# COUPLER KICK AND CAVITY TILT EFFECTS ON EMITTANCE PRESERVATION IN LINEAR ACCELERATORS

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## *Abstract*

The effects of the coupler kicks and the cavity tilts on the beam dynamics in long linear accelerator are studied. The dispersive and wakefield caused beam emittance dilution are analytically evaluated using two particle model of the beam. The numerical simulations for the European XFEL project are presented.

## INTRODUCTION

The emittance preservation in linear accelerator for X-ray free electron lasers or future $e^+e^-$ linear collider is one of the basic requirements to achieve design goals of the facilities. In an ideal linear accelerator, the required design emittance of electron beam is achieved by adiabatic damping of beam initial emittance. However, in a real accelerator the imperfections of the machine components are leading to emittance degradation along the linac [1, 2].

In this paper, we consider the emittance dilution of the beam caused by the accelerating sections random tilts and the RF coupler wake fields. The numerical study is performed for the 1.3 GHz superconducting main linear accelerator of European XFEL project [3].

The cavity random tilts leads to the beam coherent oscillations and the wake field and chromatic emittance dilution of the beam. The interactions of the beam with couplers are exciting the monopole transverse wake fields resulting on the bunch orbit kicks.

Using two-particle model of the beam, the emittance dilutions are evaluated analytically. The numerical results for the main linac of European XFEL project are presented.

## EQUATION OF MOTION, PARTICLE TRACKING AND ANALYTICAL MODEL

In our consideration of transverse beam dynamics in high-energy linear accelerator we neglect the space charge effects and assume a rigid longitudinal distribution of the beam. Under these assumptions the equation of motion for transverse coordinate $x(z,s)$ of the particle with longitudinal position $z$ in the accelerator, position $s$ in the bunch and small relative energy deviation $\delta(z,s)$ can be written as

$$x''+\frac{\gamma'}{\gamma}x'-(1-\delta)Kx=\frac{1-\delta}{\gamma}(G+F) \tag{1}$$

where derivative is over position $z$ in the linac, $\gamma$ and $\gamma'$ are the nominal beam energy and acceleration gradient respectively (in terms of relativistic Lorenz factor $\gamma$), $K$ is the normalized quadrupole fields. The function $G$ represents the on axis deflecting monopole field generated by the randomly tilted cavities and the RF coupler wakefields. The function $F$ represents the transverse dipole wakefields excited by interaction of off-axis beam with the cavities and couplers. In terms of beam and accelerator parameter the functions $G$ and $F$ can be written as

$$G(s,z)=\gamma'\alpha_k+\frac{eQ}{E_r}W_0(z,s) \tag{2}$$

$$F(z,s)=\frac{eQ}{E_r}\int_{-\infty}^{s}w_x(z,s-\bar{s})x(z,\bar{s})\rho(\bar{s})d\bar{s} \tag{3}$$

where $\alpha_k$ is the random cavity tilts, $e$, $Q$ are the unit and the total bunch charges respectively, $E_r$ is the electron rest energy, $W_0$ is the coupler monopole transverse wake potential, $w_x$ is the dipole transverse wake function in units V/pC/m$^2$ and $\rho(s)$ is the longitudinal distribution of the bunch.

The energy deviation of the particles in high-energy linear accelerators is basically determined by the initial energy spread and the beam interaction with accelerating cavities, i.e. the accelerating RF field and excited longitudinal wakefields in cavity. As the beam propagates along the accelerator, the particles within the bunch gain the energy from the external RF voltage and loss the energy due to longitudinal wakefields. The net energy gain is strongly correlated with the particle longitudinal position within the bunch thus leading to particle energy spread correlated with the longitudinal position $s$ within the bunch. The relative rms energy deviation $\delta$ of the particle within the bunch with respect to nominal energy then can be written as

$$\delta(z)=\delta_0\frac{\gamma_0}{\gamma}+\delta_c\left[1-\frac{\gamma_0}{\gamma}\right] \tag{4}$$

where $\gamma_0$ is the initial nominal energy of the beam in the entrance to the main linear accelerator, $\delta_0$ is the rms initial relative uncorrelated energy spread, and $\delta_c$ is the net rms relative correlated energy deviation due to external accelerating field and longitudinal wakefield.

The relative value of correlated energy spread induced in the linac is determined by the longitudinal wake

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potential of the bunch, the amplitude and frequency of RF voltage, the accelerating phase, the length and the total charge of the bunch. For high energy linear accelerator ($\gamma_0/\gamma << 1$) the value of $\sigma_c$ defines the maximum induced rms relative correlated energy spread in accelerator. In XFEL linac this maximum rms energy spread is at the level of $\delta_c = 0.05\%$ [3].

