ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-THPRO058

STUDY OF THE “PARTICLE IN CELL” INDUCED NOISE ON HIGH INTENSITY BEAMS

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Abstract

Numerical noise in PIC codes produces artifacts, which affects long term beam simulations needed for future heavy-ion synchrotrons as the SIS100. We present here a detailed study on the effect of numerical noise occurring in multi-particle tracking codes using a PIC scheme. The influence of the granularity of particle distributions and the fineness of the meshes of Poisson solvers on the particle dynamics is studied. These results are used to discuss the effect of the numerical noise in long term space charge benchmarking studies.

PARTICLE TRACKING WITH SPACE CHARGE

Multi-particle tracking codes for circular machines make use of so-called transfer maps $\mathcal{M}$ to track a particle from its initial 4-dimensional phase-space position $\vec{x}_i(s_0)$ to a final position $\vec{x}_f(s)$, i.e.

$$\vec{x}_f = \begin{pmatrix} x_f \\ x'_f \\ y_f \\ y'_f \end{pmatrix} = \mathcal{M} \begin{pmatrix} x_i \\ x'_i \\ y_i \\ y'_i \end{pmatrix} = \mathcal{M}\vec{x}_i, \tag{1}$$

where s (or resp. s_0) is the (initial) longitudinal position of the particle. Space charge forces can be implemented in tracking codes by using non-linear kicks

$$\begin{pmatrix} x_f \\ x'_f \\ y_f \\ y'_f \end{pmatrix} = \begin{pmatrix} x_i \\ x'_i + \Delta s F_{SC,x}(x_i, y_i, s_0) \\ y_i \\ y'_i + \Delta s F_{SC,y}(x_i, y_i, s_0) \end{pmatrix},$$

where $(F_{SC,x}, F_{SC,y})^T$ is the transverse space charge force and Δs is the integration length. The transverse self-field of the beam $(E_{SC,x}, E_{SC,y})^T(x, y, z)$, that determines the self-consistent space charge forces, is computed for long-term simulation most effectively by using a 2.5-dimensional PIC scheme. This refers to a longitudinal slicing of the beam. In each slice the particles are projected onto a transverse plane, where the self-field is calculated by a 2D Poisson solver. Then, the fields in between planes are calculated by a linear interpolation. An approximation of the 3-dimensional field is thus retrieved by a set of Poisson equations in the transverse plane.

To achieve a higher computational efficiency, the Poisson equation is only solved on a finite set of N transverse grid points and the particle distribution is approximated by M macroparticles. Since the number of grid points N is finite and the number of macro-particles M is much smaller than the number of physical particles, numerical noise is induced. A theory to understand the effects of numerical noise was first proposed in 2000 by J. Struckmeier [1], and extended recently by I. Hofmann and O.Boine-Frankenheim ([2] and this proceedings). Their approach is to use a Fokker-Planck description to predict the evolution of the particle distribution in the presence of numerical noise. A complementary approach is presented in this proceeding, in which instead the effect of numerical noise on the single particle dynamics is studied to estimate emittance growth rates.

FLUCTUATIONS IN FIELD CALCULATIONS

We consider as a transverse particle distribution $f(x, y)$ the Gaussian distribution, i.e.

$$f(x, y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}, \tag{2}$$

where σ gives the standard deviation in x and y for circular beams. The associated space charge field is given by

$$E_{SC,r}(x, y) = \frac{E_0}{\sqrt{x^2 + y^2}} \left(1 - e^{-\frac{x^2+y^2}{2\sigma^2}}\right), \tag{3}$$

where E_0 is proportional to the field gradient in the vicinity of $x = y = 0$. To proceed in the study, we adopt the random start technique (opposite to the quiet start). For the random start, the positions of M macro-particles are randomly initialized according to $f(x, y)$. The resulting self-field is not only determined by $f(x, y)$, but varies according to the random initialization of macro-particles.

A first study is done to estimate the effect of a granular approximation of the particle distribution and the fineness of the grid. To this purpose, a random start initialization and a calculation of the resulting electric field is repeated several times to study possible fluctuation for different setups of N and M at different spacial positions (x, y). With this technique, the standard deviation for the random start initialization for the electric field is found to be

$$\delta E_{SC}(x, y) = \delta_0\, \xi(x, y) \sqrt{\frac{\sqrt{N}}{M}}\, e^{-\frac{x^2+y^2}{4\sigma}}, \tag{4}$$

where the constant δ_0 can be retrieved numerically and depends on the perveance of the beam. The factor $\xi(x, y)$ describes the effect due to the bilinear interpolation, that is

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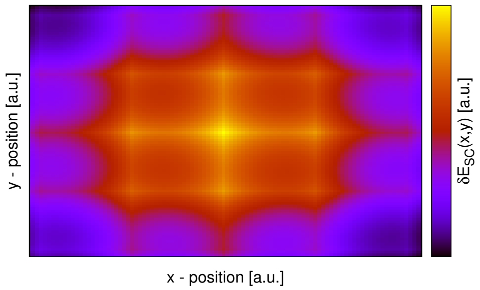

Figure 1: Standard deviation $\delta E_{SC}(x,y)$ of the transverse electric field in arbitrary units. An exponential dependence of the standard deviation on the distance to the center is found. It is superimposed by the PIC grid texture (due to the bilinear interpolation in between grid points).

used to compute the electric field in between grid points. For illustration, a sample snapshot is given in Fig. 1.

