ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-THPRO041

POSITION OF MAXIMUM IN QUANTUM SPECTRUM OF SYNCHROTRON RADIATION*

A. Burimova†, IF USP, São Paulo, Brazil
V. Bagrov, Institute of High Current Electronics and Tomsk State University, Tomsk, Russia
D. Gitman, IF USP, São Paulo, Brazil

Abstract

In the framework of quantum theory we consider the condition for radiation maximum shift between harmonics of SR spectrum for scalar and spinor particles. Since quantum spectrum is discrete and finite, one can find values of radiation parameters such that the maximum in radiation spectrum stays at highest harmonic. It turns out that there exists a "quantization" of magnetic field associated with shift of maximum from one harmonic to another.

INTRODUCTION

One of the earliest attempts to theoretically describe synchrotron radiation (SR) indicated the fact that its maximum can lie within any part of spectrum [1, 2]. More precisely, according to classical theory the position of maximum changes with energy, and in ultrarelativistic limit the number of corresponding harmonic is proportional to the cube of relativistic factor: $\nu_{max} \sim \gamma^3$. Schott formula [3] describes discrete infinite spectrum and thus classical ν_{max} doesn't meet any restrictions.

In quantum approach the number of radiated harmonic is limited by the number of initial energy level, $\nu_{max} \leq n$. Indeed, denoting initial energy of radiating particle with n and setting s to describe the energy level of final state, the number of emitted harmonic is $\nu = n - s$. Therefore, quantum transitions to the ground state ($s = 0$) are accompanied by the emission of harmonic $\nu = n$ and, clearly, quantum theory doesn't allow an infinite increase of ν_{max}. Still, one can find the conditions for $\nu_{max} = n$ and for spinless particle (boson) this analysis was performed in [4]. Our previous research conducted for bosons can be very helpful in distinguishing the influence of spin effects. In what follows we intend to consider the shift of radiation maximum from $\nu = n - 1$ to $\nu = n$ for spin one-half particle (electron) and establish the difference between results obtained for boson and electron. Spinor particle is supposed to be transversally polarized (solutions of Dirac equation are eigenfunctions of tensor-operator of transversal polarization), spin direction given by parameter $\zeta = \pm 1$. $\zeta = 1$ corresponds with spin parallel to external magnetic field and $\zeta = -1$ for antiparallel spin. Usually we use ζ to denote initial spin and ζ' for the direction of spin in final state.

In order to omit cumbersome formulae, we do not give exact expressions for radiation characteristics and restrict ourselves to specifying a number of short notations. Namely, we work in terms of radiation power or, roughly speaking, probability of quantum transitions. The power of radiation emitted by electron at transition from initial state n to final state s is expressed by $W_i^e(\zeta, \zeta', n, n-s, \beta)$, where $\beta = |\vec{\beta}|$ is the normalized velocity of emitting particle, i is standard polarization labeling index; $i = 2$ for σ-component of linear polarization, $i = 3$ for π-component and $i = 0$ for total (summed over polarization components) radiation. We start our analysis with three-level system ($n = 2$) since it is the simplest case where spectral properties show up.

* Work supported by FAPESP Foundation (SP, Brazil)
† llefrith@yandex.ru

QUANTUM TWO-HARMONIC SPECTRUM

Since we intend to track the position of radiation maximum in quantum spectrum of electron, it is convenient to study quantum transitions from second excited state. Starting with $n = 2$, as a result of emission the particle can jump either to $s = 1$ or to $s = 0$. Both types of transitions can occur with or without spin flip. Ground state allows $\zeta = -1$ only (see e.g. [5] or any other classical textbook on Quantum Mechanics), however, at $s > 0$ spin can be parallel or antiparallel to external field. Assuming initial spin direction to be fixed we can sum over possible directions of final spin ζ' and define the average power of radiation at $\nu = n - 1$

$$\overline{W}_i^e(\zeta, n, n-1, \beta) = \frac{1}{2} \sum_{\zeta'=\pm 1} W_i^e(\zeta, \zeta', n, n-1, \beta). \quad (1)$$

