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BEAMLINES WITH TWO DEFLECTING CAVITIES FOR TRANSVERSE-TO-LONGITUDINAL PHASE SPACE EXCHANGE

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Abstract

Optical systems for transverse-to-longitudinal emittance exchange involving single dipole-mode cavity were in great details studied during the last decade theoretically and experimentally. In this paper we discuss the question, if there are any advantages in usage of beamlines utilizing two deflecting cavities instead of one.

INTRODUCTION

Transverse-to-longitudinal emittance and phase space exchangers (EEXs) received large attention during the last decade and many interesting applications of such beamlines were proposed [1–6]. EEXs involving single transverse deflecting cavity (TDC) were already in great details studied theoretically and experimentally, and in this paper we consider EEXs utilizing two (identical in design and with equal in magnitude excitations) TDCs instead of one. We show that it allows not only to compensate for the so-called thick-lens effect (i.e. particle energy change in the deflecting cavity), but also gives possibilities to design mirror symmetric beamlines and beamlines which, without any further retuning of magnets, provide dispersion free beam transport when TDCs are switched off. Due to space limitation, we mostly concentrate on the necessary and sufficient conditions on the matrices of the subsections of the beamline to effect an EEX with the desired properties (i.e. give existence proofs) and the more detailed paper with the design examples will follow.

PRELIMINARIES

We consider the single particle linear dynamics in the horizontal and longitudinal degrees of freedom, describe it by the 4×4 transport matrices and ignore the motion in the vertical degree of freedom, which (on the linear level) is assumed to be decoupled from the two others. We also assume that all 4×4 matrices, which we will meet in this paper, are symplectic and will index their elements as if these matrices were extracted from the complete three degrees of freedom 6×6 beam transport matrices, where the first degree of freedom is horizontal, the second is vertical, and the longitudinal comes as the third.

Matrix of a Magnetostatic System

From energy conservation, symplecticity and absence of coupling with the vertical degree of freedom it follows that the 4×4 horizontal-longitudinal transport matrix of a magnetostatic system has the form

$$M = \begin{bmatrix} m_{11} & m_{12} & 0 & m_{16} \\ m_{21} & m_{22} & 0 & m_{26} \\ m_{51} & m_{52} & 1 & m_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where (due to symplectic conditions) its elements satisfy

$$m_{11} m_{22} - m_{12} m_{21} = 1, \tag{2}$$

$$m_{16} = m_{11} m_{52} - m_{12} m_{51}, \tag{3}$$

$$m_{26} = m_{21} m_{52} - m_{22} m_{51}. \tag{4}$$

Let B be the matrix of a beamline which is mirror symmetric to the magnetostatic beamline with the transport matrix A. Then

$$b_{52} = -a_{16}, \quad b_{51} = -a_{26}, \tag{5}$$

and if B describes a beamline which is mirror antisymmetric to the beamline described by the matrix A (reversed and then rotated by 180° around the longitudinal axis), then

$$b_{52} = a_{16}, \quad b_{51} = a_{26}. \tag{6}$$

What is also important for the further considerations, it is the fact that any symplectic matrix of the form (1) can be decomposed into the products

$$M = M^{(1)} M^{(0)} = M^{(0)} M^{(2)}, \tag{7}$$

where

$$M^{(0)} = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & 1 & m_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 & m_{16} \\ 0 & 1 & 0 & m_{26} \\ -m_{26} & m_{16} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{9}$$

$$M^{(2)} = \begin{bmatrix} 1 & 0 & 0 & m_{52} \\ 0 & 1 & 0 & -m_{51} \\ m_{51} & m_{52} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{10}$$

Matrix of a TDC

In this paper we do not go beyond the approximation for the matrix of a horizontally deflecting cavity, which was used in all papers cited above, and take it in the usual form

$$R(\kappa, l_c, q) = \begin{bmatrix} 1 & l_c & \kappa l_c/2 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & \kappa l_c/2 & q\kappa^2 l_c & 1 \end{bmatrix}, \tag{11}$$

where l_c is the cavity length, κ is its deflecting strength, and the value of the energy gain factor q depends from the particular cavity design. For example, for the n-cell pillbox resonator it is given by the expression

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$$q = \left(1 + 2n^2\right) / \left(12n^2\right) \tag{12}$$

and satisfies $1/6 < q \leq 1/4$.

Approximations made in the equations of motion in order to obtain the matrix of a TDC in the form (11) include among others the neglection of the terms of the order $O(1/\gamma_0^2)$, where γ_0 is the Lorentz factor of the reference particle. To be consistent with this, we will assume that these terms were also neglected during derivation of the matrices of all other beamline elements. With this convention the matrix of a drift of the length l has the form

$$D_0(l) = \begin{bmatrix} 1 & l & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

and for the TDC matrix one obtains

$$R(\kappa, l_c, q) = D_0(l_c/2)\, C(\kappa, ql_c)\, D_0(l_c/2), \tag{14}$$

where

$$C(\kappa, w) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & 0 & w\kappa^2 & 1 \end{bmatrix}. \tag{15}$$

Matrix of an EEX

EEX is a beamline with the transfer matrix T which, when partitioned into 2×2 submatrices

$$T = \begin{bmatrix} T_{11} & T_{13} \\ T_{31} & T_{33} \end{bmatrix}, \tag{16}$$

has the blocks $T_{11} = T_{33} = 0$. So the eight elements of the matrix T must be equal to zero, but it gives only four independent constraints because owing to the symplectic conditions equations $T_{11} = 0$ and $T_{33} = 0$ are equivalent.

