ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO037

SUPPRESSION TECHNIQUES OF CSR INDUCED EMITTANCE GROWTH IN ERL ARCS *

A.V. Bondarenko#, T. Atkinson, A.N. Matveenko, HZB, Berlin, Germany

Abstract

The Energy Recovery Linac (ERL) conception is a promising way of creating a diffraction limited synchrotron light source. The high ERL beam quality (low emittance, short bunch and low energy spread) gives an opportunity to generate high brightness photon beams. One of the main requirements for the optic in such machines is the suppression of emittance growth. An important reason for beam quality degradation is the impact of Coherent Synchrotron Radiation (CSR) in bending magnets. CSR induced emittance dilution and methods of preservation both with and without bunch compression are discussed in this article.

INTRODUCTION

One of the known methods to suppress CSR induced emittance dilution for non-compression case is using a few identical bending cells with right betatron phase advance [1]. Second known method for the case with compression is using of two chicanes with π phase advance between their centers to compensate emittance dilution [2]. In this paper we consider the case of beam compression in the identical bending cells which is usual for ERL arc.

THEORY

In this article 1D CSR model is used. It is valid if the transversal bunch size σ_t is smaller than the characteristic transverse distance [3, 4].

$$\sigma_t << 2\sqrt[3]{9\sigma^2 R} \ , \tag{1}$$

where σ is rms bunch length, R is bending radius.

Let's calculate the deflection of an electron due to the CSR in this approximation. $\delta\mathbf{X}$ is a vector (δx, $\delta x'$) of an electron position in the phase space. $\mathbf{R}_6$ is a vector (R_{16}, R_{26}).

$$\delta\mathbf{X} = \int_{s_0}^{s_N} \mathbf{R}_6(s|s_N)\frac{dE}{E_0 ds}ds \ , \tag{2}$$

where dE is a energy loss of electron due to 1 dimensional CSR wake, E_0 is beam energy, s_0 is coordinate of bending section beginning, s_N is coordinate of bending section end.

$$\mathbf{R}_6(s|s_N) = -\mathbf{M}(s|s_N)\mathbf{D}(s) \ , \tag{3}$$

where $\mathbf{D}$ is the vector of dispersion (D, D′), $\mathbf{M}$ is transport matrix [5].

The energy loss of an electron due to 1 dimensional CSR stationary wake is dE_0.

$$\frac{dE_0(ct)}{ds} = -\frac{2e^2}{3^{1/3}R^{2/3}}\int_{-\infty}^{t}\frac{1}{(ct-ct')^{1/3}}\frac{d\lambda(ct')}{cdt'}cdt' \ , \tag{4}$$

where λ is the longitudinal density distribution.

Let's introduce the dimensionless longitudinal coordinate ξ=ct/σ, that does not change during the beam compression. In this case the shape of longitudinal beam density distribution also does not change. Λ is the longitudinal density distribution depending on a dimensionless longitudinal coordinate.

$$\begin{aligned}&\Lambda(\xi) = \lambda(\xi\cdot\sigma)\cdot\sigma,\\ &\int\Lambda(\xi)d\xi = \int\lambda(\xi\cdot\sigma)\cdot\sigma d\xi = \int\lambda(ct)\cdot dct = N_e,\end{aligned} \tag{5}$$

where N_e is the number of electrons.

$$\begin{aligned}&\frac{dE_0}{ds} = -\frac{2e^2}{3^{1/3}R^{2/3}\sigma^{4/3}}\int_{-\infty}^{\xi}\frac{1}{(\xi-\xi')^{1/3}}\frac{d\Lambda(\xi')}{d\xi'}d\xi' = \frac{A(\xi)}{\sigma^{4/3}},\\ &A(\xi) = -\frac{2e^2}{3^{1/3}R^{2/3}}\int_{-\infty}^{\xi}\frac{1}{(\xi-\xi')^{1/3}}\frac{d\Lambda(\xi')}{d\xi'}d\xi'.\end{aligned} \tag{6}$$

The deflection of the electron due to the 1 dimensional CSR stationary wake $\delta\mathbf{X}_0$ is following:

$$\delta\mathbf{X}_0 = \int_{s_0}^{s_N}\mathbf{R}_6(s|s_N)\frac{A(\xi)}{E_0\sigma^{4/3}}ds \ , \tag{7}$$

$$\delta\mathbf{X}_0 = -\frac{A(\xi)}{E_0}\int_{s_o}^{s_N}\frac{\mathbf{M}(s|s_N)\mathbf{D}(s)}{\sigma^{4/3}}ds \ . \tag{8}$$

