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SPIN TUNE PARAMETRIC RESONANCE INVESTIGATION

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Abstract

The idea of resonant spin oscillation method was modernized and improved in Forschungszentrum Jülich in the proposed experiment at the COSY ring. The resonant method is based on spin tune parameterization using transverse RF magnetic or/and electric field. The spin orientation smearing due to the finite spin coherence time (SCT) plays a crucial role in the proposed experiment to search for the electric dipole moment. Our analysis is based on the T-BMT differential equations for spin together with shorten motion equations. Using well developed theory of Mathieu's differential equations we have got simplified analytic solution for prediction of spin behaviour. In this paper we have numerically evaluated all effects having fundamental contributions.

RESONANCE BUILD-UP OF EDM CONTRIBUTION

As we know the spin is a quantum value, but in the classical physics representation the "spin" means an expectation value of a quantum mechanical spin operator. Taking into account the Lorentz transformation of equation for spin with the z-direction of motion we have:

$$\frac{d\vec{S}}{dt} = \vec{d} \times \vec{E}^* + \vec{\mu} \times \vec{B}^*$$
$$E_x^* = \gamma\left(E_x + \beta \cdot B_y\right);\ E_y^* = \gamma\left(E_y - \beta \cdot B_x\right);\ E_z^* = E_z$$
$$B_x^* = \gamma\left(B_x - \beta \cdot E_y\right);\ B_y^* = \gamma\left(B_y + \beta \cdot E_x\right);\ B_z^* = B_z \qquad (1)$$

where $\vec{\mu}, \vec{d}$ are magnetic and electric dipole moments, β is the relative longitudinal velocity and $E_{x,y,z}, B_{x,y,z}$ are the electric and magnetic field components in the laboratory coordinates system.

Equations with RF Electrical Deflector

The possible method proving the existence of EDM may be the resonant build-up using the RF deflector with the horizontal electric field or considered later the RF solenoid with the vertical magnetic field [1,2]. The electric field in RF deflector can be submitted as function dependent on a time t and a curvilinear coordinate $s = c\beta \cdot t$:

$$E_x = E(t) \cdot E(s)$$
$$E(t) = E_{rf} \cos(2\pi f_{rf} \cdot t + \varphi) \qquad (2)$$
$$E(s) = \frac{l_{rf}}{L_{cir}}\left(1 + 2\sum_{n=1}^{\infty} \cdot \cos\left(2\pi\, n f_{rev} t\right)\right)$$

where l_{rf} is a deflector length, L_{cir} is orbit of length, f_{rf} is RF frequency of deflector. Taking into account $\nu_{rf} = f_{rf} / f_{rev}$, where f_{rev} is a revolution frequency, equations system (1) for all projection $S = \{S_x, S_y, S_z\}$ can be written as

$$\frac{dS_x}{d\tau} = -\left\{\nu_s + \alpha\cos\left[\left(\nu_{rf} \pm n\right)\tau + \psi\right]\right\}S_z$$
$$\frac{dS_y}{d\tau} = -\left\{-h\cos\left[\left(\nu_{rf} \pm n\right)\tau + \psi\right] + \nu_e\right\}S_z \,, \qquad (3)$$
$$\frac{dS_z}{d\tau} = \left\{-h\cos\left[\left(\nu_{rf} \pm n\right)\tau + \psi\right] + \nu_e\right\}S_y +$$
$$\left\{\nu_s + \alpha\cos\left[\left(\nu_{rf} \pm n\right)\tau + \psi\right]\right\}S_x$$

where $\nu_e = \frac{\eta}{2} \cdot \gamma \cdot \beta_z, h = \frac{\eta}{2} \cdot \gamma \cdot \frac{E_{rf} l_{rf}}{B_y L_{cir}}$,

$\alpha = \left(\frac{1}{\gamma^2 - 1} - G\right)\gamma\beta_z \cdot \frac{E_{rf} l_{rf}}{B_y L_{cir}}$ and η is dimensionless coefficient defined by the relation with $d = \eta e\hbar / 4mc$, ν_e is the spin tune of a particle moving in a magnetic field B_y due to the presumable existing electric dipole moment d. The phase ψ is very important for the longitudinal oscillations of an arbitrary particle, since it defines the revolution frequency dependence on the particle energy. At present ψ is taken to be a constant, but later the synchrotron oscillation will be taken into consideration.