Now, let us turn our attention to the concept of partial contributions. For $n = 2$ we introduce the functions

$$q_i^e(\zeta, 2, 1, \beta) = \frac{\overline{W}_i^e(\zeta, 2, 1, \beta)}{W_0^e(\zeta, 2, 2, \beta) + \overline{W}_0^e(\zeta, 2, 1, \beta)}, \quad (2)$$

$$q_i^e(\zeta, 2, 2, \beta) = \frac{W_i^e(\zeta, 2, 2, \beta)}{W_0^e(\zeta, 2, 2, \beta) + \overline{W}_0^e(\zeta, 2, 1, \beta)}, \quad (3)$$

which define the relation between the power of radiation of polarization type i associated with quantum harmonic ν ($\nu = 1$ or 2) and total (summed over spectrum and polarization components) radiation. The plots of these functions for σ-component and total radiation are shown in figures 1 and 2 where black curves correspond with $\zeta = 1$ and solid blue - with $\zeta = -1$. Dashed curves represent classical theory for reduced two-harmonic spectrum.

It is clear from figures 1 - 2 that, in accordance with the predictions of classical theory and quantum theory of radiation from bosons [4], in non-relativistic case primary harmonic $\nu = 1$ is dominating. Both $q_0^e(1, 2, 1, \beta)$ and $q_0^e(-1, 2, 1, \beta)$ are decreasing with β which means that

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-THPRO041

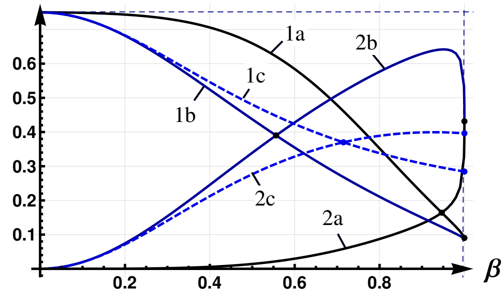

Figure 1: $q_2^e(1,2,1,\beta)$ (1a), $q_2^e(-1,2,1,\beta)$ (1b), $q_2^e(1,2,2,\beta)$ (2a), $q_2^e(-1,2,2,\beta)$ (2b).

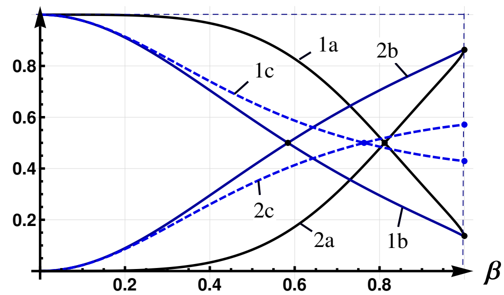

Figure 2: $q_0^e(1,2,1,\beta)$ (1a), $q_0^e(-1,2,1,\beta)$ (1b), $q_0^e(1,2,2,\beta)$ (2a), $q_0^e(-1,2,2,\beta)$ (2b).

the dominance of first harmonic vanishes at some point. Namely, at $\beta = 0.81203$ ($\gamma^2 = 2.93594$) for $\zeta = 1$ and at $\beta = 0.58403$ ($\gamma^2 = 1.51767$) for $\zeta = -1$, which are the roots of $q_0^e(\zeta,2,1,\beta) = q_0^e(\zeta,2,2,\beta)$, maximum of radiation shifts to second (last, $\nu = n = 2$) harmonic of quantum spectrum. Though it is not noticeable but at low levels, we find important to emphasize that $\beta(\zeta = -1) < \beta(\zeta = 1)$. Physically it means that electrons with initial spin antiparallel to external field switch their preferences to complete 'outpour' at lower energy. Maximum in short spectrum of scalar particle shifts to $\nu = 2$ at $\beta = 0.84618$, i.e. later than both electrons of $\zeta = \pm 1$, and classical $\beta = 0.76354$ can be called representative with respect to electron of $\zeta = 1$.

Generally, one can observe qualitative agreement between the behavior of partial contributions related to boson and electron. It allows us to assume for both $\zeta = 1$ and $\zeta = -1$ that at $n > 2$ maximum of radiation sequently shifts to higher harmonics with energy, starting with primary harmonic at $\beta \approx 0$.