Consider an arc consisting of N identical bending cells but with a different set of betatron phase advances. It can be done using different matching sections. $\mathbf{M_i}$ is a transport matrix of one bending cell with phase advance μ_i.

$$\begin{aligned}&\mathbf{J} = \begin{pmatrix}\alpha & \beta\\ -\gamma & -\alpha\end{pmatrix},\\ &\mathbf{M_i} = \mathbf{I}\cos\mu_i + \mathbf{J}\sin\mu_i = e^{\mathbf{J}\mu_i}.\end{aligned} \tag{9}$$

Transport matrix of a few bending cells from j^{th} cell to N^{th} cell is:

$$\mathbf{M}(j|N) = \prod_{n=j}^{N}\mathbf{M_n} = \prod_{n=j}^{N}e^{\mathbf{J}\mu_n} = \exp\left(\mathbf{J}\sum_{n=j}^{N}\mu_n\right). \tag{10}$$

The deflection of the electron due to the CSR in N bending cells is:

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO037

$$\begin{aligned}\delta\mathbf{X}_0 &= -\frac{A(\xi)}{E_0}\sum_{j=1}^{N}\left(\int_{s_{j-1}}^{s_j}\frac{\mathbf{M}(s|s_N)\mathbf{D}(s)}{\sigma(s)^{4/3}}ds\right)\approx\\ &\approx -\frac{A(\xi)}{E_0}\sum_{j=1}^{N}\left(\frac{1}{\sigma_j^{4/3}}\int_{s_{j-1}}^{s_j}\mathbf{M}(s|s_N)\mathbf{D}(s)ds\right)\approx\\ &\approx -\frac{A(\xi)}{E_0}\sum_{j=1}^{N}\left(\frac{1}{\sigma_j^{4/3}}\int_{s_{N-1}}^{s_N}\mathbf{M}(j|N)\mathbf{M}(s|s_N)\mathbf{D}(s)ds\right)\approx\\ &\approx -\frac{A(\xi)}{E_0}\cdot\sum_{j=1}^{N}\frac{\exp\left(\mathbf{J}\sum_{n=j}^{N}\mu_n\right)}{\sigma_j^{4/3}}\cdot\int_{s_{N-1}}^{s_N}\mathbf{M}(s|s_N)\mathbf{D}(s)ds.\end{aligned} \tag{11}$$

where σ_j is rms bunch length at j^{th} bending cell, s_j is coordinate of j^{th} cell end.

The electron shift and emittance growth is minimal if:

$$\sum_{j=1}^{N}\frac{\exp\left(i\sum_{n=j}^{N}\mu_n\right)}{\sigma_j^{4/3}}=0. \tag{12}$$

In equation we change matrix **J** to imaginary unit i (due to $\mathbf{J}^2=-\mathbf{I}$). In the non-compression case:

$$\sum_{j=1}^{N}\exp\left(i\sum_{n=j}^{N}\mu_n\right)=0. \tag{13}$$

Obvious solutions of (13) is

$$\mu_n = 2\pi\frac{k}{N}, \tag{14}$$

where k is any integer [1]. The impact of non stationary CSR wake is also cancelled out in this case.

As an example of a case with compression, consider a three bending cell section. In this case equation (12) is following:

$$\frac{e^{i(\mu_1+\mu_2)}}{\sigma_1^{4/3}}+\frac{e^{i\mu_2}}{\sigma_2^{4/3}}+\frac{1}{\sigma_3^{4/3}}=0. \tag{15}$$

This equation is equivalent to the problem of finding the supplementary angles μ_1, μ_2 of a triangle with the sides $1/\sigma_1^{4/3}$, $1/\sigma_2^{4/3}$, $1/\sigma_3^{4/3}$ (Fig. 1).

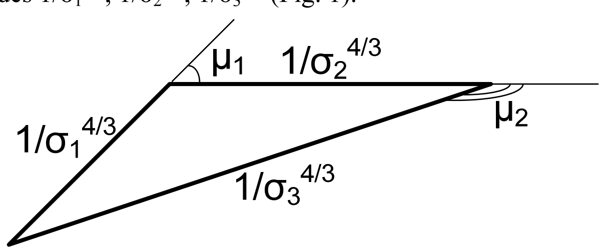

Figure 1: Triangle equivalent to the equation (15).

