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MERIT FUNCTIONS FOR THE LINAC OPTICS DESIGN FOR COLLIDERS AND LIGHT SOURCES

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Abstract

Optics matching and transverse emittance preservation are key goals for a successful operation of modern high brightness electron linacs. The capability of controlling them in a real machine critically relies on a properly designed magnetic lattice. Conscious of this fact, we introduce an ensemble of optical functions that permit to solve the often neglected conflict between strong focusing, typically implemented to counteract coherent synchrotron radiation and transverse wakefield instability, and distortion of the transverse phase space induced by chromatic aberrations and focusing errors. A numerical evaluation of the merit functions is applied to the Pohang Accelerator Laboratory free electron laser.

INTRODUCTION

Strong focusing is typically prescribed in modern high brightness electron linacs to avoid beam break up along the accelerator and to minimize coherent synchrotron radiation (CSR) effects in bunch length magnetic compressors [1]. Additional constraints are typically due to the optimization of the performance of diagnostics and beam collimation, with the additional advantage of cumulating the desired betatron phase advance in short distances, so as to save space and finally minimize the cost of the facility. Unfortunately, it may also hamper the main goal of emittance preservation through the excitation of optical aberrations and potentially leads, through focusing errors, to beam optics mismatch that can in turn corrupt the optics scheme adopted for the suppression of the instabilities. Based on these often conflicting requirements and on the partial lack, in the archival literature, of established strategies to design and optimize the optics of linear colliders and linac-driven free electron lasers (FELs), we introduce an ensemble of merit functions that offer, to our knowledge for the first time, a guidance to the definition of quadrupole strengths and Twiss functions in high brightness electron linacs. In particular, this work applies methods already published in [2] and partially revised in [1], to the PAL XFEL in Pohang, South Korea [3]. The baseline optics and other parameters of the electron beam in the PAL XFEL are shown in Fig. 1. The PAL-XFEL is designed to generate 0.1-nm hard X-ray FEL using a 10 GeV electron linac with a switch line at 3-GeV point for 1~3 nm soft X-ray FEL. The target slice emittance is 0.4 mm-mrad at 0.2 nC. A three-bunch compressor lattice is chosen so as to make more electrons in a bunch meet the requirements of emittance and correlated energy spread for SASE-FEL. The lattice also minimizes the emittance growth due to CSR.

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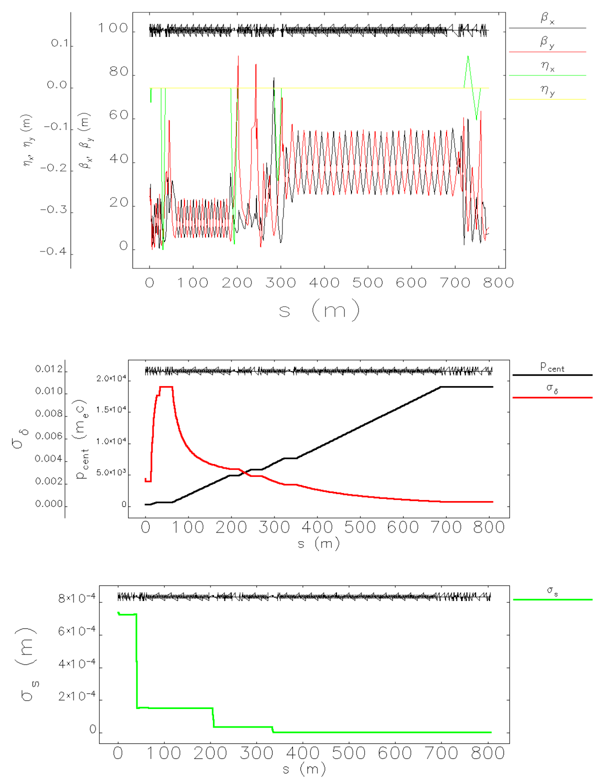

Figure 1: Baseline Twiss functions and energy dispersion from the injector end to the undulator entrance (top), mean energy, rms energy spread (middle) and rms bunch length (bottom) along the beam line.