RADIATION SPECTRUM OF BOSON

We start the analysis by briefly revising the character of evolution of radiation maximum in boson spectrum. Better description can be found in [4]. The model of scalar particle might not appear useful from practical point of view (though in some particular practical cases spin properties can be neglected), but theoretically it is an excellent tool to either simplify the study of SR characteristics or to distinguish the perturbation caused by spin properties when comparing to characteristics of spinor particle. Quantum motion of scalar particle obeys Klein-Gordon equation from which follows the relation $\gamma^2 = 1 + (2n+1)b$, where b is the value of external field in the units of Schwinger field. Obviously, for large n we have $\gamma^2 = 2nb$. Introducing the functions

$$K^b(n,\beta) = \frac{W_0^b(n,n,\beta)}{W_0^b(n,n-1,\beta)}, \quad (4)$$

for each fixed n one can find such β at which the maximum of radiation shifts from harmonic $\nu = n-1$ to $\nu = n$. The roots of equation $K^b(n,\beta) = 1$ (n,β_n) can be given in the form (n,γ_n^2), $\gamma_n^2 = (1-\beta_n^2)^{-1}$, and for boson they fit the line $\gamma^2 = 2b_0 n$ with $b_0 = 1.146129$, see figure 3.

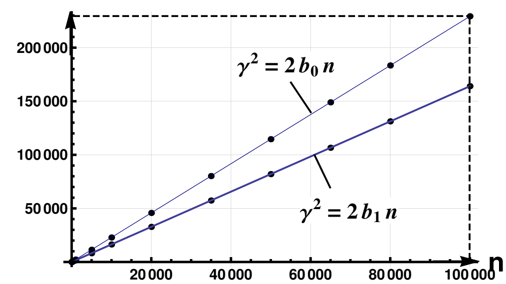

Figure 3: The roots of $K^b(n,\beta) = 1$ and $K_1^b(n,\beta) \equiv W_0^b(n,n-1,\beta) \cdot [W_0^b(n,n-2,\beta)]^{-1} = 1$.

This fact leads us to nontrivial conclusion. The maximum in radiation spectrum of boson can shift to highest harmonic only when external magnetic field is higher than critical value b_0. Otherwise maximum remains lower that $\nu = n$. Critical value exists for transition between $\nu = n-2$ and $\nu = n-1$ as well and we have found it numerically, $b_1 = 0.82009$.

A better explanation of what happens is provided by figure 4.

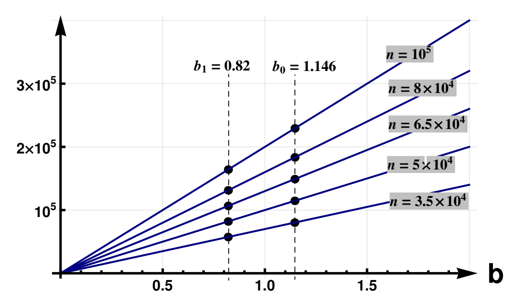

Figure 4: Lines formed by the roots $(n_a, \gamma_{n_a}^2)$.

Initially (from Klein-Gordon equation) we have $\gamma^2 = 2nb$. So, for fixed $n = n_a$ $\gamma^2(b)$ is a straight line. Now, if for n_a we find β_{n_a}, and, therefore, γ_{n_a}, satisfying $K^b(n_a,\beta_{n_a}) = 1$, we immediately see that points (n_1,γ_{n_1}), (n_2,γ_{n_2}), form a vertical line that corresponds with b_0. Same effect can be observed for the roots of $K_1^b(n,\beta) = 1$ (dashed line with b_1 mark) and so on, for the shift between each pair of neighboring harmonics.

This information provides a possibility to make a statement about the "quantization" of external field in the fol-

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-THPRO041

lowing sense: for a spinless particle the discrete sequence of critical values of field intensity controls the position of radiation maximum. Only when external field exceeds a fixed critical value, maximum shifts to corresponding harmonic.

INFLUENCE OF SPIN

Scalar equation of motion can't serve well for an electron, so we turn our attention to Dirac equation in this case. Dirac equation gives $\gamma^2 = 1 + 2nb$, or for large n, $\gamma^2 = 2nb$.

Similarly to boson, we introduce the functions

$$K^e(\zeta, n, \beta) = \frac{W_0^e(\zeta, n, n, \beta)}{\overline{W}_0^e(\zeta, n, n-1, \beta)} . \qquad (5)$$

Their behavior is very much alike the behavior of $K^b(n, \beta)$ both for $\zeta = 1$ and $\zeta = -1$, see figure 5 and [4].