Solution of it is following:

$$\cos\mu_1=\frac{\frac{1}{\sigma_3^{8/3}}-\frac{1}{\sigma_1^{8/3}}-\frac{1}{\sigma_2^{8/3}}}{\frac{2}{\sigma_1^{4/3}\sigma_2^{4/3}}},\quad \cos\mu_2=\frac{\frac{1}{\sigma_1^{8/3}}-\frac{1}{\sigma_2^{8/3}}-\frac{1}{\sigma_3^{8/3}}}{\frac{2}{\sigma_2^{4/3}\sigma_3^{4/3}}}. \tag{16}$$

Since R_{56} is the same in all bending cells

$$\sigma_2=\frac{\sigma_1+\sigma_3}{2}. \tag{17}$$

Let $\kappa=\sigma_1/\sigma_3$ is a compression factor.

$$\begin{aligned}\cos\mu_1&=\frac{1}{2}\left(\left(\frac{\kappa+\kappa^2}{2}\right)^{4/3}-\left(\frac{1+\kappa}{2\kappa}\right)^{4/3}-\left(\frac{2\kappa}{1+\kappa}\right)^{4/3}\right),\\ \cos\mu_2&=\frac{1}{2}\left(\left(\frac{1+\kappa}{2\kappa^2}\right)^{4/3}-\left(\frac{1+\kappa}{2}\right)^{4/3}-\left(\frac{2}{1+\kappa}\right)^{4/3}\right).\end{aligned} \tag{18}$$

Maximal bunching is achieved when $\mu_1=0$, $\mu_2=\pi$, and numerical solution of κ is approximately 2.

Let's discuss limitations of this method.

First we do not consider the impact of non stationary wake, which is not mentioned in the equations above.

The second effect is the longitudinal motion in bending cells which is not similarly under compression. Bunch compressed in bending cells at the same value, but relative compression is different from cell to cell.

Third effect is related to a CSR induced longitudinal motion δct. The longitudinal dynamics in the bending cells are alike if CSR induced longitudinal motion is much smaller then rms bunch length.

$$\delta ct << \sigma, \tag{19}$$

$$\delta ct=\int_{s_1}^{s_2}\mathrm{R}_{56}(s|s_2)\frac{dE}{E_0 ds}ds<\frac{\Delta E_{csr}}{E_0}R_{56}=\delta_{csr}R_{56}, \tag{20}$$

where R_{56} is the longitudinal dispersion in the bending section, δ_{csr} is CSR induced relative energy spread produced in the bending section. Condition (19) is valid if δ_{csr} is small enough.

$$\delta_{csr} << \frac{\sigma}{R_{56}}. \tag{21}$$

SIMULATION

As a numerical example consider a case of a factor two compression in three cells arc. Consider a 90° arc consisting of three identical 30° TBA cells. TBA dipole angle is 10°, magnetic length is 1.16 m. Two sextupoles are used in each TBA cell to suppress second order dispersion. X betatron phase advance between first and second cell is $\mu_1=2\cdot2\pi$, between second and third cell is $\mu_2=2.5\cdot2\pi$. R_{56} in each cell is 16.7 mm.

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO037

Table 1: Bunch parameters in simulation

Beam energy	1 GeV
Bunch charge	0.4 nC
Initial energy spread rms	$4 \cdot 10^{-3}$
Bunch length rms	0.2 mm
Transversal normalized emittances, $\varepsilon_x/\varepsilon_y$	0.1/0.1 mm·mrad

Simulation was made using elegant in 1D non stationary model. In simulation CSRCSBEND and CSRDRIFT element were used. To accurately calculate the cancelation of emittance growth due to CSR, it is necessary to "adapt" the recommendation in the elegant manual [6] and use a greater number of particles when implementing the Savitzky-Golay filter half-width for smoothing. Parameter BINS was equal 600 and number of particles was 10^6, half-width of Savitzky-Golay filter was equal 2.

