

# EFFECT OF LASER-PLASMA CHANNELING ON THIRD-HARMONIC RADIATION GENERATION

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## Abstract

An intense Gaussian laser beam, propagating through a magnetized plasma, becomes self-focused due to the ponderomotive force on the electrons. The magnetic field reduces the radius of the laser beam and enhances the self focusing of the laser beam. The self-sustained plasma channel can affect the efficiency of harmonic generation of the interacting laser beam. The radial density gradient of the channel beats with the oscillatory electron velocity to produce density perturbation at laser frequency. The ponderomotive force at second-harmonic frequency produces electrons density oscillations that beat with the oscillatory velocity to create a non-linear current, driving the third harmonic radiation. Our results show that the efficiency of third-harmonic generation of the laser beam is affected significantly due to the self-sustained plasma channel. The strength of magnetic field play a crucial role in efficiency enhancement of third-harmonic generation.

## INTRODUCTION

Currently, the laser intensities in excess of  $10^{20}$  W/cm<sup>2</sup> are available. The electric field of laser pulses is very high, such a large electric field can force the electrons in the plasma to oscillate with relativistic velocity. Although a number of high-order harmonic generations [1-2] were analyzed but the third harmonic generation [3-4] has its unique place in the research related to laser plasma interactions. From last few years a great deal of research has been focused on second and third-harmonic in laser produced plasma [5-7]. In process of harmonic generation, two photons of energy  $\hbar\omega_1$  and momentum  $\hbar\vec{k}_1$  combine to produce a photon of higher-harmonic radiation with energy  $\hbar\omega_2$  and momentum  $\hbar\vec{k}_2$ , where  $(\omega_1, \vec{k}_1)$  are the frequency and wave vector of the fundamental wave and  $(\omega_2, \vec{k}_2)$  are the frequency and the wave vector of the second harmonic wave, which satisfy the linear dispersion relation for electromagnetic waves. During third-harmonic generation phenomenon, the fundamental laser beam generates a beam of frequency with three times of the fundamental beam.

A plasma channel is produced from the radial expulsion of plasma ions due to charge separation created in the displacement (or cavitation) of plasma electrons by the large ponderomotive force of the laser. An intense trailing laser pulse ( $I \approx 5 \times 10^{16}$  W/cm<sup>2</sup>) is observed to be guided throughout the length of this channel for about 20 Rayleigh lengths, approximately equal to the propagation length of the self-guided pump laser pulse [8]. This kind of plasma channel can be utilized in enhancement of the generation of second and third-harmonics radiation of a

fundamental frequency of the laser beam. We emphasize to use a laser-plasma channel to study the third harmonic generation in the presence of a magnetic field. Salih *et al.* [9] have studied the second-harmonic generation of a Gaussian laser beam in a self created magnetized plasma channel. If the laser intensity is quite enough then the plasma electron motion can be at relativistic level. We considered the case, where the relativistic effects are dominant in laser-plasma interactions. Rax *et al.* [10] demonstrated that when an intense plane polarized laser pulse interacts with a plasma, the relativistic nonlinearities induce third harmonic radiation. Kant *et al.* [11] observed the resonant third-harmonic generation of a short pulse laser from electron hole plasmas in the presence of wiggler magnetic field. It was observed that for a specific wiggler wave number value, the phase matching conditions for the process are satisfied, leading to resonant enhancement in energy conversion efficiency.

In this article, we examine the effect of a laser-plasma channeling and a longitudinal magnetic field on third-harmonic generation of a Gaussian laser beam in a plasma. The nonlinearity arises through the ponderomotive force and relativistic mass variation, while the ions are taken to be immobile. The second harmonic ponderomotive force produces electrons density oscillations that beat with the oscillatory velocity due to the laser to create a non linear current, driving the third harmonic.