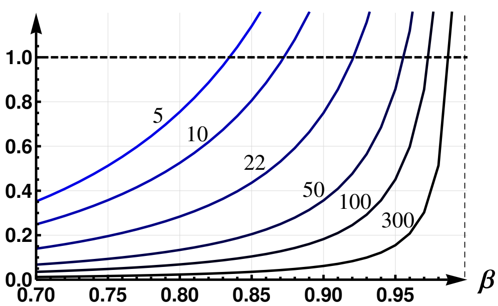

Figure 5: Plots of functions $K^e(1, n, \beta)$ at $n = 5, 10, 22, 50, 100, 300$.

However, the roots of $K^e(\zeta, n, \beta) = 1$ (n, γ_n^2) do not form a line, and this is exactly when the properties of spin enter the picture. Figure 6 shows that the supposed straight line is perturbed significantly. Though we only present the data related to $\zeta = 1$, the discrepancy between boson and electron is the same for $\zeta = -1$. Obviously, one can find a slight difference between $K^e(1, n, \beta)$ and $K^e(-1, n, \beta)$ for low n, and increasing the number of initial energy level this difference becomes negligible.

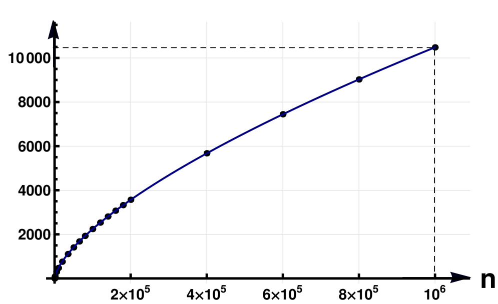

Figure 6: Roots of equation $K^e(1, n, \beta) = 1$, $(\gamma = (1 - \beta^2)^{-1})$, and numerical approximation.

Using numerical methods we have found an approximation for the points (n, γ_n^2) of the form

$$\gamma_n^2(n) = k \cdot n^m . \qquad (6)$$

We performed calculation up to $n \sim 10^6$, and the values of k and m for $\zeta = \pm 1$

$$k = 1.0012284 , \; m = 0.6699919 \;\; \text{for} \;\; \zeta = 1 , \qquad (7)$$

$$k = 1.0028489 , \; m = 0.6698787 \;\; \text{for} \;\; \zeta = -1 . \qquad (8)$$

CONCLUSIONS

As was predicted by classical theory, the maximum of radiation in quantum spectrum shifts successively to higher harmonics, starting with primary harmonic in non-relativistic case. This result remains valid independently of presence of spin or its direction.

For a scalar particle there exists a condition for radiation maximum to lie at highest harmonic. Moreover, we can assume that it is possible to find an ordered set of numbers, which are the critical values of external field, such that the shift of radiation maximum in the spectrum of boson can only happen when the intensity of external field is greater than certain critical value related to corresponding harmonic. If this condition is not satisfied, the position of maximum remains unchanged.

It turned out that the presence of spin perturbs the result obtained for scalar particle. For an electron no linear dependence between γ^2 and n can be observed. We have found an approximation for this dependence, where the power of n is close to 2/3 both for initial spin parallel and antiparallel to external field.

ACKNOWLEDGEMENT

Authors acknowledge the financial support of FAPESP Foundation (SP, Brazil) and greatly appreciate excellent work performed by the Organizers of IPAC'14.

REFERENCES

[1] L.A. Artzimovich, I. Ya. Pomeranchuk "Radiation from Fast Electrons in Magnetic Field", JETP, 1946, V. 16, N 5, p. 379-389.

[2] V.L. Ginzburg, Academy of Science USSR Journal, 1947, V. 11, Physics Series, p. 165-181.

[3] G.A. Schott, "Electromagnetic Radiation", Ann. d. Phys. 1907, V. 24, p. 635.

[4] A.N. Burimova, V.G. Bagrov, D.M. Gitman "New results in fundamental theory of synchrotron radiation", 2013, J. Phys.: Conf. Ser., 425, 162011, http://iopscience.iop.org/1742-6596/425/16/162011

[5] J.J. Sakurai, *Modern Quantum Mechanics*, (Addison-Wesley Publishing Company, 1994), p. 76.