ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUOAB01

COMPUTATION OF EIGENMODES IN LONG AND COMPLEX ACCELERATING STRUCTURES BY MEANS OF CONCATENATION STRATEGIES*

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Abstract

The computation of eigenmodes for complex accelerating structures is a challenging and important task for the design and operation of particle accelerators. Discretizing long and complex structures to determine its eigenmodes leads to demanding computations typically performed on super computers. This contribution presents an application example of a method to compute eigenmodes and other parameters derived from these eigenmodes for long and complex structures using standard workstation computers. This is accomplished by the decomposition of the complex structure into several single segments. In a next step, the electromagnetic properties of the segments are described in terms of a compact state-space model. Subsequently, the state-space models of the single structures are concatenated to the full structure. The results of direct calculations are compared with results obtained by the concatenation scheme in terms of computational time and accuracy.

INTRODUCTION

Eigenmode computations are important to design superconducting resonators used in state-of-the-art high energy particle accelerators. While eigenmode computations for single cavities can be performed with desktop computers, the brute-force approach for computing modes (often referred to as multi-cavity modes) in long cavity chains typically requires the usage of expensive computer infrastructure. The computational demand of the direct eigenmode computations of the full chain can be avoided by decomposing the structure into single elements. Subsequently, a set of eigenmodes of the individual segments is determined. Finally, the eigenmodes of the individual segments are superposed such that the field distributions of the eigenmodes are continuous at the decomposition planes. The method allowing for these steps is referred to as State-Space Concatenations (SSC) [1]. SSC is in fact a generalization of the Coupled S-Parameter Calculations (CSC) approach [2] used for the scattering parameter computations in e.g. [3]. In contrast to CSC, SSC directly delivers eigenmodes as well as field distributions.

The current contribution is focussed on the computation of eigenmodes in rotational symmetric arrangements of third harmonic cavities (see Fig. 1). The two-cavity arrangement in Fig. 1a is predominantly used for validation with direct computations performed with CST Studio Suite® [4], whereas the chain in Fig. 1b models the cavity chain in the module ACC39 at FLASH and the structure with eight cavities in Fig. 1c reflects the arrangement of third harmonic cavities designed for the European X-FEL. It is stressed that all computations presented in this article are performed without the consideration of HOM couplers, so that the structures are rotational symmetric. Basically, the rotational symmetry allows for using a 2D field solver for the eigenmode computations. However, the current study is only an intermediate step as for future investigations the consideration of HOM couplers is in progress. In that case, the application of a 2D field solver for the straightforward eigenmode computations fails.

GENERAL NUMERICAL ASPECTS

All presented eigenmode computations neglect intrinsic losses of the structures, so that the tangential components of the electric field on the boundary vanishes. At the ends of the beam pipes, the normal component of the electric field is enforced to vanish. Figure 1 shows the decomposition of the geometries into three segments, namely beam pipe, cavity, and bellow. At the black lines waveguide ports accounting for the TM_{01} 2D port mode are assigned (cutoff frequency $f_{co} = 5.7371\,\mathrm{GHz}$). All three segments are discretized with [4] based on a hexahedral mesh using all three symmetry planes. All eigenmodes of the segments with frequencies below 8.5 GHz are computed with the Jacobi-Davidson Method (JDM). In addition, a set of *residual* modes is determined to account for not considered higher order modes. Based on the performed computations, the individual segments are described by the reduced second order state-space equation

$$\frac{\partial^2}{\partial t^2}\mathbf{x}_r(t) = \mathbf{A}_r\,\mathbf{x}_r(t) + \mathbf{B}_r\frac{\partial}{\partial t}\mathbf{i}_r(t), \tag{1}$$