## DYNAMICS OF FUNDAMENTAL BEAM

We consider the propagation of a high-power Gaussian laser beam of spot size  $r_0$  in a plasma of uniform density  $n_0$  along the direction of a static magnetic field  $B_z \parallel \hat{z}$ . The electric field is  $\vec{E}_1 = (\hat{x} + i\hat{y})A_1(r, z)\exp[-i(\omega t - k_1 z)]$ , where  $k_1 = (\omega/c)[1 - \{\omega_p^2/(\omega(\gamma\omega - \omega_c))\}]^{1/2}$ ,  $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ ,  $\omega_c = eB_s/mc$ ,  $-e$  and  $m$  are the electron charge and mass, respectively, and  $c$  is the velocity of light. The laser imparts an oscillatory velocity to the electrons and exerts a ponderomotive force,  $\vec{F}_p = e\nabla\phi_p$ . The ponderomotive force pushes the electrons radially outwards creating a space charge field. On the time scale of plasma period  $\omega_p^{-1}$ , a quasi-steady state is reached. In this snerio, the modified electron density is  $n_e/n_0 = 1 - (\nabla \cdot \vec{F}_p / 4\pi n_0 e^2)$ , where  $n_0$  is the ion density.

Following Singh and Gupta [12], the equation of band width parameter (i.e., the evolution of the laser spor size) ( $f$ ) as a function of the  $z = \xi(\omega r_0^2/c)$  can be written as

$$\frac{\partial^2 f}{\partial \xi^2} = \frac{1}{4} \left( 1 + \frac{\varepsilon_0}{\varepsilon_p} \right)^2 \frac{1}{\varepsilon_0 f^3} - \frac{1}{2} \left( 1 + \frac{\varepsilon_0}{\varepsilon_p} \right) \left( \frac{r_0^2 \omega^2}{c^2} \right) \frac{\phi}{\varepsilon_0} f \quad (1)$$

In Eq. (1), the first term on the right hand side is due to the diffraction effects and the second term is the nonlinear term that is responsible for laser self focusing. For self-focusing, the two terms on the right hand side balance each other exactly with  $f = 1$  giving  $R_0^2 = (1/4)(1 + \varepsilon_0/\varepsilon_p)^2 (\varepsilon_0 \Omega_p^2 / \phi)$ , where  $R_0 = r_0 \omega_p / c$  and  $\Omega_p = \omega_p / \omega$ .

The solution of Eq. (1) gives the laser spot size determination. Laser guiding can be confirmed from our results. In our results, it is noted that the radius of the laser beam decreases with the intensity of the fundamental laser, as shown in Fig. 1 i.e., the laser beam gets more self-focused at higher intensities. The magnetic field reduces the radius of the laser beam, i.e., enhances the self focusing of the laser beam.

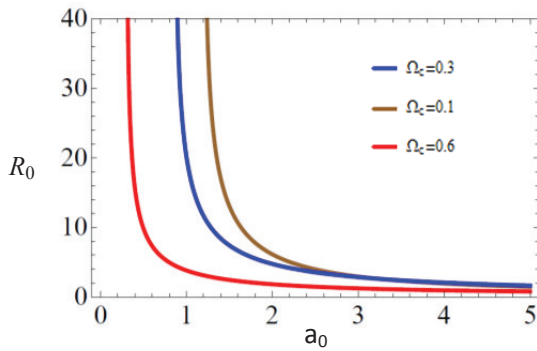


Figure 1: Variation of normalized laser beam radius ( $R_0$ ) with normalized fundamental laser intensity ( $a_0$ ) for plasma density  $\Omega_p^2 = 0.1$  at different values of the magnetic field ( $\Omega_c = 0.1, 0.3, 0.6$ ).

### THIRD HARMONIC GENERATION

The laser exerts ponderomotive force on electrons  $\vec{F}_2 = -(1/2)e\vec{v}_1 \times \vec{B}_1$  at  $(2\omega, 2\vec{k})$ , which gives rise to oscillatory velocity  $\vec{v}_2 = -\hat{z} e^2 E_1^2 \vec{k}_1 / 4m^2 \gamma \omega^2 (\gamma \omega - \omega_c)$ , which couples with density perturbation at laser frequency through equation of continuity to produce density perturbations at  $(2\omega, 2\vec{k})$ ,  $n_2 = -e^2 n_e E_1^2 k_1^2 / 4m^2 \gamma \omega^3 (\gamma \omega - \omega_c)$ ,  $n_2$  couples with  $\vec{v}_1$ , to produce nonlinear current density at the third harmonic frequency  $J_3^{NL} = e^4 n_e E_1^2 k_1^2 \vec{E}_1 / 8m^3 i \gamma \omega^3 (\gamma \omega - \omega_c)^2$ , whereas, linear current density can be written as  $J_3^L = -n_e e^2 \vec{E}_3 / 3m \gamma i \omega$ . Using the electromagnetic wave equation, and taking the fast variation  $\vec{E}_3$  as

