ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-THPRO060

BEAM-BEAM EFFECT ON THE BTF IN BUNCHED BEAMS

Paul Andreas Görgen*, Oliver Boine-Frankenheim, TEMF, TU-Darmstadt, Germany
Wolfram Fischer, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

We present studies on the transverse baseband Beam Transfer Functions (BTFs) in bunched beams at high energies. The goal of the work is to evaluate whether transverse BTFs can be used to diagnose the tune spread arising from transverse nonlinearities such as the beam-beam effect and space charge. We employ an analytic expression to the BTFs of beams under a transverse nonlinear lens arising from a bi-Gaussian charge distribution. We obtain agreement between a simulation model of an electron-lens like configuration and the analytic results. The tune spread for this scenario can be recovered by means of a fit against the analytic expectation. The results are compared with measurements where the beam-beam effect acts as a substitute for the electron lens. A similar behaviour of the BTF is observed. This allows the conclusion that the transverse BTF can be used to diagnose tune spread from an electron-lens. Finally we discuss the problems that arise when trying to recover the tune spread from BTFs of arbitrary non-Gaussian beams and in the presence of coherent beam-beam modes.

ANALYTIC EXPECTATIONS

In 2014, two electron lenses were put in operation at Brookhaven National Laboratorys Relativistic Heavy Ion Collider (RHIC) [1], with the goal of partly compensating the incoherent tune shift from the beam-beam effect in proton operation. We discuss measuring the incoherent transverse tune spread induced by an electron lens via transverse BTF.

The BTF is defined as the ratio $R(\Omega)$ of the amplitude of the beams linear response A_{resp} to an excitation at frequency Ω of amplitude A_{drive}.

$$R(\Omega) = \frac{A_{\text{resp}}}{A_{\text{drive}}} \tag{1}$$

Transverse BTFs can be measured in the horizontal and vertical direction. For coasting beams with a particle frequency distribution $\psi(\omega)$ that is independent of the particle amplitude in the direction of the BTF, the BTF can be calculated analytically by integration over the single particle response functions modeled as harmonic oscillators [2].

$$R(\Omega) \propto -i\pi\psi(\Omega) + \text{P.V.} \int_{-\infty}^{\infty} \frac{1}{\omega - \Omega}\psi(\omega)\mathrm{d}\omega \tag{2}$$

The imaginary part of R is proportional to ψ, allowing to read the tune distribution directly from the BTF.

However the situation for the BTF of a beam undergoing incoherent tune spread from a transverse source such as an electron lens is different: The particle betatron frequency is a function of the particle amplitude. The BTF can be calculated as [3]:

$$R_i(\Omega) = c \cdot \iint_0^{\infty} \frac{1}{\Omega - \omega_i\left(J_x, J_y\right)} \frac{J_i \mathrm{d}\psi}{\mathrm{d}J_i} \mathrm{d}J_x \mathrm{d}J_y \tag{3}$$

Wherein J_x, J_y are the transverse action angle variables, ψ the distribution function in action angle variables, c a constant absorbing any constant factors and $\omega_i(J_x, J_y)$ a particles mean betatron frequency as a function of its action angle amplitude. i is either x or y, the direction of the BTF measurement.

For Eq. 3 we can not give a general like Eq. 2. If the shape of the nonlinearity giving rise to the tune spread and the transverse beam distribution are known, the amplitude of the tune spread (e.g. ξ_{bb}, the peak tune shift from the beam-beam interaction) can be recovered by fitting the BTF against the analytic equation for the BTF with the amplitude of the nonlinearity as a fit parameter.

