ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPME021

THEORETICAL FRAMEWORK TO PREDICT EFFICIENCY OF IONIZATION COOLING LATTICES*

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Abstract

Reduction of the 6-dimensional phase-space of a muon beam by 6 orders of magnitude is a key requirement for a Muon Collider. Recently, a 12-stage rectilinear ionization cooling channel has been proposed to achieve that goal. In this paper, we establish the mathematical framework to predict and evaluate the cooling performance of the proposed channel. We predict the system effectiveness, by deriving key lattice parameters such as the lattice quality factor which describes the rate of cooling versus the surviving particles and the longitudinal and effective partition numbers for each stage. Main theoretical findings, such as the equilibrium emittances and effective cooling length, are compared against findings from numerical simulations.

INTRODUCTION

A key challenge in the development of a Muon Collider is that the phase space of the beam that comes from pion decay greatly exceeds the acceptance of the downstream accelerator system and therefore, a cooling channel is required. Given the short life time of a muon particle, ionization cooling is the only practical method that can be realized [1]. Recently, a 12-stage tapered [2] rectilinear channel to achieve this goal has been proposed [3].

The primary goal of this work is to establish the key mathematical framework to evaluate the efficiency and predict cooling performance of an ionization cooling channel. This is of great interest since a well-designed channel must accurately follow the theoretical predictions. Deviations are usually associated with poor dynamic acceptance, chromatic effects, or poor matching into a cooling channel.

THEORETICAL BACKGROUND

In ionization cooling, particles pass through a material medium and lose energy through ionization interactions, and this is followed by beam reacceleration in rf cavities. The differential equation for rms transverse cooling is [4]:

$$\frac{d\varepsilon_N}{ds} = -\frac{g_t}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta\gamma\ \beta_t}{2}\frac{d\left\langle \theta_{rms}{}^2 \right\rangle}{ds}, \quad (1)$$

where the first term is the energy-loss cooling effect and the second is the multiple-scattering heating term. Here ε_N is the normalized rms emittance, β_t is the transverse betatron function at the absorber, dE/ds is the energy loss rate, and θ_{rms} is the rms multiple scattering angle:

$$\frac{d\left\langle \theta_{rms}^2 \right\rangle}{ds} = \frac{E_s^2}{\beta^4\gamma^2 L_R \left(m_\mu c^2\right)^2}, \quad (2)$$

where L_R is the material radiation length, and E_s is ~13.6 MeV, g_t is the transverse cooling partition number; $g_t = 1$ without transverse-longitudinal coupling.

Longitudinal cooling depends on having the energy loss mechanism such that higher-energy muons lose more energy. The equation is:

$$\frac{d\varepsilon_L}{ds} = -\frac{g_L}{\beta^2 E_\mu}\frac{dE_\mu}{ds}\varepsilon_L + \frac{\beta\gamma\ \beta_L}{2}\frac{d\left\langle (\delta p/p)^2{}_{rms} \right\rangle}{ds}, \quad (3)$$

where β_L is the longitudinal betatron function, g_L is the longitudinal partition number, which is approximately given by [5]:

$$g_{L,0} = -\frac{2}{\gamma^2} + 2\frac{(1-\frac{\beta^2}{\gamma^2})}{\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I(Z)}\right) - \beta^2}. \quad (4)$$

This factor must be > 0 for cooling, but is negative for muon momentum P_μ < ~ 350 MeV/c, and only weakly positive for larger P_μ. The factor is enhanced by placing the absorbers where transverse position depends upon energy (nonzero dispersion) and the absorber density or thickness also depends upon energy, such as in a wedge absorber (Fig. 1). With wedge cooling the longitudinal and dispersion-coupled transverse partition numbers are modified to $g_L \rightarrow g_{L,0} + D\rho'/\rho_0$ and, $g_x \rightarrow 1 - D\rho'/\rho_0$ where ρ'/ρ_0 is the change in density with respect to the transverse position, ρ_0 is the reference density associated with dE/ds and D is the dispersion. More generally, coupling of transverse and longitudinal damping mixes the cooling rates under the constraint that the sum of the cooling rates (damping partition numbers) Σ_g is constant, with a momentum dependence:

$$\Sigma_g = g_x + g_y + g_L = 2 + g_{L,0}$$

$$= 2\beta^2 + 2\frac{(1-\frac{\beta^2}{\gamma^2})}{\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I(Z)}\right) - \beta^2}, \quad (5)$$