The scaling law Eq. (4) is derived for circular beams for random start initializations. It is valid, as long as the beam shape is reasonably resolved, which is the case for $N \geq 16$ grid points within $[-2\sigma, 2\sigma]$ of the particle distribution $f(x,y)$.

The field fluctuations in the longitudinal plane are derived for arbitrary transverse fluctuations. Assume, we know the transverse fluctuations of two slices situated at $s = s_i$ and $s = s_{i+1}$, given by $\delta E_{SC}(x,y,s_i)$ and $\delta E_{SC}(x,y,s_{i+1})$. Then, due to the *Gaussian law of error propagation*, the field fluctuation at the positon s is given by

$$\delta E_{SC}(x,y,s) = \sqrt{d_s^2 \delta E_{SC}(x,y,s_i) + d_{s+1}^2 \delta E_{SC}(x,y,s_{i+1})} \quad (5)$$

where $d_s = (s_{i+1} - s)/(s_{i+1} - s_i)$ and $d_{s+1} = (s - s_i)/(s_{i+1} - s_i)$ for $s_i < s < s_{i+1}$. This scaling is confirmed by sytematic simulation studies, carried out in the same manner as for the transverse plane. The results for a sample bunched beam are presented for $x = y = 0$ in Fig. 2. As expected, in between two transverse planes the standard deviation declines due to the linear interpolation.

TRACKING OF BEAMS AFFECTED BY NOISE

For the study of the effect of numerical noise on the single particle beam dynamics, it is most important to study the correlations of fluctuations of the electric field while tracking. Therefore a spectral analysis of the electric field for a choosen setup (beam and machine parameters) has to be done. A flat spectrum refers to random (white) numerical noise, while peaks in the spectrum (f.e. due to resonances) refer to directed (coloured) noise. If the fluctuations are solely random, the effect of noise can be understood in the context of a random walk model: At every position where a space charge kick is applied on a single particle, the noise randomly perturbs the particles position in phase space, i.e.

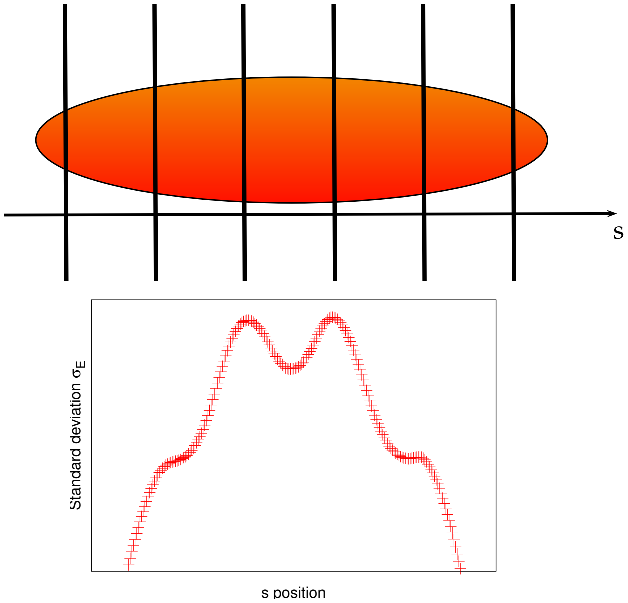

Figure 2: Standard deviation $\delta E_{SC}(x,y,s)$ in the longitudinal plane (bottom) for $x = y = 0$ for a sample bunched beam. At the position of a each transverse plane (top), a local maximum is reached, that is given by the 2-dimensional problem, as stated in Eq (4). In between two planes the standard deviation declines.

$$x \rightarrow x \pm \Delta x \frac{\beta x}{\sqrt{x^2 + (\beta x')^2}} \quad (6)$$

$$x' \rightarrow x' \pm \Delta x \frac{\beta x'}{\sqrt{x^2 + (\beta x')^2}} \quad (7)$$

and equivalently for y, y'. The multiplicative factors proportional to Δx in Eq. (6) and Eq. (7) ensure that the particles normalized phase space positions stay always on a circle and are thus dependent on the β- function of the machine. The shift Δx is given by

$$\Delta x = \frac{q \delta E_{SC}(x,y)}{pv} \Delta s, \quad (8)$$

for a macro-particle of charge q, velocity v and momentum p. The fluctuations of the electric field $\delta E_{SC}(x,y)$ are given by Eq. (4) and depend on the β-function of the machine. Thus they have to be derived for any longitudinal positions, where a kick is applied.

In terms of single particle emittances ε_x, ε_y for $\Delta x, \Delta y << 1$ it becomes

$$\varepsilon_x \rightarrow (1 \pm \Delta)\, \varepsilon_x, \quad (9)$$

$$\varepsilon_y \rightarrow (1 \pm \Delta)\, \varepsilon_y, \quad (10)$$

with a shift parameter $\Delta = 2\beta \Delta x / \sqrt{x^2 + (\beta x')^2}$.

