) and vertical (y) polarization, emitted by a single electron during one passage through the undulator per solid angle $d\Omega$ per relative bandwidth $d\lambda/\lambda$ is given by the Fourier transform of the electric field given be Eq. (4):

$$dN_{x,y} = d\Omega\left(\frac{d\lambda}{\lambda}\right)\frac{\alpha c^2}{4\pi^2 e^2}|X^*|^2 |\tilde{E}_{x,y}(\lambda)|^2, \tag{7}$$

$$\tilde{E}_{x,y}(\lambda) = \int_{-\infty}^{\infty} \exp(i\,2\pi c\tau/\lambda)E_{x,y}(\tau)d\tau . \tag{8}$$

By changing the integration variable from τ to retarded time t and using the electron longitudinal coordinate z instead of t, we obtain the following results:

$$\tilde{E}_{x,y}(\lambda) = \int_0^{N\lambda_u} \exp(i\Phi(z))F_{x,y}(z)dz , \tag{9}$$

$$F(z) = \frac{e}{c^2|X^*|} \cdot \frac{[n \times [(n - \beta(z)) \times \dot{\beta}(z)]]}{\beta_z(z)(1 - (n \cdot \beta(z)))^2}, \tag{10}$$

$$\Phi(z) = \frac{2\pi}{\lambda}\int_0^z (1 - (n \cdot \beta(z'))) \frac{dz'}{\beta_z(z')} . \tag{11}$$

For more details see [10]. It is significant that the outlined above expressions are very general in nature and can be applied for electrons which slowly change their energy.

For simplicity, we will consider here the shape of the radiation spectral line along the undulator axis: $n_{x,y} = 0$. For high-energy electrons ($\gamma >> 1$, $K/\gamma << 1$) we have:

$$\Phi(z) = \frac{\pi}{\lambda}\int_0^z \frac{1}{(\gamma(z'))^2}\left(1 + (\gamma\beta_x(z'))^2\right)dz' \tag{12}$$

Let $z_i = \lambda_u \cdot (i-1)$, $i = 1, 2, ...N$ are the initial points of the i -th period. We can conveniently split the integral in Eq. (9) into integrals over periods:

$$\tilde{E}_{x,y}(\lambda) = \sum_{i=1}^{N} \int_{z_i}^{z_i+\lambda_u} \exp(i\Phi(z))F_{x,y}(z)dz , \tag{13}$$

$$\Phi(z_i + \varsigma) = \Phi(z_i) + \Delta\Phi_i(\varsigma), \tag{14}$$

$$\Delta\Phi_i(\varsigma) = \frac{\pi}{\lambda}\int_{z_i}^{z_i+\varsigma} \frac{1}{(\gamma(z'))^2}\left(1 + (\gamma\beta_x(z'))^2\right)dz' . \tag{15}$$

Substituting (14) and (15) we will get:

$$\tilde{E}_{x,y}(\lambda) = \sum_{i=1}^{N} \exp(i\Phi(z_i))\tilde{E}_{(i)x,y}(\lambda), \tag{16}$$

$$\tilde{E}_{(i)x,y}(\lambda) = \int_0^{\lambda_u} \exp(i\Delta\Phi_i(\varsigma))F_{x,y}(z_i + \varsigma)d\varsigma . \tag{17}$$

Numerical analysis shows that practically the functions $\tilde{E}_{(i)x,y}(\lambda)$ do not depend on the number of period (i) in far-field approximation. As a result we have from (16):

$$\tilde{E}_{x,y}(\lambda) = I_{\text{int}}(\lambda) \cdot \tilde{E}_{(1)x,y}(\lambda), \tag{18}$$

$$I_{\text{int}}(\lambda) = \sum_{i=1}^{N} \exp(i\Phi(z_i)) . \tag{19}$$

Let γ_i be the electron reduced energy at the initial point of the i -th period,

$$\gamma_i = \gamma_1(1 - \delta(i-1)), \tag{20}$$

where δ is given by Eq. (3).