Equations (1) and (3) have exponential solutions:

*Work supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with U.S. Department of Energy
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$$\varepsilon_i\,(s) = (\varepsilon_{i,0} - \varepsilon_{i,eq})e^{-s\frac{g_i dP_\mu/ds}{P_\mu}} + \varepsilon_{i,eq}\,, \qquad (6)$$

where i = x, y or L, for the appropriate dimension, $\varepsilon_{i,0}$ is the initial emittance, and $\varepsilon_{i,eq}$ is the equilibrium emittance found from balancing the heating and cooling terms. The equilibrium emittances are:

$$\varepsilon_{t,eq} \cong \frac{\beta_t E_s^2}{2 g_t \beta m_\mu c^2 L_R \frac{dE}{ds}}\,, \qquad (7)$$

for the transverse motion and,

$$\varepsilon_{L,eq} \cong \frac{\beta_L m_e c^2 \gamma^2 \left(1 - \frac{\beta^2}{2}\right)}{2 g_L \beta m_\mu c^2 \left[\frac{\ln[\frac{2 m_e c^2 \gamma^2 \beta^2}{I(Z)}]}{\beta^2} - 1\right]}\,, \qquad (8)$$

for longitudinal motion. Another critical component of the cooling solutions is the cooling length given by:

$$L_{cool,i} = \left[\frac{g_i}{P_\mu}\frac{dP_\mu}{ds}\right]^{-1} = \left[\frac{g_i}{\beta^2 E_\mu}\frac{dE_\mu}{ds}\right]^{-1}\,, \qquad (9)$$

where the energy loss is averaged over the full transport length. The cooling length must be much less than the decay length (660 $\beta\gamma$ m); preferably < ~100m.

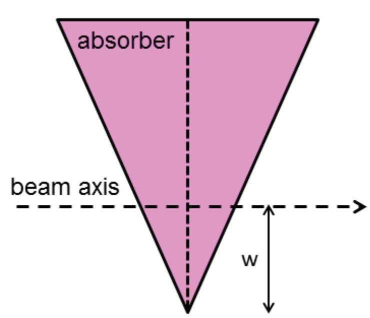

Figure 1: The beam passes from left to right with its center along the axis (δ=0), with a dispersion D at the wedge. The wedge opening angle is α and the distance from the beam centroid (closed orbit center) to the apex is w. With wedge absorbers: $\delta g_L = D\,\rho'/\rho_0 = D/w$.

At ~200 MeV/c the preferred focusing magnets are solenoids, which focus both x and y and also couple x and y motion. Solenoidal focusing lattices have been developed with relatively small betatron function at the absorbers and with dispersion that can be combined with wedges to obtain 6-D cooling [5]. Typically, x and y motion is so tightly coupled that they cannot be separated, even though the wedge/dispersion is predominantly in one plane. The partition numbers are approximated by: $g_L \to g_{L,0} + \delta g_L$ and $g_x = g_y = 1 - \delta g_L/2$.

With solenoidal focusing, particles have angular momentum. To some extent this complication can be ignored if there are periodic field flips. The intrinsic angular momentum is damped by the cooling absorbers. Without field flips the transverse modes are not as tightly coupled.