$(\hat{x} + i\hat{y})A_3(r, z) \exp\{-i(3\omega t - k_3 z)\}$ , we obtain in the WKB approximation,

$$2ik_3 \frac{\partial A_3}{\partial z} + \left( \frac{\partial^2 A_3}{\partial r^2} + \frac{1}{r} \frac{\partial A_3}{\partial r} \right) + \left[ \frac{9\omega^2}{c^2} - \frac{\omega_p^2 \left( \frac{n_e}{n_0} \right)}{\gamma^2} - k_3^2 - \frac{\omega^2}{c^2} \phi \frac{r^2}{r_0^2} \right] A_3 = \frac{3}{8} \frac{\omega_p^2 e^2 \left( \frac{n_e}{n_0} \right) k^2 A_1^3}{m^2 \gamma \omega^2 (\gamma \omega - \omega_c)^2} \quad (2)$$

The radial eigenmodes of  $A_3$  can be deduced from (2), if one ignores the first and the last term. Eq. (2) has the Laguerre polynomial solution  $A_3 = F(r)g(z)$ , where  $F = (r/r_0') \exp[-(r^2/2r_0'^2 f^2)]$ , and  $r_0'^2 = (c^2 r_0^2 / \omega^2 \phi)^{1/2}$  is the third harmonic beam width. On retaining the remaining terms of (2), we presume that the radial profile remains unmodified, only the beam width parameter changes, as the beam propagates in the magneto-plasma. Substituting for  $A_3$  in (2), multiplying the resulting equation by  $F^* r dr$ , and integrating over  $r$  from 0 to  $\infty$ , we get [7, 8]

$$\left| \frac{G_3}{A_1} \right| = \frac{\omega_p^2 a_0^2 \left( \frac{n_e}{n_0} \right) k^2}{8\gamma (\gamma \omega - \omega_c)^2} \frac{(r_0'^2 f^2 r_0'^2)}{(3r_0'^2 f^2 + r_0'^2)} \xi \quad (3)$$

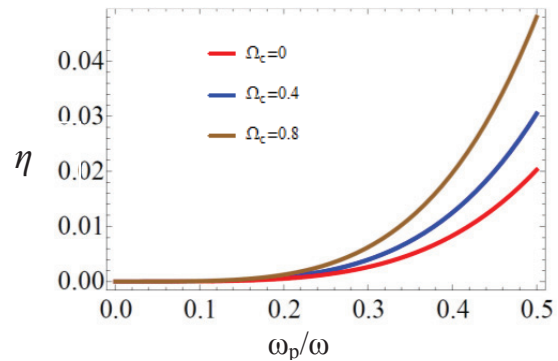


Figure 2: Variation of conversion efficiency of third-harmonic generation with the normalized plasma density for different strengths of the magnetic field ( $\Omega_c = 0, 0.4, 0.8$ ) at  $a_0 = 4$ .

The conversion efficiency of THG in a self-sustained magnetic-plasma channel can be calculated by squaring of Eq. (3). Laser beam gets more self-focused at higher densities, therefore, the efficiency of third-harmonic generation increases with plasma density, as shown in Fig. 2.

The efficiency of third-harmonic generation is affected by the strength of magnetic field. The presence of magnetic field alters the dynamics of oscillating electrons in the plasma which modifies the plasma wave. Still the plasma electrons try to be with the oscillating field but the presence of magnetic field modifies the electric field generated by the plasma wave. In a result, the third-harmonic is significantly affected by the strength of magnetic field. The efficiency of third-harmonic increases with the strength of the magnetic field.

## CONCLUSIONS

An intense Gaussian laser beam, propagating through a magnetized plasma, gets self focused due to the radial ponderomotive force on the electrons. When the laser power equals the critical power of the self focusing, the laser propagates without convergence or divergence and a depressed density plasma channel is created. The magnetization of the plasma enhances the self-focusing of the laser. A plasma channel has radial electron density gradient in electron density that plays a crucial role in third harmonic generation. The radial density gradient of the channel beats with the oscillatory electron velocity to produce density perturbation at laser frequency. The second-harmonic ponderomotive force produces electrons density oscillations that beat with the oscillatory velocity due to the laser to create a non-linear current, driving the third harmonic radiation. The magnetic field enhances the efficiency of the third-harmonic generation to a significant level.

## ACKNOWLEDGEMENT

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