OPTICS SENSITIVITY TO FOCUSING ERRORS

Our treatment basically applies well known concepts of linear and nonlinear accelerator physics. Some of them however, like the tune-shift-with-amplitude and the chromaticity, are not suited to describe local sources of phase space distortion. This fact justifies a less conventional approach in which we define a sensitivity coefficient that identifies sources of optics mismatch along the lattice. We adopt the definition of the mismatch parameter introduced in [4]. In addition, we assume a relative focusing error $k\tau$, due to a perturbation τ in the quadrupole strengths, so that the final mismatch parameter is computed in each plane as follows:

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$$m_\delta^f \cong 1+\frac{1}{2}\left[\left(\sum_{i=1}^{N} k_i\beta_i L_i\tau_i \cos(2\Delta\mu_i)\right)^2+\left(\sum_{i=1}^{N} k_i\beta_i L_i\tau_i \sin(2\Delta\mu_i)\right)^2\right] \quad (1)$$

The sum in Eq. 1 is over N quadrupoles, L is the quadrupole magnetic length, k its strength, τ the fractional strength error and Δμ the betatron phase advance. We now evaluate Eq. 1 one quadrupole at the time, while keeping all other magnets at their nominal strength (τ = *0*). After averaging over many betatron oscillations, we estimate the mismatch induced at the *end* of the line by each *individual* quadrupole magnet:

$$m_\delta^q - 1 \cong \frac{1}{2}(k\beta L\tau)_q^2 \equiv \xi_q(\tau) \quad (2)$$

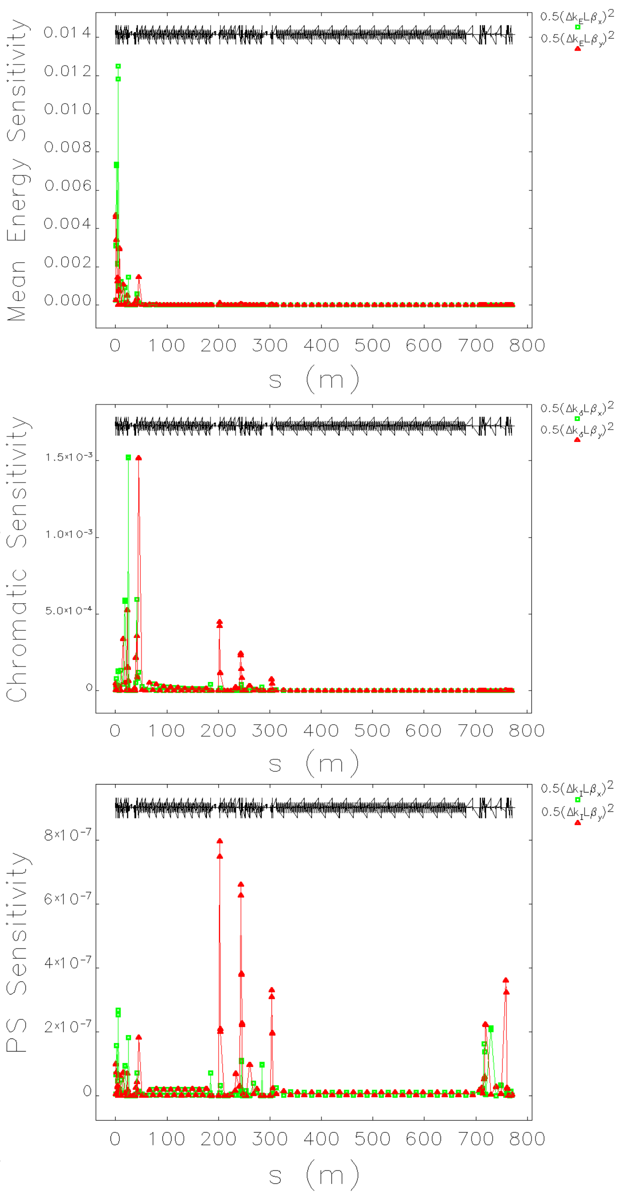

Figure 2: Optics sensitivity to a local mean energy error of 1 MeV (top), rms energy spread (middle) and a magnets' supply calibration error of 0.01% in the PAL XFEL.