Some yardsticks have been developed for evaluating the performance of cooling channels. One is the quality factor Q, which is defined locally by the expression:

$$Q = \frac{\frac{1}{(\varepsilon_x\varepsilon_y\varepsilon_z)}\frac{d(\varepsilon_x\varepsilon_y\varepsilon_z)}{ds}}{\frac{1}{N}\frac{dN}{ds}}\,, \qquad (10)$$

which is a useful guideline for cooling rate evaluation, and good cooling channels have large Q (preferably Q> ~10, since collider luminosity L is proportional to $N^2/(\beta_\perp (\varepsilon_x\varepsilon_y)^{1/2})$, Q > ~6 is needed to break even).

Another criterion with some validity is g_{eff} a effective total cooling rate generalized from the partition numbers. For a cooling channel segment, g_{eff} is given by:

$$g_{eff} = \frac{\ln\left[\frac{(\varepsilon_x\varepsilon_y\varepsilon_z)_{start}}{(\varepsilon_x\varepsilon_y\varepsilon_z)_{end}}\right]}{\frac{L}{P_\mu}\frac{dP_\mu}{ds}}\,, \qquad (11)$$

or,

$$g_{eff,L}(s) = \frac{\frac{1}{(\varepsilon_x\varepsilon_y\varepsilon_z)}\frac{d(\varepsilon_x\varepsilon_y\varepsilon_z)}{ds}}{\frac{1}{P_\mu}\frac{dP_\mu}{ds}}\,, \qquad (12)$$

as a local value L. Note that dP_μ/ds is the total P_μ loss in absorbers. From the above analysis, for an optimum cooling channel g_{eff} >1.0 and when the initial emittance is close to the equilibrium emittance, g_{eff} becomes small.

APPLICATION TO A MUON COOLING SCENARIO

The above discussion can be applied to the analysis of cooling channels designed for collider scenarios. The proposed 6-D cooling scenario [3] starts with 4 segments of transverse and longitudinal cooling (A1-A4), following which 21 bunches are recombined to 1. The enlarged beam is then cooled in 8 further segments (B1-B8). Each segment consists of a sequence of identical cooling cells with tilted solenoidal focusing, rf and absorbers (H_2 or LiH). Parameters of these segments are presented in Table 1. In each segment $\delta g_L = D\rho'/\rho_0 = D/w$. D and β_t are obtained by lattice evaluation at the central momentum.

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Table 1: Components and Performance of the Rectilinear 6-dimensional Cooling Channel

Stage	L m	Cell m	β_t cm	D cm	L_{abs} cm	Θ_{abs}	g_L	g_t	L_{co} m	$\varepsilon_{t,eq}$ mm	$\varepsilon_{t,sim}$ mm	$\varepsilon_{L,sim}$ mm	g_{eff}	$\varepsilon_{t,cal}$ mm	$\varepsilon_{L,cal}$ mm
A1	132	2	82	10.7	2·12.6	39	.37	.70	52	4.7	17	49	1.09	17	49
A2	171.6	1.32	55	6.8	2·8.6	44	.40	.68	50	2.9	6.3	14.4	.77	6.9	19
A3	107	1	38	4.2	15.2	100	.40	.68	42	2.0	3.4	4.7	.66	3.4	4.3
A4	70.4	0.8	30	1.85	10.9	110	.22	.75	46	1.5	2.1	2.6	.56	2.3	2.0
											1.47	2.34		1.7	2.0
B1	55	2.75	42	5.25	33.6	120	.25	.73	49	2.3	5.1	10.	.87	5.1	10
B2	64	2	27.4	5.0	30.7	117	.24	.74	39	1.5	3.76	7.8	.70	3.5	7.9
B3	81	1.5	20.2	4.6	25.3	113	.26	.72	36	1.1	2.4	6.1	.54	2.2	5.6
B4	63.5	1.27	14	4.0	24.4	124	.32	.69	31	0.8	1.55	4.3	.45	1.4	3.8
B5	73	.806	8.1	1.4	3.9	61	.12	.79	23	0.6	1.1	3.4	.34	1.0	2.6
B6	62	.806	5.9	1.2	4.6	90	.23	.74	20	0.5	.68	3.0	.30	.65	2.2
B7	40.3	.806	4.2	1.1	4.0	90	.25	.73	23	0.35	.50	2.2	.36	.5	1.9
B8	49	.806	3.0	0.6	3.8	120	.24	.73	24	0.25	.38	1.94	.38	.39	1.7
											.29	1.6		.28	1.5