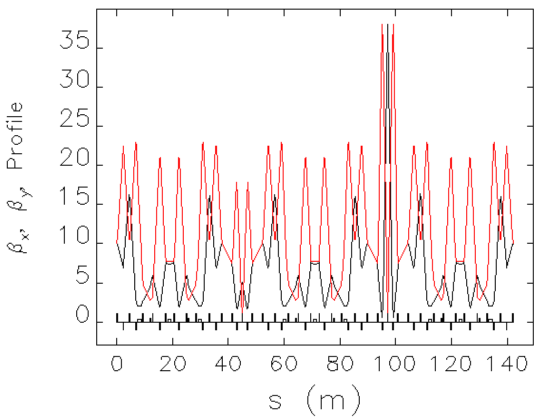

Figure 2: Beta function in 90° arc.

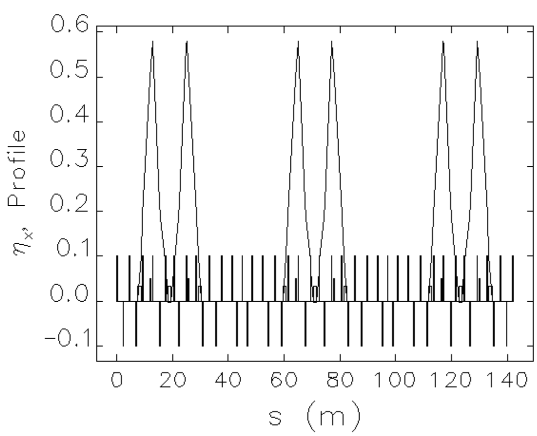

Figure 3: Transversal dispersion in 90° arc.

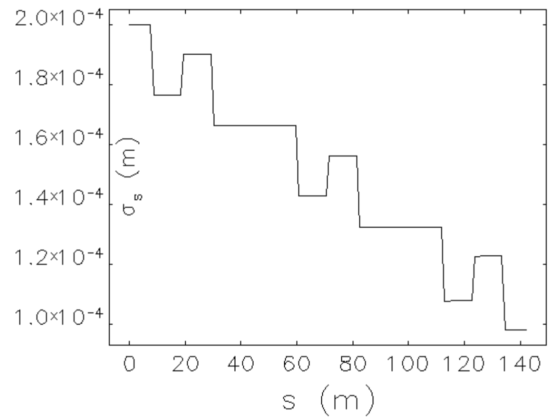

Figure 4: Bunch length 90° arc.

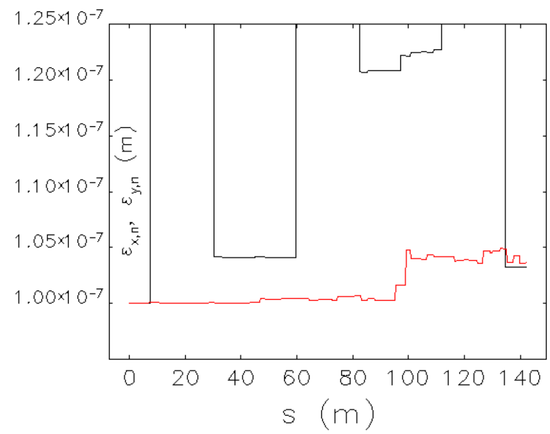

Figure 5: Emittance in 90° arc. (Large step-like increase/decrease in the horizontal emittance in the picture is due to the non-zero horizontal dispersion inside each TBA section).

REFERENCES

[1] R. Hajima. R-matrix analysis of the CSR effect in a future ERL light source. Proceedings of APAC 2004, Gyeongju, Korea, pp. 125-127.

[2] P. Emma. et al., Emittance Dilution Through Coherent Energy Spread Generation in Bending Systems, SLAC-PUB-7554, May 1997.

[3] Ya.S. Derbenev, V.D. Shiltsev, Transverse Effects of Microbunch Radiative Interaction, SLAC Report 7181, June, 1996.

[4] R.D. Ryne et al, Large-scale Simulation of Synchrotron Radiation Using a Lienard-Wiechert Approach, Proceedings IPAC 2012, New Orleans, USA.

[5] A.W. Chao, M. Tigner, Handbook of Accelerator Physics and Engeneering, 1998, Chapter 2, Sec. 2.1, p. 50.

[6] http://www.aps.anl.gov/Accelerator_Systems_Division/Accelerator_Operations_Physics/manuals/elegant_latest/node111.html