In case the ω_i and ψ are not known we can still identify a property in Eq. 3 that allows to recover the tune spread (the width interval of particle betatron frequencies): In the calculation of the BTF, the imaginary part arises from the pole in the integration when the denominator in the fraction becomes zero. Whenever there are no J_x, J_y such that $\Omega - \omega_i(J_x, J_y) = 0$, the integral has a real solution and $\text{Im}(R_i)(\Omega) = 0$. This lets us conclude that wherever $\text{Im}(R_i)(\Omega) \neq 0$, there are particles with $\omega_i = \Omega$ and therefore the tune density is nonzero. The observation that $\text{Im}(R_i(\Omega)) = 0$ does not imply that no particles oscillate at Ω: It can be that the contributions of different J_x, J_y cancel or $\mathrm{d}\psi/\mathrm{d}J_i$ is zero.

Application to an Electron Lens

In the case of an electron lens ω_i is defined by the electron gun system. For the electron lens in operation at RHIC since 2014 [1], ω_i will be approximately due to a Gaussian charge distribution to match the ion beam shape. For this case we can calculate the BTF by solving the integral numerically. An equation for tune shift caused by a circular Gaussian charge distribution for space charge in these coordinates was given by Burov and Lebedev [4]. We use it as follows:

$$\omega_x(J_x, J_y) = \omega_{0,x} + \xi_{\text{bb}} \int_0^1 \frac{\left(I_0(\frac{J_x z}{2}) - I_1(\frac{J_x z}{2})\right) I_0(\frac{J_y z}{2})}{\exp\left(z(J_x + J_y)/2\right)} \mathrm{d}z \tag{4}$$

The BTF with $\omega_i = \omega_x$ for a matched electron beam in Eq. 3 is shown in Fig. 1. For this transverse nonlinearity the tune density $\psi(\omega)$ is not proportional to $\text{Im}(R)(\omega)$ anymore. Additionally in contrast to the strong requirement $\text{Im}(R) \neq 0$ we chose the weaker requirement $|\text{Im}(R)| < t \cdot |R|$. We do

* goergen@temf.tu-darmstadt.de

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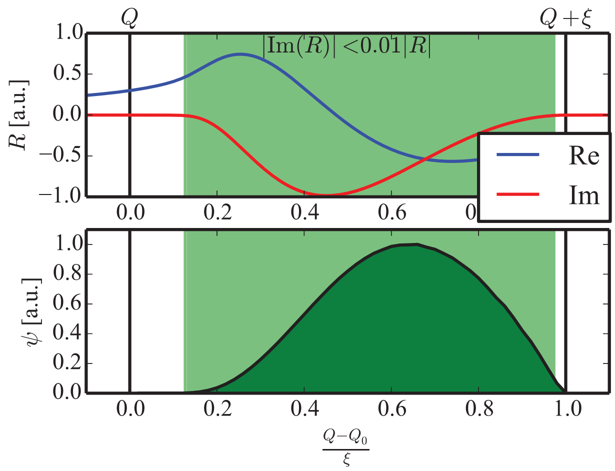

Figure 1: Analytic BTF R and on-axis tune density ψ for incoherent tune shift due to a Gaussian matched electron lens acting on a proton beam. The area shaded in green is where the condition in the plot is fulfilled. At maximum tune shift $(Q - Q_0)/\xi = 1$ particles have nearly zero amplitude leading to a negligible $\mathrm{Im}(R)$ in Eq. 3

so because our numeric solution of Eq. 3 was calculated with an accuracy of 1% which is why we chose a threshold $t = 0.01$. A similar condition has to be chosen in measured or simulated BTF where noise leads to a non-zero $\mathrm{Im}(R)$.

SIMULATION

We used simulations to assess how applicable the condition $|\mathrm{Im}(R)| < t \cdot |R|$ is in measurements for detection of tune spread. The objectives of the simulations are to adress two questions: How does the BTF change compared to the analytic coasting beam picture without coherent modes when there is slow synchrotron motion and/or coherent modes from beam-beam interactions? Furthermore how can the beam-beam effect be used as a stand-in for the electron lens while they are not yet available? The simulation is implemented as a particle-in-cell (PIC) code based on Patric [5]. The simulation uses a two-dimensional Poisson solver for the beam-beam interaction and analytic equations for the fields of the round Gaussian matched electron lens. The machine is approximated by a one-turn map.