and the corresponding output equation

$$\mathbf{v}_r(t) = \mathbf{B}_r^{\mathrm{T}}\,\mathbf{x}_r(t). \tag{2}$$

Here, r is the index of the segment (i.e. $1 \le r \le 3$), $\mathbf{i}_r(t) \in \mathbb{R}^{2\times 1}$ comprises the two modal currents of the TM_{01} 2D port modes at port (or cut) planes, and $\mathbf{v}_r(t) \in \mathbb{R}^{2\times 1}$ the respective modal voltages. The state-matrices $\mathbf{A}_r \in \mathbb{R}^{N_{\mathrm{sta}}\times N_{\mathrm{sta}}}$ are diagonal matrices dependent on the resonant frequencies of the eigenmodes and the residual modes of the segment r. The input-matrices $\mathbf{B}_r \in \mathbb{R}^{N_{\mathrm{sta}}\times 2}$ hold interaction integrals between 3D eigenmodes and 2D port modes. The state-vector $\mathbf{x}_r \in \mathbb{R}^{N_{\mathrm{sta}}\times 1}$ contains transient weighting factors for the 3D eigenmodes. The electric field distributions due to the modal current excitation at the waveguide ports in the

* The work is part of EuCARD-2, partly funded by the European Commission, GA 312453.
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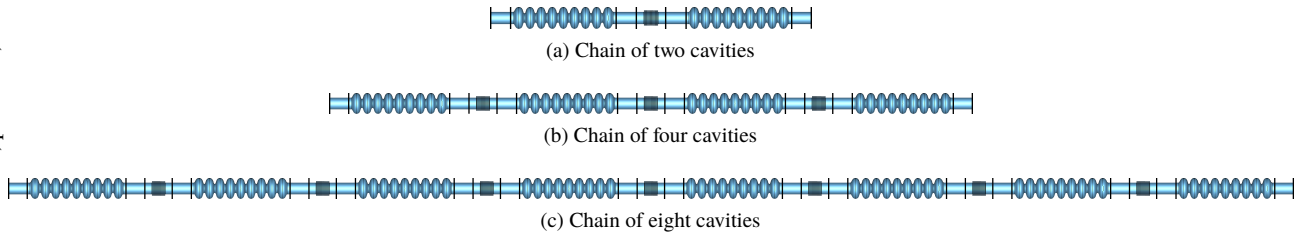

Figure 1: Investigated cavity chains with bellows in between. Structure (a) is solely used for validation purposes, (b) models the third harmonic cavity chain in FLASH, and (c) reflects the chain for the European X-FEL respectively.

segments can be determined by

$$\mathbf{E}_r(\mathbf{r},t) = \sum_{n=1}^{N_{\text{sta}}} \tilde{\mathbf{E}}_{r,n}(\mathbf{r})\, x_{r,n}(t), \tag{3}$$

where $\tilde{\mathbf{E}}_{r,n}(\mathbf{r})$ is the spatial dependent field distribution of the n-th eigenmode in the r-th segment. Table 1 collects details of the three state-space models and details related to the creation of the models. Here, N_{mc} is the number of mesh cells used for the discretization and T_{gen} the total time needed to generate the reduced order model. All computing times refer to an Intel(R) Core(TM) i5-2400 CPU @ 3.10 GHz processor with a RAM size of 8 GB.

Table 1: Details on State-Space Models for Segments

Segment	r	N_{sta}	N_{mc}	T_{gen}
Pipe	1	8	1,100	< 1 min
Cavity	2	33	44,436	51 min
Bellow	3	15	16,758	5 min

After generating the state-space models of the individual segments, they are combined to the three structures shown in Fig. 1 using the approach proposed in [1]. Consequently, three different state-space systems, which describe the properties of the entire chains arise from the concatenations. The eigenvalues of the resulting state-matrices determine the resonant frequencies of the eigenmodes in the coupled arrangements, whereas the eigenvectors contain weighting factors to construct field distributions of the eigenmodes of the full structures based on the eigenmodes of the individual segments, e.g. refer to (3). Therefore, the field distributions of the eigenmodes of the full structure are available by means of SSC. However, often only the longitudinal field profiles of the eigenmodes on the symmetry axis are of interest. In particular the so-called R over Q parameter of the eigenmodes, which is defined by

$$(R/Q)_n = \frac{1}{\tilde{\omega}_n W_n} \left| \int_{z_{\min}}^{z_{\max}} E_{n,z}(0,0,z)\, \mathrm{e}^{j\tilde{\omega}_n z/c}\, \mathrm{d}z \right|^2 \tag{4}$$

is of crucial importance as it specifies the interaction between the bunch of charged particles and the 3D eigenmodes in the resonator chain or vice versa. In (4), $\tilde{\omega}_n$ denotes the resonant frequency of the n-th mode, W_n the energy stored in the n-th mode, c the speed of light, and $E_{n,z}(0,0,z)$ the longitudinal on-axis field profile of the n-th mode.

