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FCC-ee/CepC BEAM-BEAM SIMULATIONS WITH BEAMSTRAHLUNG*

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Abstract

Beamstrahlung, namely synchrotron radiation emitted during the beam-beam collision [1], can be an important effect for circular high-energy lepton colliders such as FCC-ee (TLEP) [2] and CepC [3]. In this paper we study beam-beam effects in the presence of energy spreading and bunch lengthening due to beamstrahlung.

BEAMSTRAHLUNG

Beamstrahlung (BS) introduces an additional source of steady-state energy spread, which lengthens the bunches [5]. The strength of the beamstrahlung is characterized by the parameter $\Upsilon \equiv B/B_c$, with $B_c = m_e^2 c^2/(e\hbar) \approx 4.4$ GT the Schwinger critical field. The average value of Υ during the collision of Gaussian beams is [6, 7] $\Upsilon \approx (5/6) r_e^2 \gamma N_b/(\alpha\sigma_z(\sigma_x^* + \sigma_y^*))$, where α denotes the fine structure constant ($\alpha \approx 1/137$). For all proposed circular colliders Υ is much smaller than 1. Then we can approximate the average number of photons per collision as [7] $n_\gamma \approx 2.1\,\alpha r_e N_b/(\sigma_x + \sigma_y)$, the average relative energy loss as $\delta_B \approx 0.86\, r_e^3 \gamma N_b^2/(\sigma_z(\sigma_x + \sigma_y)^2)$, and the standard deviation of the energy loss as [6]

$$\sigma_{\delta,B} \approx \delta_B \left(0.333 + \frac{4.583}{n_\gamma}\right)^{1/2} . \tag{1}$$

The additional steady-state energy spread due to beamstrahlung (added in quadrature) can be estimated from [5]

$$\Delta\sigma_{\delta,B} \approx \frac{1}{2}\sqrt{\frac{\tau_z n_{IP}}{T_0}}\sigma_{\delta,B} \equiv \frac{A}{\sigma_z} , \tag{2}$$

with τ_z the damping time, T_0 the revolution period, n_{IP} the number of interaction points, and, in the last step, we have singled out the dependence on σ_z. Adding the natural rms energy spread from synchrotron radiation, $\sigma_{\delta,SR}$, yields the total relative energy spread of

$$\sigma_\delta = \sqrt{(\Delta\sigma_{\delta,B})^2 + \sigma_{\delta,SR}^2} . \tag{3}$$

Using $\sigma_{z,\text{tot}} = \sigma_{\delta,\text{tot}}\sigma_{z,SR}/\sigma_{\delta,SR}$, self-consistency requires

$$\sigma_{\delta,\text{tot}}^2 - \sigma_{\delta,SR}^2 = \left(\frac{\sigma_{\delta,SR}}{\sigma_{\delta,\text{tot}}}\frac{A}{\sigma_{z,SR}}\right)^2 , \tag{4}$$

where the subindex "SR" refers to the bunch length or energy spread computed with arc synchrotron radiation only. The explicit solution for the total energy spread is

$$\sigma_{\delta,\text{tot}} = \left(\frac{1}{2}\sigma_{\delta,SR}^2 + \left(\frac{1}{4}\sigma_{\delta,SR}^4 + A^2\frac{\sigma_{\delta,SR}^2}{\sigma_{z,SR}^2}\right)^{1/2}\right)^{1/2} . \tag{5}$$

SIMULATION APPROACHES

The upgraded weak-strong and strong-strong beam-beam codes BBWS and BBSS take into account the combined effect of both standard synchrotron radiation and beamstrahlung in a semi- or fully self-consistent manner. Both codes were used to simulate the beam-beam behavior for the various proposed running modes of FCC-ee and CepC, considering the beam and machine parameters of Ref. [4].

Figure 1 illustrates the recipe employed for modeling the beamstrahlung. The collision is divided into many small steps. Individually tracked particles randomly emit synchrotron radiation according to their local bending radius $1/\rho = |\Delta\sqrt{x'^2 + y'^2}/\Delta s|$. The probability of the emission of a photon is proportional to $\Delta s/\rho$.

Two models for the random emission were implemented. The first represents the photon emission as a Gaussian fluctuation with the correct rms value (including the average energy loss). The second model generates the exact full photon spectrum as described by the $K_{5/3}$ Bessel function, by inverting a pre-computed table for $\mathcal{N}_\gamma(\omega)$. These two approaches yield about the same simulated luminosity and bunch length (see Figs. 7 and 8), whereas the beam lifetime is sensitive to the detailed photon spectrum.