Electron Lens

A first validation of simulation against analytic results was carried out using a Gaussian beam interacting with a matched electron lens and comparison with analytic results. A sample is shown in Fig. 2. For RHIC-typical high-energy proton operation conditions the introduction of synchrotron motion did not significantly change the BTF compared to the analytic result for coasting beams.

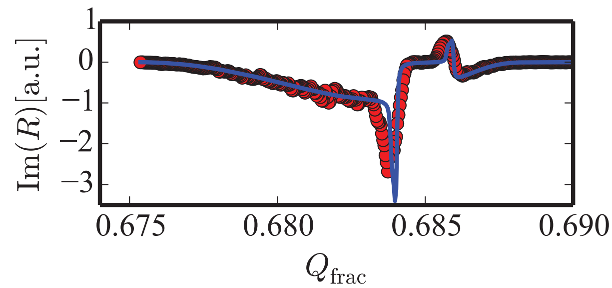

Figure 2: The BTF from PIC simulation (dots) agrees with the analytically calculated one (solid line) for an ion beam with a hole in action-angle phase space interacting with an electron lens. To make the effect of the derivative in Eq. 3 more apparent we punched a hole into the Gaussian beam by setting the particle density to zero for $3 < J_x + 0.6 \cdot J_y < 5$ for illustration. The peaks in the BTF correspond to the high derivative of ψ at the edges of the hole.

Beam-beam

We investigated the applicability of a beam-beam interaction as a stand-in for an electron lens. This was necessary in order to produce electron-lens-like BTFs to test the theory before the commissioning of the RHIC electron lens in 2014. Beam-beam differs from the electron lens in that it produces the well-known coherent π and σ modes with two beams nearly on the same working point [6] .

Beam-beam without coherent modes To find conditions where coherent modes are absent for testing our analytic theory on measurement data we followed Alexahin [7]. He mentions two methods of moving coherent modes into the incoherent spectrum and subsequently Landau-damping them: Firstly intensity splitting, having beams of an intensity difference of a factor of at least about 2 will move the π mode into the incoherent spectrum. Secondly tune splitting: Introducing a sufficiently large difference in tune between the two beams will move both modes into the incoherent spectrum. Simulation showed that for an intensity ratio of 1:10 and tune splitting the simulated BTF with beam-beam were in agreement with those of an electron lens. Furthermore introduction of synchrotron motion at RHIC-typical frequencies for high-energy proton operation (where the electron lens is applied) did not significantly change the BTF in comparison to the coasting-beam theory.

Threshold method on beam-beam with coherent modes One can argue that even in the presence of coherent modes, wherever the beam provides Landau damping the imaginary part of the BTF should be non-zero. So apart from the coherent modes at which we can expect a non-zero imaginary part of the BTF, our threshold method should still show areas where tune spread provides Landau damping. Simulated BTFs with two beams of same intensity and tune show that this assumption holds true. The picture can become more complicated when Landau damping from the