A single particle is thus subjected to position dependent fluctuations of the electric field (given by Eq. (4) or (5)) and to the main space charge field given by Eq. (3). From the description by the random walk model, it can be seen, that the amplitude of a particle in phase space has thus a higher probability to rise than to shrink. Therefore the rms emittances $\varepsilon_{x,rms}$, $\varepsilon_{y,rms}$ grow constantly in the presence of numerical noise.

The random walk model is used to estimate the emittance growth of a coasting beam in a FODO lattice, see Fig. 3 and 4. A transverse grid of $N = 128$ and $N = 64$ grid points is used and 45 space charge kicks per betatronic wavelength are applied. The setup is choosen such that the numerical noise is artificially enhanced in order to enable us to see stronger effects. Thus the number of macro-particles is varied in the range of $M = 1000$ and $M = 5000$ while the rms emittance is set to $\varepsilon_{x,rms} = \varepsilon_{y,rms} = 75$ mm mrad and the tune shift is choosen to reach $\Delta Q_x/Q_x \approx \Delta Q_y/Q_y \approx 0.32$. The integration length Δs and the longitudinal positions of space charge kicks are kept the same during all simulations. The tracking is done using the MICROMAP library [3], that makes use of a Poisson solver described in [4], [5], [6].

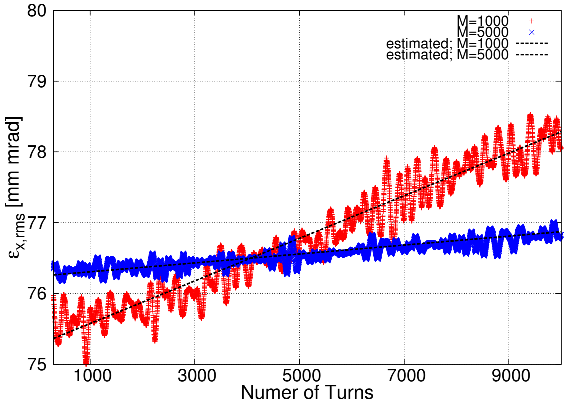

Figure 3: Artificial emittance growth in a PIC tracking code simulations and in theory for 10000 turns with different numbers of macro-particles M and fixed $N = 128$. The dashed lines represent the expected emittance growth, given by the random walk model introduced in this chapter. The artificial emittance growth in a computer simulation is represented by red ($M = 1000$) and blue ($M = 5000$) points.

As it can be seen in Fig. 3 and 4, the emittance growth rate is well recovered by our model. The slope of the emittance growth rate $d\varepsilon/ds$ changes according to the step length Δx in the random walk model, that depends on the number of macroparticles M and grid points N . By varying the step length Δx, see Eq. (4) and Eq. (6-8), the dependence of N and M on the emittance growth rate, is studied. The dependence is given by

$$\frac{d\varepsilon}{ds} \propto (\Delta x)^2 \propto \frac{\sqrt{N}}{M} \tag{11}$$

as expected for a quantity that is quadratic in x, x' as the emittance. The scaling law Eq. (11) has to be understood as an estimate on the emittance growth rate in the case of non-correlated electric field fluctuations. For correlated electric fields, the random walk modeling is not valid, but can be used as an approximation in case of weakly correlated noise.

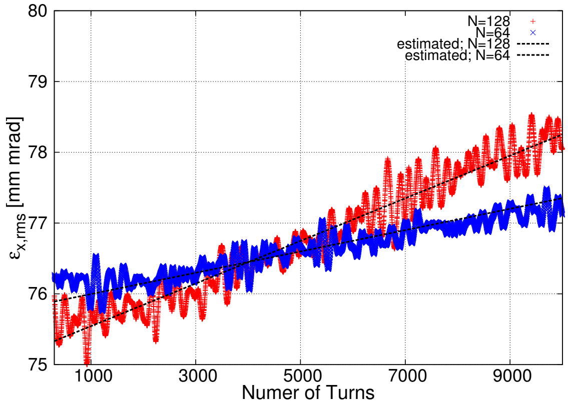

Figure 4: Artificial emittance growth in PIC tracking code simulations and in theory for 10000 turns with different numbers of grid points N and fixed $M = 1000$. As in Fig. 3, the dashed lines represent the expected emittance growth, given by the random walk model introduced in this chapter. The artificial emittance growth in a computer simulation is represented by red ($N = 128$) and blue ($N = 64$) points.

SUMMARY

A full understanding of the electric field fluctuation for random starts in the 2.5-dimensional approach is obtained, and a scaling on the number of macro-particles M and grid points N is given. This scaling corresponds to uncorrelated self-field fluctuations of tracked beams . Therefore it can be applied to predict the effect of space charge fields affected by numerical noise on single particle dynamics. Finally, with this approach, estimates on emittance growth rates are achieved.

The modeling of the single particle dynamics with a random walk model gives thus an understanding of the effect of numerical noise on the single particle dynamics and provides a tool to estimate unwanted numerical artifacts.

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