In case of the weak-strong simulation the bunch length of the strong bunch is regularly updated (every 100 turns) so as to correspond to the bunch length of the weak beam, which is evolving under the influence of the beamstrahlung. When simulating the beam lifetime, similar self-consistent updates are applied to the horizontal beam size.

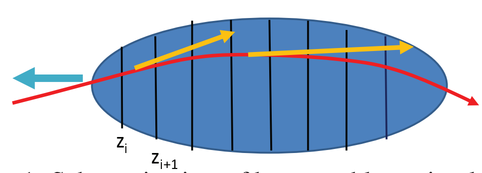

Figure 1: Schematic view of beamstrahlung simulation.

Equilibrium values are quickly reached for the bunch lengths, the luminosity, and the transverse beam sizes, as is illustrated in Fig. 2, which shows the result of a weak-strong simulation for CepC without and with beamstrahlung (including the self-consistent bunch length).

In the strong-strong simulation with BBSS the bunches of both beams are divided into 15–20 slides. Each slice contains many macroparticles (of order 10^5). The collision is calculated slice by slice. Using a 3D symplectic integrator the beam potential ϕ is computed on each slice boundary z_i, and then interpolated longitudinally for the next tracking step of the macroparticles. The interpolation is important. The macroparticles also suffer energy

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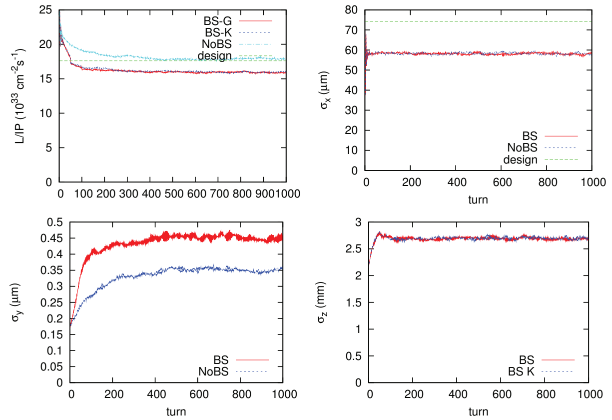

Figure 2: CepC luminosity, rms bunch length, horizontal and vertical beam sizes vs. turn number, from a weak-strong simulation without and with beamstrahlung.

changes in proportion to $\partial\phi/\partial z$. The calculation procedure is repeated several times during a collision, until all slices of two bunches have passed through each other, at each step updating the trajectories and the potentials.

LUMINOSITY PERFORMANCE

Figure 3 presents weak-strong simulation results for TLEP/FCC-ee at four different collision energies. The simulated luminosities are close to the analytically expected values, as is illustrated in Table 1, which also compares calculated and simulated equilibrium bunch lengths. The simulations reveal extended vertical beam tails (Fig. 4).

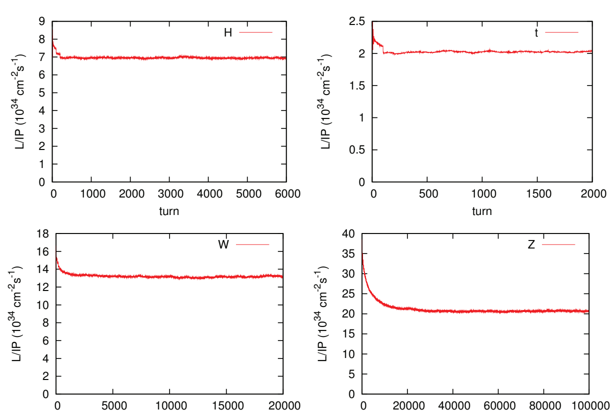

Figure 3: Weak-strong simulation of luminosity for TLEP/FCC-ee at four different c.m. energies.

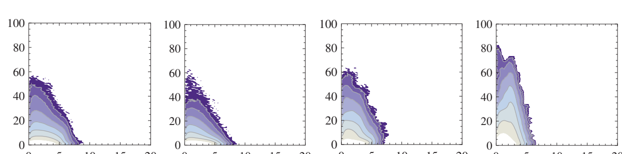

Figure 4: Contour of transverse beam tails from weak-strong simulation for H, t, W and Z (the colour code represents a log scale). The unit is $\sqrt{2J_i/\varepsilon_i}$

Figure 5 displays weak-strong simulation results for the special TLEP low-emittance crab-waist scenario at the Z pole [8]. The bunch length is almost tripled due to the beamstrahlung, both with and without the crab waist. However, switching on the crab waist reduces the vertical beam size by a factor of 5 and the horizontal one by a factor of 2; most importantly, it increases the luminosity about 5-fold.