To study the transverse beam dynamics in linear accelerator one need the wake functions for the cavity. The expression for the transverse wake function per unit active length of TESLA cavity is derived in Ref. [4]. The coupler wakefields [5] effects have been evaluated based on the average $k_0$ and rms $k_{rms}$ kick factors given as

$$\begin{aligned} k_0 &= \int W(s)\rho(s)ds \\ k_{rms} &= \left[\int \left(W(s)-k_0\right)^2 \rho(s)ds\right]^{1/2} \end{aligned} \tag{5}$$

The rms kick factors for bunch rms length of $\sigma_z = 25\mu m$ are presented in Table 1.

Table 1: Kick Factor for the Couplers and Cavities

| Kick factors | Main linac |
|---|---|
| Cavity wake rms kick ( dipole) V/nC/mm/cavity | 0.77 |
| Input coupler wake rms kick (monopole) V/nC/cavity | 8.2 |
| Input coupler wake rms kick ( dipole) V/nC/mm/cavity | 1.25 |

The average and rms kick factors are related as $k_0 = \sqrt{3}k_{rms}$, thus the longitudinal distribution of the wake potentials can be linearly approximated as

$$W_\perp(s,r) = k_0 + k_{rms}\frac{s}{\sigma} + \left(k_0^D + k_{rms}^D\frac{s}{\sigma}\right)r \tag{6}$$

For tracking simulation the bunch is divided longitudinally into "slices" $s_k$ distributed equidistantly over the range over which we choose to model the bunch. Each of these slices represents a fraction of all particles in the bunch and must reflect the distribution of its particles in the 3-dimensional coordinate space ($x, x', \Delta E/E$). Then it follows that a slice $s_k$ is characterized by its transverse coordinates ($x_k, x'_k$) and the mean energy of its particles $E_k$. The distribution of the longitudinal slice energy defines the correlated energy spread within the bunch caused by the longitudinal wakefields and particles energy exchange with accelerating RF voltage. In addition, a slice has a total charge $q_k$. The coordinates of a slice are the mean values of the particles coordinates represented by the slice. The particle distribution in each slice is represented by the subslices in energy region $E_{km}$ with corresponding charge $q_{km}$. The variation of the sublice energy defines an uncorrelated energy spread of the bunch. Furthermore, the particle distribution in transverse phase space ($x, x'$) in each subslice $\nu = (km)$ is described by the second order moments that defines the emittance of the subslice.

In two-particle model of the beam, the beam is modelled by two particles with offsets $x_1$ and $x_1 + \Delta x$. The relative emittance dilution in this model is then derived as

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{\left\langle \Delta x^2 \right\rangle}{4\varepsilon\beta(z)} \tag{7}$$

## CAVITY AND RANDOM TILTS

For the cavity random tilts the particles of the beam experience extra transverse Lorenz force and undergrowth coherent betatron oscillations along the linac. Fig. 1 presents the beam orbit in the main linear accelerator with randomly tilted cavities with rms tilted angle of 0.5mrad.

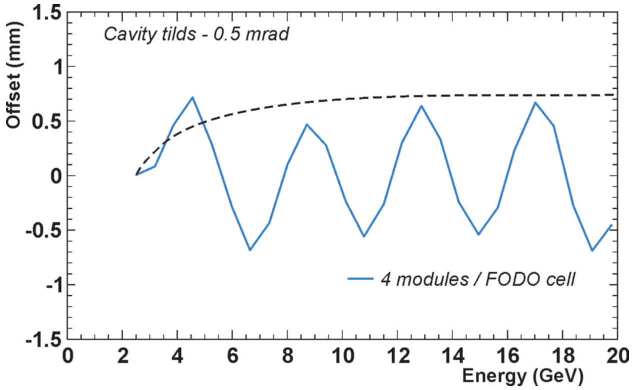

Figure 1: The coherent oscillations of the beam due to random cavity tilts.

For small tilt angle $\alpha$ of the cavity, in first approximation, the transverse force is given by the transverse component of accelerating electric field with respect to linac axis. Induced coherent oscillation of the beam in turn lead to transverse wake field excitation in cavities. In addition the beam will experience the chromatic emittance dilution due to both uncorrelated and correlated energy spread. We will use a two-particle model to evaluate the effects. In two-particle model the leading charge orbit $x_1$ and the orbit difference $\Delta x$ obey the equations

$$\begin{aligned} & x_1'' + \frac{\gamma'}{\gamma}x_1' + Kx_1 = \frac{\gamma'}{\gamma}\alpha_k \\ & \Delta x'' + \frac{\gamma'}{\gamma}\Delta x' + K(1-\delta)\Delta x = \delta K x_1 + \frac{eQW_D}{2E_r}C_w\frac{\gamma_0}{\gamma}x_1 \end{aligned} \tag{8}$$

where $C_w = eQW_d/2E_0$, $W_D = W_\perp(2\sigma_z)$ and $\sigma_z$ is the bunch rms length.