Let us consider a beamline with several TDCs and let us point out two technical tricks, which essentially simplify derivation of the conditions which the matrices of the subsections of the beamline must satisfy to effect an EEX:

- Let A and B be the matrices of the type (1) and let M be an arbitrary 4×4 symplectic matrix. Then the matrix BMA is an EEX matrix if and only if the same is the matrix $B^{(2)} M A^{(1)}$. It means that only the elements a_{16} and a_{26} of the matrix A and only the elements b_{51} and b_{52} of the matrix B are of importance for an EEX design.
- Drifts from the formula (14) can be included into the matrices of the neighboring to the TDC magnetostatic beamline parts and therefore the matrix (15) can be used instead of the matrix (11) during calculations.

EEX with a Single TDC

General requirements on the subsections of the beamline before and after a TDC and on the TDC itself to effect an EEX were obtained for the first time in [3]. In our notations they can be obtained from the analysis of the matrix

$$T = B^{(2)}\, C(\kappa, w)\, A^{(1)} \tag{17}$$

and are as follows

$$b_{52} = a_{16}, \quad b_{51} = -a_{26}, \tag{18}$$

$$w = ql_c = 0, \quad a_{16}\,\kappa = -1. \tag{19}$$

Ignoring for a while the first of Eqs. (19) and comparing (18) with (5) one sees that the beamline after the TDC can not be a mirror symmetric image of the beamline before the TDC (see also [6]). On the other hand, looking at (6) one observes that if $a_{26} = 0$ and the beamline described by the matrix B is an antisymmetric image of the beamline associated with the matrix A, then the conditions (18) are fulfilled automatically, which includes as partial case the two dogleg solution [2].

COMPENSATION OF THICK-LENS EFFECT WITH QUADRUPOLE OPTICS

The first of Eqs. (19) tells us that the emittance exchange employing single TDC can not be made exact even in a linear sense if the effect of the particle energy change due to the thickness of the deflecting cavity (thick-lens effect) is not completely negligible. Until now two approaches to overcome this problem were known: either use of an additional accelerating mode cavity operated at zero-crossing [4] or application of the scheme involving three TDCs [5]. In this section we present new solution utilizing two TDCs by showing that for given $w \neq 0$ and $\delta = \pm 1$ there exist æ $\neq 0$ and matrices A, N and B of the type (1) such that the equation

$$B\, C(\delta\kappa, w)\, N\, C(\kappa, w)\, A = C(æ\kappa, 0) \tag{20}$$

holds for an arbitrary value of κ.

Skipping the solution process, we state that the general solution of Eq. (20) requires

$$a_{16} = a_{26} = a_{56} = 0, \quad a_{12} \neq 0, \tag{21}$$

$$n_{16} = n_{26} = n_{56} = 0, \quad n_{12} = -2\,\delta\, w, \tag{22}$$

$$b_{16} = b_{26} = b_{56} = 0, \tag{23}$$

which, in particular, means that the matrices A, N and B can be created by using quadrupole magnets only.

The general solution depends from four parameters, and if one takes as such free parameters the quantities

$$n_{11}, \quad n_{22}, \quad a_{11}, \quad a_{12} \neq 0, \tag{24}$$

then

$$æ = 2w / a_{12}, \tag{25}$$

$$a_{22} = a_{12}\,(1 + \delta n_{11}) / (2w), \tag{26}$$

$$b_{11} = \delta a_{12}(1 + \delta n_{22}) / (2w), \quad b_{12} = \delta a_{12}, \tag{27}$$

$$b_{22} = \delta\,(2w / a_{12} - a_{11}), \tag{28}$$

and the remaining matrix elements a_{21}, n_{21}, and b_{21} are uniquely determined according to Eq. (2).

The above requirements on the matrices A, N, and B are actually not so complicated as it might seem at the first sight. They can be satisfied even by using only one quadrupole per matrix, but unfortunately all these quadrupoles must be

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vertically defocusing. As one more example, let us point out that the matrices

$$A = D_0(w), \quad N = I_\delta D_0(-2w), \quad B = I_\delta D_0(w), \tag{29}$$

where $I_\delta = \mathrm{diag}(\delta, \delta, 1, 1)$, also solve Eq. (20) with æ = 2.