VALIDATION AND RESULTS

Validation of Scheme

To ensure that the SSC scheme delivers reasonable resonant frequencies and field distributions, the two-cavity chain shown in Fig. 1a is entirely discretized with [4] taking into account all symmetry planes. In total, the eigenmode computations with the JDM solver require ≈ 29 h to compute 136 3D eigenmodes. These modes are computed based on the generated state-space models using the SSC approach as well. Here the total computational time is ≈ 1 h (compare Table 1). The relative error of the resonant frequencies obtained from the SSC method is in the order of 10^{-4}, compared to the solution from the direct computation. Figure 2a shows the comparison between the $(R/Q)_n$ parameters determined by the direct computation with [4] (red circles) and the concatenation approach with SSC (black crosses). The diagram conveys that SSC delivers reasonable $(R/Q)_n$ parameters, which agree with the results arising from the straightforward computations.

Comparison of R/Q for Different Arrangements

Figure 2b depicts the $(R/Q)_n$ parameters of TM_{01} monopole modes arising from the SSC concatenation scheme for the three structures shown in Fig. 1. The black crosses indicate the $(R/Q)_n$ parameters for the two-resonator arrangement (Fig. 1a), orange plusses for the chain with four resonators (Fig. 1b), and blue circles for the eight-resonator structure (Fig. 1c). Modes belonging to the fundamental TM_{01} monopole band are observable in Fig. 2 in the frequency range 3.7461 GHz – 3.9 GHz, whereas modes in the frequency range 6.9799 GHz – 8.1645 GHz belong to the second TM_{01} monopole band. The modes in between these two bands are beam pipe and bellow modes. Figure 2b shows that the larger the chain becomes, the more modes exists in the structure within a finite frequency interval. Furthermore, enlarging the chain leads to larger maximal $(R/Q)_n$ factors. In addition, the diagram reveals that modes in the second TM_{01} monopole band have a relatively large $(R/Q)_n$ so that these modes can be excited in an unwanted manner by bunches of charged particles. The red curve in Fig. 3 shows the longitudinal field profile of the mode in the eight-cavity chain with the largest $(R/Q)_n$ parameter in the second monopole band. The field energy is not localized in individual segments of the chain so that a good coupling of the mode to the beam pipes ends (and therefore a small external

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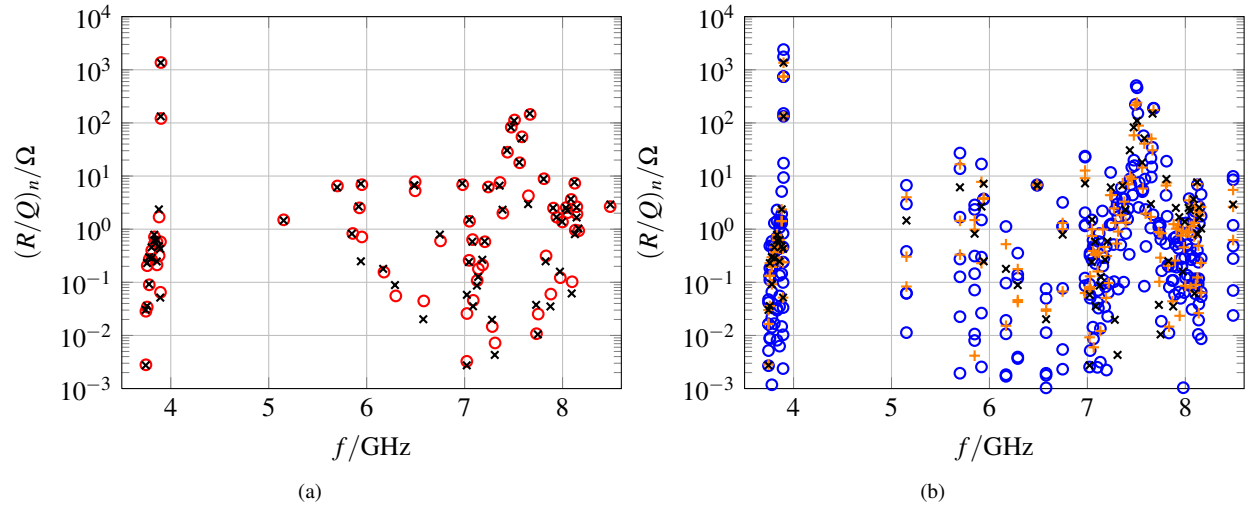