We have from (14) and (15): $\Phi(z_1) = 0$,

$$\Phi(z_{i+1}) = \Phi(z_i) + \frac{\pi\lambda_u}{\lambda\gamma_i^2}\left(1 + \frac{K^2}{2}\right), \tag{21}$$

The direct solution for Eqs. (20) and (21) is equal to:

$$\Phi(z_i) = \frac{\pi\lambda_u}{\lambda\gamma_1^2}\left(1 + \frac{K^2}{2}\right)(i-1)(1 + \delta(i-2)) . \tag{22}$$

If we neglect by the radiation energy lost that is putting $\delta = 0$ into (22), we obtain from (19) and (22) the standard interfering function for undulator radiation with perfectly periodical trajectory:

$$I_{\text{int}}(\lambda) = \frac{\sin(Np)}{\sin(p)}, \tag{23}$$

where $p = \frac{\pi\lambda_u}{2\lambda\gamma_1^2}\left(1 + \frac{K^2}{2}\right)$.

The additional term in the phase (22), which describes the radiation energy loss at the wavelength of fundamental, is equal to $2\pi\delta(i-1)(i-2)$. We can find the critical number of undulator periods N_{loss} from the relation $2\pi\delta(N_{loss}-1)(N_{loss}-2) = 2\pi$, obtaining the estimation for N_{loss}

$$N_{loss} \cong \frac{1}{\sqrt{\delta}} . \tag{24}$$

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NUMERICAL SIMULATIONS

The simulations were performed with the European XFEL parameters listed in the Table 1 (see [11]).

Table 1: European XFEL Parameters for Simulation

Electron beam energy	17.5 GeV
Undulator period λ_u	40 mm
Undulator deflection parameter K	4
Energy of fundamental harmonic ε_1	8078 eV
Number of undulator periods N	124

We can readily calculate from the foregoing equations: The energy loss per one undulator period is equal to:

$$\Delta E_r(period) = 8889 \text{ eV}.$$

Relative energy loss δ per one period is equal to:

$$\delta = \frac{\Delta\gamma(period)}{\gamma} = 5\cdot 10^{-7}.$$

Critical number of undulator periods N_{loss} is equal to:

$$N_{loss} \cong \frac{1}{\sqrt{\delta}} \cong 1414.$$

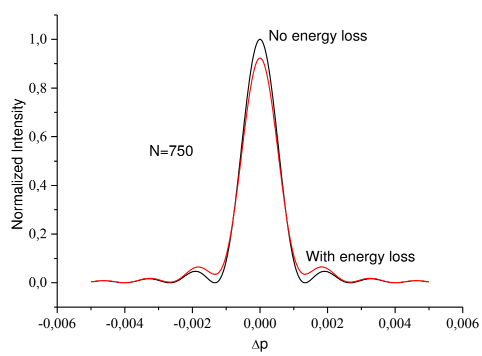

Figure 2: Normalized undulator radiation intensity without energy loss (black curve) and with energy loss (red curve). N = 750 periods.

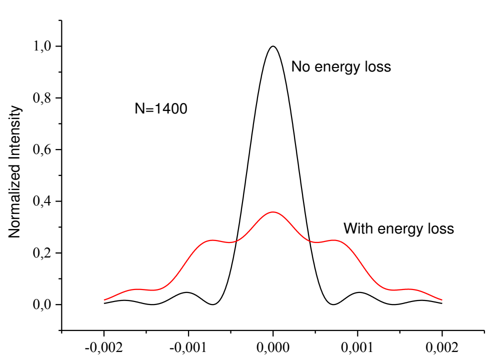

Figure 3: Normalized undulator radiation intensity without energy loss (black curve) and with energy loss (red curve). N = 1400 periods.

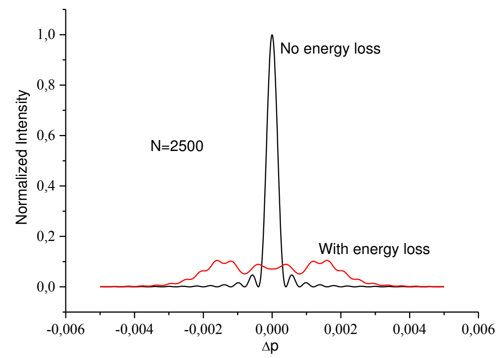

Figure 4: Normalized undulator radiation intensity without energy loss (black curve) and with energy loss (red curve). N = 2500 periods.

Figures 2–4 show the numerically calculated spectral intensities along the axis of the undulators with 750, 1400 (which is close to N_{loss}) and 2500 periods correspondently.

Calculations were carried out in the framework of approach described above. The European XFEL undulator parameters were used for simulations, see Table 1. These numerical results clearly show that for undulator spontaneous radiation the energy loss should be taken properly into account if the number of undulator periods is large enough. The simple estimation for critical number of periods, given by Eq. (24), is in a good agreement with results of numerical simulations, see Figures 2 - 4.

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