It is clear that the beamline satisfying Eq. (20) can be used in any situation where the TDC with the compensated thick-lens effect is needed, in particular in EEXs. But, as concerning EEXs, additional simplification is possible, because, as discussed in the previous section, the property of the beamline to be an EEX constrains only dispersive elements of the matrices representing the beamline parts before the first and after the last TDC. So, for an EEX design, it is sufficient to consider the matrix

$$T = B^{(2)}\, C(\delta\kappa, w)\, N\, C(\kappa, w)\, A^{(1)}, \tag{30}$$

where the matrix N is such as required by the solution of Eq. (20). One can show that this matrix is an EEX matrix if and only if

$$b_{52} = -\delta\,[1 + (a_{16}\kappa)] \,/\, \kappa, \tag{31}$$

$$a_{26} = [1 + (1 + \delta n_{11}) \cdot (a_{16}\kappa)] \,/\, (2w\kappa), \tag{32}$$

$$b_{51} = -[n_{22} + (\delta + n_{22}) \cdot (a_{16}\kappa)] \,/\, (2w\kappa), \tag{33}$$

where a_{16} and $\kappa \neq 0$ are the free parameters.

Let us consider the partial case when the beamline after the second TDC is the mirror symmetric image of the beamline before the first TDC and let us assume additionally that for the entrance beamline $a_{26} = 0$. These requirements lead to the conditions

$$\delta = -1, \quad 2a_{16}\kappa = -1, \quad N = I_{-1} D_0(-2w), \tag{34}$$

which, for example, can be satisfied by employing symmetric quadrupole triplet for the construction of the matrix N and for the simultaneous control of the vertical focusing. It gives, in particular, a mirror symmetric EEX where the two TDCs separated by the symmetric quadrupole triplet are inserted in the middle of the four bend magnetic chicane.

EEX WORKING FOR AN ARBITRARY VALUE OF THE ENERGY GAIN FACTOR

In the design of an exact (in the linear sense) EEX one do not have to consider Eq. (20) first, but can start straight from the matrix (30) and look for the conditions under which this matrix is an EEX matrix. Unfortunately, it is not the easy task to find the general solutions of this problem in the analytical form, and we restrict ourselves to the search for the partial solutions with special additional properties. One such partial solution in which the matrix N is the matrix of a drift-quadrupole system was provided in the previous section, and in this section we give a partial solution which is valid for an arbitrary value of the energy gain factor q.

Because the elements of the submatrices T_{11} and T_{22} are second order polynomials with respect to the variable $w = ql_c$, the matrix T will be an EEX matrix independently of the particular q value if and only if all coefficients of these polynomials are equal to zero. It, in the next turn, can be achieved if and only if the following conditions on the elements of the matrices A, N and B are fulfilled

$$n_{11} \neq -\delta, \quad n_{12} = 0, \quad n_{16} \neq 0, \quad n_{56} = 0, \tag{35}$$

$$n_{16}\,\kappa = -\frac{2\,n_{11}}{1 + \delta\, n_{11}}, \tag{36}$$

$$a_{16} = \frac{1 - n_{11}^2}{1 + n_{11}^2}\,\frac{n_{52}}{2}, \quad a_{26} = -\frac{1 - n_{11}^2}{1 + n_{11}^2}\,\frac{n_{51}}{2}, \tag{37}$$

$$b_{51} = \frac{1 - n_{11}^2}{1 + n_{11}^2}\,\frac{n_{26}}{2}, \quad b_{52} = -\frac{1 - n_{11}^2}{1 + n_{11}^2}\,\frac{n_{16}}{2}, \tag{38}$$

which gives a general solution of the problem considered.

EEX WHICH TURNS INTO DISPERSION FREE BEAMLINE WITH CAVITIES OFF

Unfortunately, for an arbitrary choice of their free parameters, the EEXs described in the two preceding sections can not provide dispersion free beam transport when TDCs are switched off, but all other elements in the beamline (magnets) remains untouched. To prove that such beamline is possible at all, one has to point out the conditions under which the matrix (30) is an EEX matrix and, simultaneously,

$$b_{51} = n_{26} + n_{21}\, a_{16} + n_{22}\, a_{26}, \tag{39}$$

$$b_{52} = -n_{16} - n_{11}\, a_{16} - n_{12}\, a_{26}, \tag{40}$$

which are requirements for the matrix $B^{(2)} M A^{(1)}$ to be dispersion free. Because in such a general formulation the problem is still too complicated for finding a general solution, in order to complete the existence proof we provide the partial solution

$$a_{26} = n_{21} = n_{26} = n_{56} = b_{51} = 0, \tag{41}$$

$$n_{11} = \frac{\delta}{2}(n_{16}\kappa)^2, \tag{42}$$

$$n_{12} = -\frac{1 + [(n_{16}\kappa) + \delta]^2}{(n_{16}\kappa)}\, w, \tag{43}$$

$$a_{16} = -\frac{(n_{16}\kappa) + \delta}{1 + [(n_{16}\kappa) + \delta]^2}\,\frac{2}{(n_{16}\kappa)}\,\delta n_{16}, \tag{44}$$

$$b_{52} = -\frac{(n_{16}\kappa) + 2\delta}{1 + [(n_{16}\kappa) + \delta]^2}\,\delta n_{16}. \tag{45}$$

where $n_{16} \neq 0$ and $\kappa \neq 0$ are the free parameters.

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