Resonant Condition for RF Electrical Deflector

The equations system (3) does not have analytical solutions, therefore it will be solved by the numerical integration. Nevertheless in order to understand how to build-up a spin resonance we first solve (3) in approach of $S_{x,z} << 1$. Actually, this approach works well, since in future experiments the maximum growth of components $S_{x,z}$ is expected to be up to value of $10^{-3} \div 10^{-6}$.

First, we solve the system of equations under the assumption of the smallness of parameters $\alpha, h << 1$ and synchrotron oscillating absent:

$$\frac{d^2 S_z}{d\tau^2} + \left(\nu_s^2 + \nu_e^2\right)S_z - \qquad (4)$$
$$-\left[2\left(h\nu_e - \alpha\nu_s\right)\cos\Psi(\tau) - (h^2 + \alpha^2)\cos^2\Psi(\tau)\right] \cdot S_z -$$
$$-h\left(\nu_{rf} \pm n\right) \cdot \sin\Psi(\tau) \cdot S_y + \alpha\left(\nu_{rf} \pm n\right) \cdot \sin\Psi(\tau) \cdot S_x = 0$$

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The equation describes the oscillation of the longitudinal component S_z at the parameterized tune dependent on $h,\alpha,\nu_e,\nu_{rf},\nu_s,S_{x,z}$. Since the parameters h,α,ν_e are very small, we can transfer the terms responsible for parametric excitation to the right side of equation:

$$\frac{d^2 S_z}{d\tau^2}+\nu_s^2 S_z = f(\tau,S_{x,y,z})$$
$$f(\tau,S_{x,y,z}) = -\left[2\alpha\nu_s\cos\Psi(\tau)+\alpha^2\cos^2\Psi(\tau)\right]\cdot S_z + h(\nu_{rf}\pm n)\cdot\sin\Psi(\tau)\cdot S_y - \alpha(\nu_{rf}\pm n)\cdot\sin\Psi(\tau)\cdot S_x \quad (5)$$

To solve it we use the asymptotic methods in the theory of nonlinear oscillations of Bogoliubov and Mitropolsky [3], which works well for the case of $S_{z,x} << 1$ and $S_y \approx 1$. In first approach, the solution is sought in the form of:

$$S_z(\tau) = a(\tau)\cdot\cos\vartheta, \qquad \vartheta = \nu_s\tau+\theta$$
$$\frac{da(\tau)}{d\tau} = \frac{1}{2\pi\nu_s}\cdot \int_0^{2\pi}\left\{\begin{matrix}\left[2\alpha\nu_s\cos\Psi(\tau)+\alpha^2\cos^2\Psi(\tau)\right]\cdot S_z - \\ h(\nu_{rf}\pm n)\cdot\sin\Psi(\tau)\cdot S_y + \alpha(\nu_{rf}\pm n)\cdot\sin\Psi(\tau)\cdot S_x\end{matrix}\right\}\cdot \sin\vartheta\, d\tau \quad (6)$$

In case of resonance it one of the conditions is met

induced by S_z:

1. $\underset{\tau}{M}\left\{2\alpha\nu_s\cos\left[(\nu_{rf}\pm n)\tau+\psi\right]\cdot\sin\nu_s\tau\cdot\sin(\nu_s\tau+\theta)\right\}\neq 0$
2. $\underset{\tau}{M}\left\{\alpha^2\cos^2\left[(\nu_{rf}\pm n)\tau+\psi\right]\cdot\sin\nu_s\tau\cdot\sin(\nu_s\tau+\theta)\right\}\neq 0$

induced by S_y

3. $\underset{\tau}{M}\left\{h(\nu_{rf}\pm n)\cdot\sin\left[(\nu_{rf}\pm n)\tau+\psi\right]\cdot\sin(\nu_s\tau+\theta)\right\}\neq 0$

induced by S_x

4. $\underset{\tau}{M}\left\{\alpha(\nu_{rf}\pm n)\cdot\sin\left[(\nu_{rf}\pm n)\tau+\psi\right]\cdot\sin(\nu_s\tau+\theta)\right\}\neq 0$
5. $\underset{\tau}{M}\left\{\alpha(\nu_{rf}\pm n)\cdot\sin\left[(\nu_{rf}\pm n)\tau+\psi\right]\cdot\cos\nu_s\tau\cdot\sin(\nu_s\tau+\theta)\right\}\neq 0$ (7)

where $\underset{\tau}{M}$ is an operator of averaging over τ. Each of 1-5 is performed correspondently at:

1. $\nu_{rf}\pm n = 2\nu_s$; 2. $2\cdot(\nu_{rf}\pm n) = 2\nu_s$ 3. $\nu_{rf}\pm n = \nu_s$
4. $\nu_{rf}\pm n = \nu_s$ 5. $\nu_{rf}\pm n = 2\nu_s$ (8)

We see the resonance conditions are satisfied for two cases: $\nu_{rf}\pm n = 2\nu_s$ (in 1 and 5) or/and $\nu_{rf}\pm n = \nu_s$ (in 2-4). In the case $\nu_{rf}\pm n = 2\nu_s$ the longitudinal component S_z grows up proportionally $\frac{da(\tau)}{d\tau}\sim\alpha\nu_s$ and/or $\frac{da(\tau)}{d\tau}\sim\alpha\nu_{rf}$. At second condition $\nu_{rf}\pm n = \nu_s$ we should observe the growth $\frac{da(\tau)}{d\tau}\sim\alpha^2$, or/and $\frac{da(\tau)}{d\tau}\sim h(\nu_{rf}\pm n)$, or/and $\frac{da(\tau)}{d\tau}\sim\alpha(\nu_{rf}\pm n)$.

RESONANT BUILD-UP ANALYSIS

Now we analyse the influence of various factors entering into the general equations system of spin oscillation (3). In order to detect the EDM signal we should fulfil the conditions for the resonant build-up of S_z component growth.

Fundamental Features of Spin Oscillation Equation

From this point of view we have investigated numerically the parametric dependence of equation system (3) on $\varepsilon_y,\varepsilon_x,f_e,\delta$ in resonance and non-resonance $\nu_{RF} = \nu_s(1\pm\delta)$. In results we have got

versus $f_e = \nu_e/\nu_s$: the period of envelope decreases with increasing the factor f_e in different ways depending on $\varepsilon_y,\varepsilon_x$;

versus $\varepsilon_y = \alpha/\nu_s$ and $\varepsilon_x = -h/\nu_e$: the period of envelope is proportional to $1/|\varepsilon_y-\varepsilon_x|$ and $S_{z\max}\sim|\varepsilon_y-\varepsilon_x|$

versus $|\varepsilon_y-\varepsilon_x|$: due to symmetry of the equation system (3) relatively of X and Y planes at $\varepsilon_y=\varepsilon_x$ it is no resonant build-up growth of amplitude and let us call it a fundamental feature for later reference (*);

versus δ: the period of envelope is proportional to $T\sim T_{res}/(1+\delta\cdot T_{res})$ and $S_{z\max} \rightarrow S_{z\max}(t=0)$.

Difference between RFE and RFB Deflectors

A very important feature of symmetry of the equations system (3) gives a clear explanation of difference in the effects of RFE and RFB deflectors on the spin. In case of RFE deflector $\varepsilon_y-\varepsilon_x = \alpha/\nu_s + h/\nu_e$ and we can always create the resonant condition for the build-up of S_z amplitude, but for RFB deflector we can't create the resonant condition, since $\varepsilon_y-\varepsilon_x = \varepsilon_{RFB}-\varepsilon_{RFB}=0$. Thus, in case of both deflectors switched on the resonant growth completely coincides with case of RFE deflector alone.

Factor of Resonance Detuning

Figure 1 shows results of the numerical integration of equation system (4) for protons with the energy 100 MeV: the S_z component versus the turn number for the resonant

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$\nu_{rf} \pm n = \nu_s$ and the non-resonant $\nu_{rf} \pm n \neq \nu_s$ cases. In both cases, the character of dependence is the same. But in the resonant case, the final value of S_z leads to the maximum $S_z = 1$ during a very long time ~100 years, and in the off-resonance S_z is limited by a very small value reached much faster.

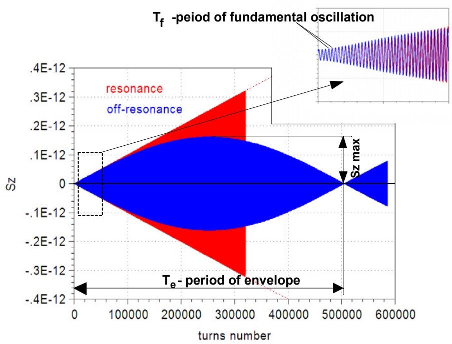

Figure 1: S_z vs turns number for the resonant (red) and non-resonant (green) cases

The figure 2 shows the $S_{z\,max}$ dependence on the resonance detuning $\delta = \nu_{RF}/\nu_s \pm 1$. You can see that at bigger RF detuning than $\delta = 10^{-11}$ the maximum possible registered signal $S_{z\,max}$ does not exceed 10^{-6}.