In Table 1, $\varepsilon_{t,sim}$ and $\varepsilon_{L,\,sim}$ are calculated using ICOOL [6] simulations of the system. Thus, $\varepsilon_{t,\,cal}$ and $\varepsilon_{L,cal}$ are calculated using the rms cooling equations with the simulated initial conditions. L_{co} is $L_{cool,i}$ at g_i=1. L_{abs} is the absorber length for the beam centroid. A1-2 have two H_2 absorbers per cell, A3-4 and B1-4 have 1 H_2 per cell and B5-8 have LiH.

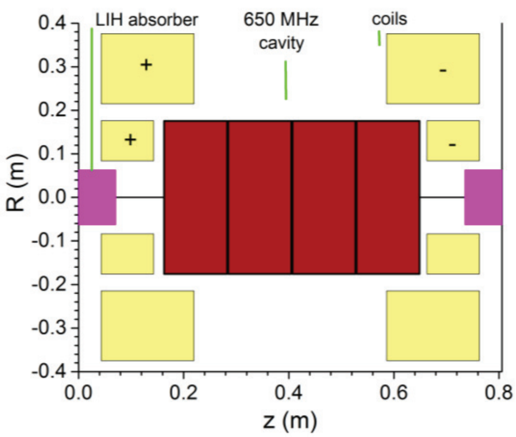

Figure 2: Side view of a cell of the rectilinear cooling channel (from B6) showing cavities (red), focusing coils (yellow) and wedge absorbers (magenta). Compared to earlier stages, stronger coils, higher-frequency rf, and smaller apertures are used. Details can be found Ref. 3.

PERFORMANCE EVALUATION

In each sequence (A1-A4) and (B1-B8), the focusing increases from section to section as the beam is cooled transversely and longitudinally (see Fig. 2). Initially the beams are much larger than the equilibrium emittances, and the cooling is more efficient for the earlier segments (A1-A4) with g_{eff} ranging from 1.09 to 0.56. In the later segments the beams are more closely matched to the equilibrium and the cooling is less efficient (for B5-B8 g_{eff} ~ 0.35). The loss of efficiency is also seen in a drop in the Q-factor evaluated in simulations, that is: Q ~ (7.1,17,16,13.5) for sections A1-A4 but drops over B1-B8 Q ~ (7.6,12,10.5, 8.5,7.5,7.3,5.8,4.6).

The wedge absorbers and dispersion are sufficient to enable longitudinal cooling in each segment. While there is good agreement between the simulated cooling channels and the simplified models there are some discrepancies that need to be examined.

The longitudinal cooling in simulation tends to be somewhat less than expected from simple application of the rms cooling equations. The linear model may overestimate the wedge/dispersion and it omits transverse – longitudinal dynamic coupling. Both A1-A4 and B1-B8 are ~450 m long with μ decay losses of ~30% (dynamic losses are similar). A shorter channel with larger g_{eff} could be better. On the other hand, the present rectilinear channel does meet the basic cooling goals of the project.

CONCLUSION

The Muon Accelerator Program is developing a number of ionization cooling channels for possible use for a Muon Collider. In this study, we have presented the mathematical framework to study the effectiveness of a ionization cooling channel. The main theoretical findings were compared against results from numerical simulations. The authors are very grateful to J. S. Scott, R. B Palmer and V. Balbekov for many fruitful discussions.

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