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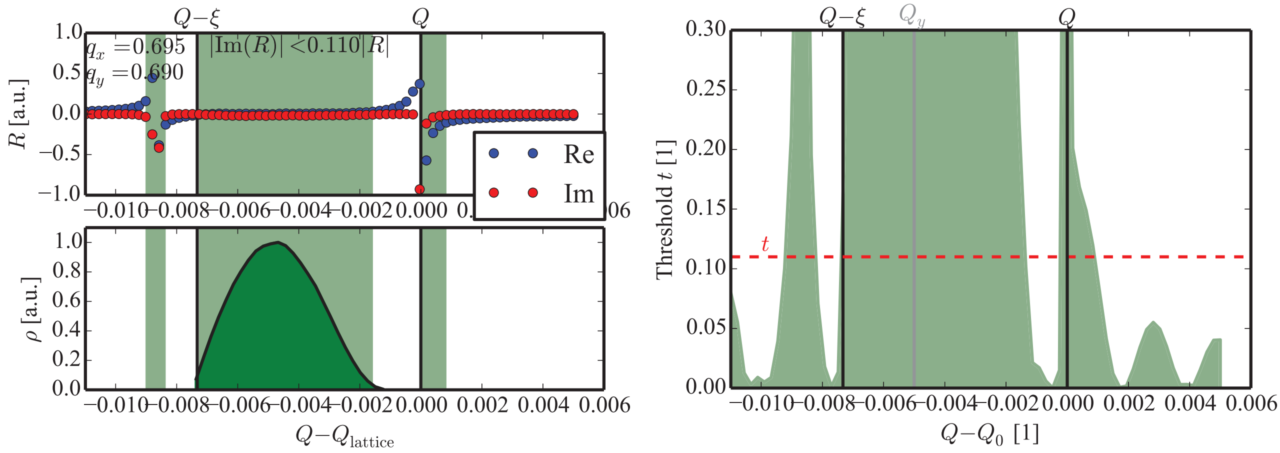

(a) BTF and tune density **(b)** Threshold dependency of the method.

Figure 3: Example of the threshold method used on BTF of beams with identical tunes and intensities. Part (a) shows the BTF R and tune density together with the area where $|\mathrm{Im}(R)| < t \cdot |R|$ with $t = 0.11$. part (b) shows how the recovered area changes as a function of t. We see that the tune spread can be recovered from the threshold method.

other plane gets coupled into the plane of the BTF. A BTF with coherent beam-beam modes and both beams on identical tunes is shown in Fig. 3. After neglecting the coherent π and σ modes of the beam-beam interaction one is left with the tune spread from the beam-beam interaction, which gives an indication that the threshold method is still applicable in the presence of coherent beam-beam when taking into account the positions of the coherent modes.

MEASUREMENT

In measurements in 2012 and 2013 we used proton beams with split tunes and split intensities to have BTF like those of an electron lens as discussed above. We recovered the tune spread and the beam-beam parameter using the threshold method described above and fitting against the analytic BTF calculated for a matched Gaussian lens. The results of the

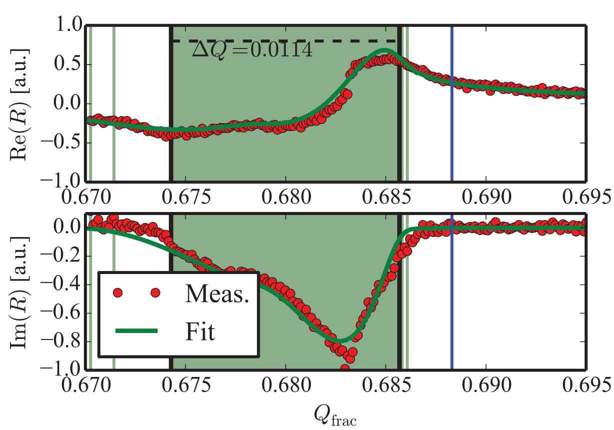

Figure 4: Measured BTF in the horizontal plane for a weak beam in a weak-strong beam-beam interaction. Fit against Eq. 3 is shown, the shaded area is where $|\mathrm{Im}(R)| < 0.3 \cdot |R|$.

fit and threshold method were in agreement with each other and with the expectation from beam profile and current measurements in the horizontal plane, see Fig. 4. In the other plane bumps of unknown origin were observed in the BTF indicating that other damping processes might be at work. Still the fit and the threshold method were in agreement and indicated a tune spread about twice as high as expected from beam-beam alone.

We conclude that in absence of coherent modes our method allows recovery of tune spread from BTF for coasting beams and bunched beams of low synchrotron frequency. In favourable conditions, recovery may still be possible in presence of coherent modes but needs further investigation. The threshold method will detect any sources of damping so tune spread recovery requires Landau damping to be dominant.

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