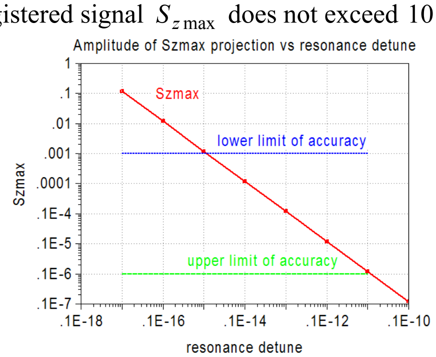

Figure 2: $S_{z\,max}$ vs the resonance detuning together with the measurement accuracy limit

Next, we tested the effect of frequency deviation of the spin precession from the RF tune $\nu_s = \nu_{RF}(1+\delta)$. Figure 3 shows for comparison the build-up process for two cases: fixed and random RF detuning. You can see that in the first case the process is periodic, and in the second case it increases and then remains almost constant.

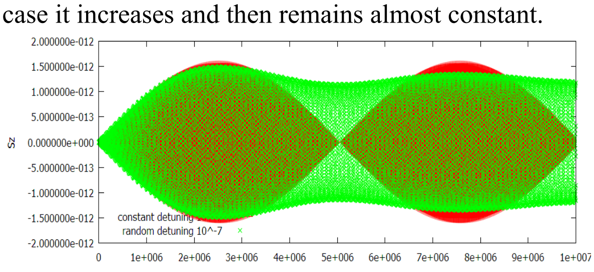

Figure 3: The S_z build-up process for two cases: fixed (red) and random (green) RF detuning

The same has been modeled for the case of random deviation of the spin tune, for instance due to random changes in the energy of a particle. It is appeared the build-up process for the random tune spin deviation coincides with the ideal case, when we have no spin tune spread and observe the resonant build-up with $\delta = 0$.

Dependence of Spin Build-up Process on Longitudinal Oscillation

To simplify the analysis of the longitudinal oscillation contribution in the build-up process with $\psi(\tau) = \Psi_m \sin(\nu_l \tau + \theta)$ we take $\alpha = 0$ and (5) is reduced into:

$$\frac{d^2 S_z}{d\tau^2} + \nu_s^2 S_z = h\nu_{RF} \cdot \sin[\nu_{RF} \cdot \tau + \Psi_m \sin(\nu_l \tau + \theta)], \quad (9)$$

or using the representation of trigonometric functions with the trigonometric argument in terms of the Bessel functions, we can write the right side (9):

$$\frac{d^2 S_z}{d\tau^2} + \nu_s^2 S_z =$$

$$h\nu_{RF} \begin{Bmatrix} J_0(\Psi_m) \cdot \sin\nu_{RF}\tau + J_1(\Psi_m)\cos[(\nu_{RF} + \nu_l)\tau + \theta] + \\ J_1(\Psi_m)\cos[(\nu_{RF} - \nu_l)\tau - \theta] \end{Bmatrix}$$

We see that in addition to the fundamental mode two neighbor modes appear at a distance of frequency $\nu_{RF} \pm \nu_l$.

CONCLUSION

We have shown that for the resonant build-up of transverse spin component we need as minimum an electrical RF deflector with extremely high frequency stability relative to the frequency of spin precession. Having studied the properties of the spin equations, we have shown that the vertical magnetic RF deflector alone is not able to build up the transverse component of the spin due to spin interaction with the magnetic field only. We have studied the fundamental difference between a random perturbation of the spin tune and a random perturbation of the RF tune in the deflector.

REFERENCES

[1] W. Morse, Resonance method with RF E-fields, Bad Honnef, Germany.

[2] A. Lehrach et al., Precursor experiments to search for permanent electric dipole moments of protons and deutrons at COSY, PSTP Proceedings, Dubna, 2011

[3] N. Bogolyubov, Yu. Mitropolskij, Asymptotic Methods in the Theory of Non-linear Oscillations, Hindustan Publishers, Delhi, 1961.