We define ξ_q as the optics sensitivity to focusing errors. If we assume that the mismatch parameter at the end of the line, $m_\delta^f = 1 + C$, is the result of many identical, uncorrelated and small contributions $\xi_q << 1$, the maximum sensitivity allowed to each quadrupole (tolerance) is of the order of $m_\delta^q - 1 = \xi_q \approx C/\sqrt{N}$. As an example, for $C = 5\%$ and $N = 100$, lattice regions characterized by $\xi_q \geq 0.5\%$ should be deemed worthy of stricter tolerances in strength value or in need of weaker focusing. It can be shown through the beam matrix formalism that Eq. 2 also describes the final relative emittance growth induced by each individual error kick [2]. The optics sensitivity to several kind of focusing errors was computed for the PAL XFEL and shown in Fig. 2. The total number of quadrupoles in the lattice under consideration is 222.

LOCAL CHROMATICITY

The sensitivity coefficient introduced in Eq. 2 measures the cumulative effect of focusing errors on the beam optics at the *end* of the line, which might be the undulator entrance in FELs or the interaction point in colliders: it is not well-suited to describe *local* optics mismatch. This complementary process can be monitored by looking at the *local* chromatic dependence of the Twiss functions. In practice, the beam mean energy is assumed to be varied by a small amount ΔE=δ·E, in small steps. The Twiss function *f(s)* is computed at each step of the reference trajectory. A linear fit provides the coefficient $df/d\delta$ at any location along the lattice. The local mismatch due to an energy error is estimated by $\Delta f(s) \cong \frac{df(s)}{d\delta}\delta$, where *f(s)* is one of the Twiss functions β(s) or α(s) and δ is typically not larger than a few percent. The chromatic derivative of the betatron funtions in the PAL XFEL is shown in Fig. 3.

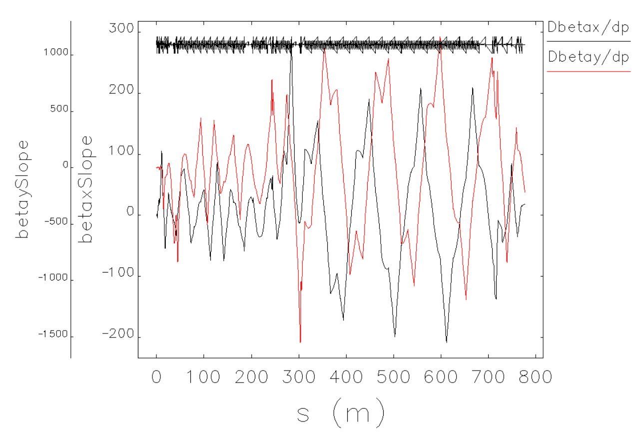

Figure 3: $d\beta/d\delta$ in units of meter in the PAL XFEL.

The value of the slope $d\beta/d\delta$ at any location is the linear fit applied to β as a function of δ, where δ is the energy deviation relative to the local reference energy.

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H-FUNCTION IN COMPRESSORS

CSR-induced phase space distortion in magnetic compressors is a critical aspect of any high brightness linac design. CSR-induced emittance growth is primarily due to betatron oscillations of the longitudinal slices of the bunch around different dispersive trajectories. Even in the case of achromatic lines, the increased betatron invariant of the slices' centroid is not recovered as the dispersion function collapses to zero. If we describe the CSR effect in a short magnet with the single-kick approximation, we are allowed to use the beam matrix formalism to obtain a simple expression for the projected emittance growth [2]:

$$\varepsilon \cong \left[\det \begin{pmatrix} \varepsilon_0\beta + \eta^2\sigma^2_{\delta,CSR} & -\varepsilon_0\alpha + \eta\eta'\sigma^2_{\delta,CSR} \\ -\varepsilon_0\alpha + \eta\eta'\sigma^2_{\delta,CSR} & \varepsilon_0\frac{1+\alpha^2}{\beta} + \eta'^2\sigma^2_{\delta,CSR} \end{pmatrix} \right]^{1/2} = \varepsilon_0\sqrt{1+\frac{H}{\varepsilon_0}\sigma^2_{\delta,CSR}}, \quad (3)$$

where ε and ε_0 are the geometric RMS emittance after and before the CSR kick, respectively, $H = \left[\eta^2 + (\beta\eta' + \alpha\eta)^2\right]/\beta$ in the bending plane, η is the energy dispersion function, η' is its first derivative with respect to the curvilinear longitudinal coordinate *s* and $\sigma_{\delta,CSR}$ is the CSR-induced RMS relative energy spread [5]. Since $\sigma_{\delta,CSR}$ is inversely proportional to the bunch length, the transverse CSR effect in a four dipoles magnetic chicane is dominated by the radiation emission in the second half of the system, where the bunch reaches its shortest duration. Thus, Eq. 3 prescribes to shrink the *H*-function in proximity of the fourth dipole to suppress CSR-induced emittance growth. When the bending angle is small ($\theta \ll 1$) and in the presence of a beam waist, this prescription involves shrinking β in proximity of the 4^{th} dipole magnet. This solution was in fact proposed in [6] and adopted, *e.g.*, in [7]. We stress that the prescription of a small *H*-function is in conflict with that of an equally desirable large *H*-function needed to wash out the microbunching-induced charge density modulation via path length differences [2]. The *H*-induced damping, however, is expected to be effective for the microbunching gain generated in the chicane only, which is typically negligible with respect to the total gain of the instability (i.e., developing along the entire accelerator) [1]. Figure 4 shows the *H*-function and the CSR-induced normalized emittance growth in the third PAL XFEL magnetic compressor, where the largest impact on beam quality is expected with respect to the preceding compressors. Eq. 3 was evaluated via the ELEGANT code [8] in the compressor's dipole magnets, each split into 4 slices. Then, the total emittance growth can be estimated with the square-root sum of all individual contributions shown in Fig. 4. It turns out to be of the order of 0.01 μm.

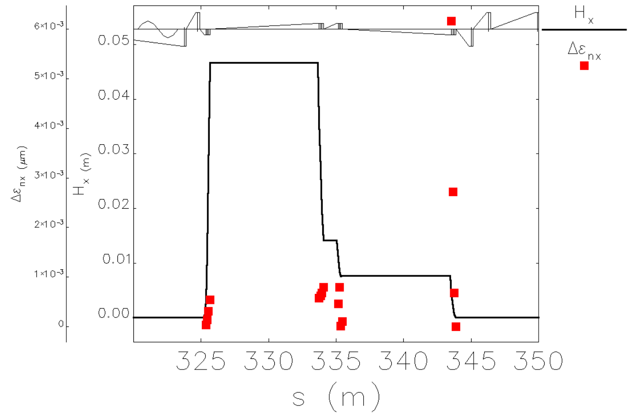

Figure 4: Chromatic *H*-function and CSR-induced normalized emittance growth in the third magnetic compressor of PAL XFEL.

CONCLUSIONS

The analysis presented in [2] revealed a relatively low sensitivity of the PAL XFEL optics design to perturbations to the nominal focusing strength. Major sources of optics mismatch are identified at the injection point and in proximity of the first magnetic compressor, where a lower beam rigidity and larger energy spread are present. A local variation of the betatron functions smaller than 1 m is expected along the main linac, for energy/calibration errors of the order of 0.1% or smaller. Finally, CSR in magnetic compressors is expected to degrade the normalized projected emittance in total not more than a few percent of its value at the injection. These semi-analytical evaluations shall be verified with more accurate and massive simulations including CSR and statistically meaningful sets of machine errors.

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