To evaluate the rms difference orbit we assume uncorrelated random tilts ($<\alpha_k\alpha_m>= 0,\ k \neq m$), the

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small betatron phase advance per cavity and neglect the fast oscillating terms in solutions. Replacing the beta value in cavity by the beta average, in thin lens approximation for initial rms uncorrelated energy spread $\delta_0$ we get the following emittance dilution

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{4}{3}\frac{d^2 N_{cav}\delta_0^2\alpha_{rms}^2}{\varepsilon_0 L_{cell}\sin\mu} tg^2\frac{\mu}{2}\frac{\gamma_0}{\Delta\gamma}\left[\ln\frac{\gamma}{\gamma_0}\right]^3 \qquad (9)$$

where $\mu$ is the phase advance per FODO cell, $\Delta\gamma$ is the energy gain per FODO cell, $\gamma_0, \gamma$ are the initial and actual energy of design particle, $L_{cell}$ is the FODO cell length, $\varepsilon_0$ is the beam initial natural emittance, $N_{cav}$ is the number of cavities per FODO cell, $d$ is the cavity length.

For correlated chromatic emittance dilution of the beam $\sigma_{cor}(z) = \sigma_c\left[1 - \frac{\gamma_0}{\gamma(z)}\right]$ we get

$$\frac{\Delta\varepsilon}{\varepsilon} \approx 4\frac{d^2 N_{cav}\delta_c^2\alpha_{rms}^2}{\varepsilon_0 L_{cell}\sin\mu} tg^2\frac{\mu}{2}\frac{\gamma^2}{\gamma_0\Delta\gamma}\ln\frac{\gamma}{\gamma_0} \qquad (10)$$

For the wake field emittance growth we get

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{4}{3}C_W^2\frac{d^2 N_{cav} L_{cell}\alpha_{rms}^2}{\varepsilon_0\sin^3\mu}\frac{\gamma_0}{\Delta\gamma}\left[\ln\frac{\gamma}{\gamma_0}\right]^3 \qquad (11)$$

Figure 2 presents the uncorrelated emittance dilution along the European XFEL main linac for 0.125% initial energy spread.

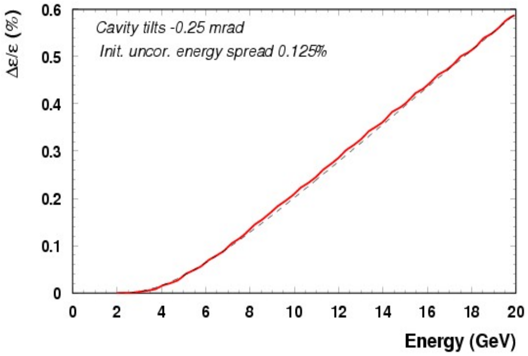

Figure 2: The uncorrelated chromatic emittance dilution along the XFEL linac. Solid line – tracking simulations, dashed line analytical prediction (9).

## COUPLER WAKEFIELDS EFFECT

In this section the particle tracking simulation results are presented for the coupler wakefield effects. Due to the asymmetry of the coupler geometry, the wake fields induced by the electron beam have the transverse monopole component. As a consequence the beam experiences the average deflection given by the kick factor. As the beam propagating along the linac, it performs the coherent oscillation due to distributed couplers monopole transverse wakefield kicks (Fig. 3) .

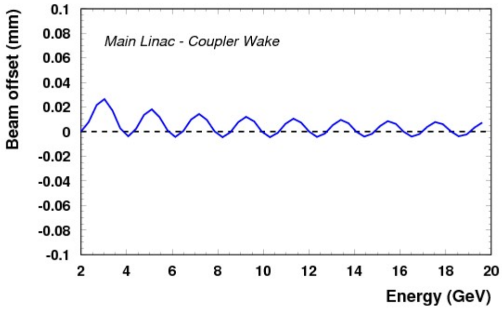

Figure 3: The beam coherent oscillations due to coupler monopole transverse wakes.

As a result the beam emittance along the linac will be diluted due to the chromatic and wake field effects. Fig.4 presents the chromatic emittance dilution along the XFEL linac due to initial uncorrelated energy spread.

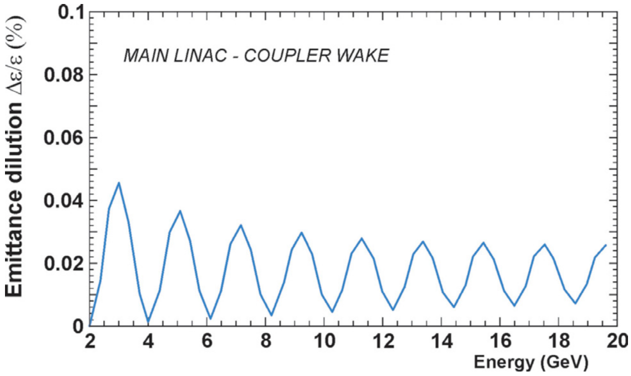

Figure 4: The uncorrelated emittance dilution along the main linac due to couplers wakefield kicks.

## SUMMARY

The cavity tilts and coupler kicks effects to the beam orbit and the emittance dilution in linear accelerators are studied analytically and numerically. Although the emittance dilutions for the XFEL main linac are small, the effects can be essential for the smaller beam emittance.

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