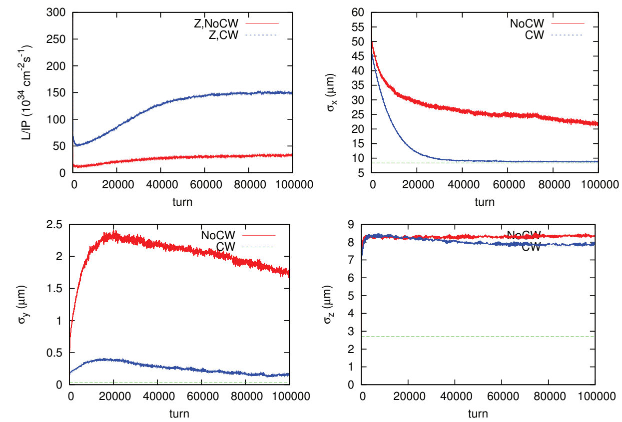

Figure 5: Weak-strong simulation for TLEP-Z low-emittance parameters with (blue) and without crab waist (red): luminosity (top left), vertical beam size (top right), horizontal beam size (bottom left), and bunch length (bottom right). Green dashes indicate beam sizes without BS.

The simulated performance, in terms of luminosity and beam size, varies with the betatron tune, while the bunch length is nearly independent of the working point. Figure 6 presents the results of a horizontal tune scan for fixed vertical tune, revealing synchro-betatron resonances close to the half integer.

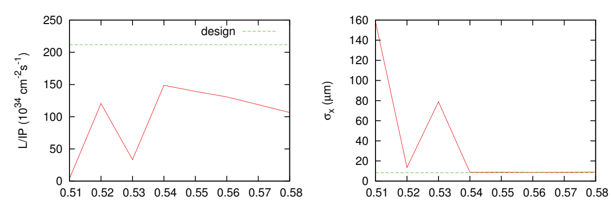

Figure 6: Luminosity (left) and horizontal beam size (right) vs Q_x at $Q_y = 0.61$ (tunes/IP), for TLEP-Z crab waist scenario, from a weak-strong simulation ($Q_s = 0.062$).

Strong-strong results for TLEP-H and -t are shown in Figs. 7 and 8, which also illustrate the difference between a simple Gaussian fluctuation and the exact photon spectrum. The much weaker radiation damping for TLEP-W and TLEP-Z would render the corresponding strong-strong computations more difficult. Computing demands are further aggravated for the TLEP-Z crab-waist scheme, the proper modeling of which would require a significantly larger number of slices.

BEAM LIFETIME

The beam lifetime due to beamstrahlung can be calculated in a number of ways. Two alternative analytical for-

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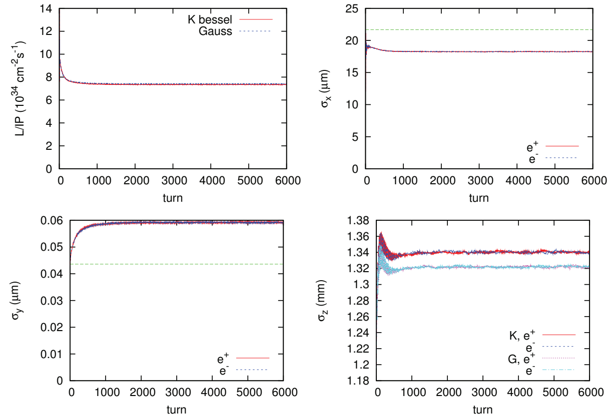

Figure 7: Strong-strong simulation for TLEP-H: luminosity (top left), vertical beam size (top right), horizontal beam size (bottom left), and bunch length (bottom right).

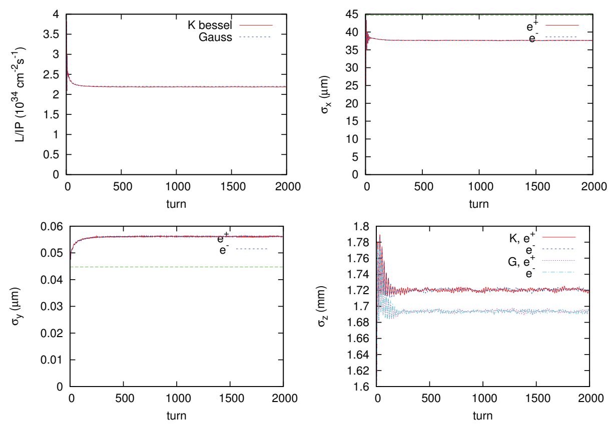

Figure 8: Strong-strong simulation for TLEP-t: luminosity (top left), vertical beam size (top right), horizontal beam size (bottom left), and bunch length (bottom right).