Figure 2: Diagram (a) shows the $(R/Q)_n$ of monopole modes in the chain depicted in Fig. 1a. The red circles arising from a direct eigenmode computation with [4], whereas the black dots are obtained from the SSC approach. Both methods deliver comparable results. Diagram (b) presents the $(R/Q)_n$ determined by SSC for the arrangements shown in Fig. 1. Black crosses correspond to a chain of two, orange plusses to a chain of four and blue circles to a chain of eight cavities.

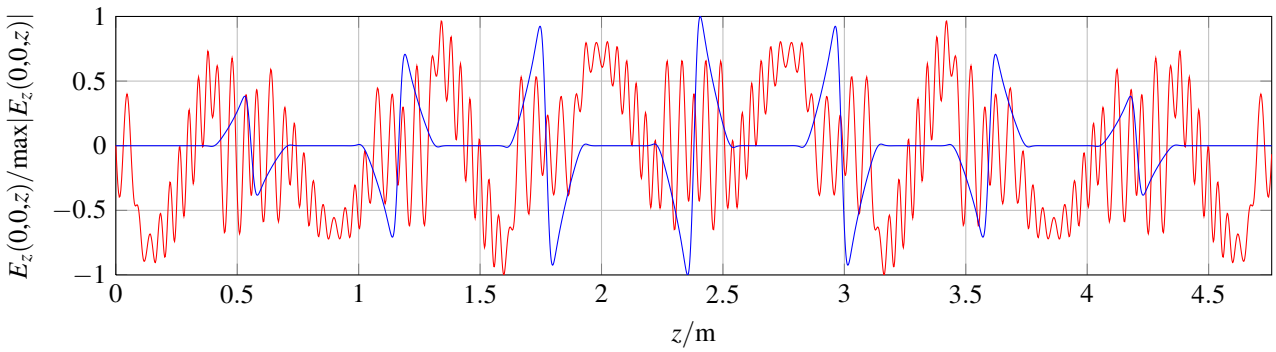

Figure 3: Normalized longitudinal on-axis electric field profiles of two eigenmodes in the eight-cavity chain (refer to Fig. 1c). Red shows the profile of the mode with the largest R/Q parameter in the second TM_{01} monopole band, i.e. $\tilde{\omega}_n/2/\pi = 7.4983\,\text{GHz}$, $(R/Q)_n = 503.81\,\Omega$. Blue shows the profile of the monopole mode in between the first and the second monopole band with the largest R/Q parameter, i.e. $\tilde{\omega}_n/2/\pi = 5.6987\,\text{GHz}$, $(R/Q)_n = 27.085\,\Omega$.

quality factor) can be expected. The blue curve in Fig. 3 represents the longitudinal field profile of the beam pipe (or bellow) mode with the largest considered $(R/Q)_n$ parameter. Here, the field energy is predominantly localized in the seven bellows, so that the coupling of the modes to the two ends of the beam pipe is expected to be weak and the external quality factor to be large. Modes with a large $(R/Q)_n$ and a weak coupling to the ends of the beam pipe are potentially dangerous for the operation as the amplitude of the modes decays slowly (if not damped additionally) after the excitation by the bunch of charged particles.

SUMMARY AND OUTLOOK

The current contribution introduces the SSC scheme for the computation of multi-cavity modes. The validation shows that the direct computation of eigenmodes and the computation by means of the concatenation method SSC delivers similar results (rel. error in resonant frequency of modes is 10^{-4}). The concatenation approach however is computationally more efficient for large and complex structures than straightforward computations. Further studies based on SSC are in preparation which account for rotational symmetry breaking HOM couplers. Aside from $(R/Q)_n$ parameters external quality factors are also of interest for future considerations.

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