mulae were proposed in Refs. [8] and [9]. Several methods are also available for inferring the beam lifetime from the simulations. One approach is to directly compute the particles lost by exceeding a limiting momentum acceptance of, e.g., 1.5%, or a vertical aperture limit, taken to be $40\sigma_y$, i.e., $\tau_{\rm BS,1} = T_{\rm sim} N_{\rm tot}/(\Delta N)_{\rm lost}$, with $T_{\rm sim}$ the simulated time interval, $N_{\rm tot}$ the total number of macroparticles, and $(\Delta N)_{\rm lost}$ the number of lost macroparticles. A second approach is to calculate the incoming flux due to radiation damping at the limiting amplitude (longitudinally $\sqrt{2J_z}$, where J_z denotes the action variable, or $\hat{y}$ transversely) from the equilibrium beam-tail distribution simulated without acceptance limit, in close analogy to the lifetime calculation for conventional synchrotron radiation [10]. E.g. for the longitudinal plane one has $\tau_{\rm BS,2} = \tau_z/(2\xi\rho(\xi))$, with $\rho(\xi)$ the density, and $\xi \equiv J_z/\epsilon_z = \delta_{\rm max}^2/(2\sigma_\delta^2)$ the normalized acceptance.

Table 1: Calculated Luminosity and Bunch Length

	TLEP/FCC-ee					Ce-pC
	Z	Z (cr. w.)	W	t	H	
	luminosity [10^{34} cm^{-2}s^{-1}]					
analyt.	28	219	12	6.0	1.7	1.8
w-s.	21	150	13	6.9	2.0	1.6
s-strong	—	—	—	7.5	2.2	1.6
	σ_z [mm]					
w/o BS	1.64	1.9	1.01	0.81	1.16	2.3
analyt.	2.56	6.4	1.49	1.17	1.49	2.7
w-s.	2.8	7.9	1.5	1.2	1.6	2.7
s-strong	—	—	—	1.3	1.72	2.9

Table 2 compares simulated beam lifetimes, as computed by the aforementioned two approaches, with predictions from the analytical formulae of Refs. [9] or [8]. Reassuringly, the direct beam loss simulations and the calculation from the equilibrium distribution at the acceptance limit (Figs. 9 and 10) yield consistent results. However, at a given value of $\delta_{\rm max}$ the simulated lifetimes are a factor 10–20 shorter than the analytical estimates. They are dominated by the longitudinal plane, with at most a few per cent contribution from the vertical. The lifetime varies strongly with $\delta_{\rm max}$ (Fig. 10), but it is almost independent of the value of β_y^* [11].

Table 2: Expected and Simulated BS Lifetime

$\tau_{\rm BS}$ [min]	TLEP-H	TLEP-t	CepC
analytical [9]	310	3.6	113
analytical [8]	1400	3.3	619
weak-strong (loss)	26	0.3	5.5
weak-strong (distr.)	33	0.3	—

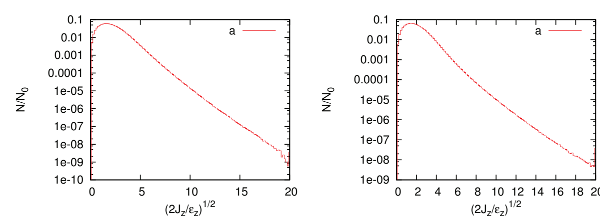

Figure 9: Equilibrium distribution for TLEP-H (left) and TLEP-t (right) from tracking 100 particles over 10^8 turns.

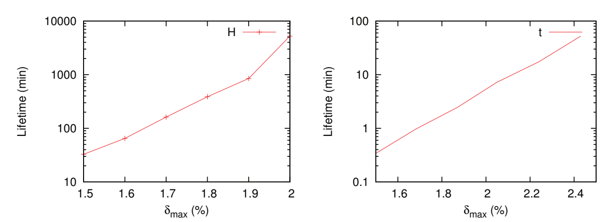

Figure 10: Lifetime vs. momentum acceptance inferred from the equilibrium distributions in Fig. 9 for TLEP-H (left) and TLEP-t (right).

CONCLUSIONS

Both weak-strong and strong-strong simulations confirm the analytically expected luminosities for TLEP (FCC-ee) and CepC. The analytical expression (5) is consistent with the steady-state bunch length obtained in strong-strong simulations, with differences at the few per cent level. The lifetime values predicted by the simulated losses or equilibrium distributions are considerable shorter than those pre-

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dicted by the available analytical expressions. Similar discrepancies were reported previously [8, 12].

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