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RADIATION OF A CHARGED PARTICLE BUNCH MOVING IN THE PRESENCE OF PLANAR WIRE STRUCTURE *

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Abstract

The structure under consideration represents a set of long thin parallel wires which are placed in a plane with fixed spacing. The wires can exhibit a limited conductivity. If the period of the structure is much less than the typical wavelength, the structure‘s influence can be described with help of the averaged boundary conditions [1]. The main attention is given to the case when the bunch flies through the grid in the orthogonal direction. Radiation of charged particle bunch which have small transversal size and limited longitudinal one is studied. Analytical expressions for volume and surface waves are given for the bunches with arbitrary longitudinal profile. A separate analysis is performed for the particular case of the plane which is ideally conducting in only one direction. It is shown that the surface wave is similar, in some way, to the radiation field of the bunch moving in a wire metamaterial [2]. It is demonstrated that the detection of surface waves can be used to estimate the longitudinal sizes of bunches. Typical numerical results for bunches of different shapes and structures with different parameters are given.

INTRODUCTION

Radiation of charged particles in the presence of periodic structures is perspective for development of new technics for bunch diagnostics. Recent investigations [2–5] show, that particular interest represent so-called “wire metamaterial”, a periodic structure comprised of thin parallel conducting rods. It was found, that the radiation generated by charged particle bunch moving perpendicularly to the wires has unique properties, which allow the determination of the length, shape and velocity of the bunch.

However, implementation and operation of three-dimensional wire metamaterial in practice meets a series of engineering difficulties. Therefore, the investigation of radiation generated by a bunch passing through a planar (two-dimensional) wire stricture (Fig. 1) was undertaken. We note that similar problems were considered earlier [6–9], but only radiation of a point charge was analyzed and the main attention was paid to a volume radiation. In our research we consider a charged particle bunch of an arbitrary longitudinal profile and take into account “non-ideality” of the wires. Moreover, a special attention is paid to a surface wave, which, as will be shown, can play the key role for the bunch diagnostics.

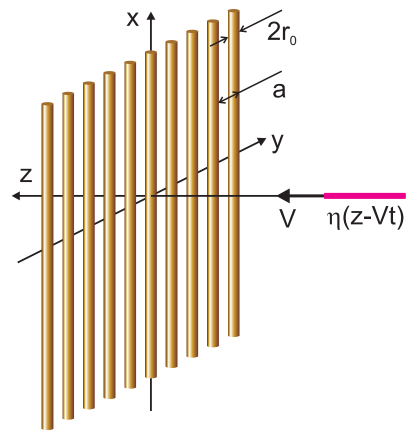

Figure 1: Planar wire grid.

We consider a charged particle bunch flying through wire grid perpendicularly to its surface. Wires are directed along x axis and placed periodically in the plane $z = 0$, period of the structure is a and wires’ radius is r_0. Charged particle bunch has infinitesimal size in the transverse direction and possess longitudinal charge distribution $\eta(z - c\beta t)$. Thus, electrical current of the bunch has the form $\vec{j} = c\beta\delta(x)\delta(y)\eta(z - c\beta t)\vec{\mathbf{e}}_z$. The total electromagnetic field can be represented as a sum of “incident” field of the bunch and the one which is induced due to the presence of the grid. In order to describe the field induced by the wire grid we use the averaged boundary conditions [1]

$$\begin{aligned} E_{x\omega}|_{z=0} &= -\frac{c}{4\pi}\left(A + B\frac{\partial^2}{\partial x^2}\right)\left\{H_{y\omega}\right\}, \\ \{E_{x\omega}\} &= \left\{E_{y\omega}\right\} = \{H_{x\omega}\} = 0, \end{aligned} \tag{1}$$

where {} denotes the jump of the corresponding value at $z = 0$, i.e., $\{f(z)\} = f(+0) - f(-0)$. Parameters A and B are defined by the properties of the wire grid

$$\begin{aligned} A &= a\left[Z - 2i\frac{\omega}{c^2}\ln\left(\frac{a}{2\pi r_0}\right)\right], \\ B &= -2i\frac{a}{\omega}\ln\left(\frac{a}{2\pi r_0}\right). \end{aligned} \tag{2}$$

The impedance of the wires Z has a simple approximation for two cases [1]

$$Z \approx \frac{1}{\sigma_e}\begin{cases} \left(\pi r_0^2\right)^{-1}, & r_0 \ll 2d, \\ \left(\sqrt{2}\pi r_0 d\right)^{-1}\exp\{-i\pi/4\}, & r_0 \gg 2d. \end{cases} \tag{3}$$

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where σ_e and d are the conductivity and the thickness of the skin layer for the wire material.

The description above is valid under assumption that the wavelength and the scale of the spatial variation of the incident field are greater than the grid's period, which is much greater than the wires' thickness:

$$a \ll c\beta/\left(\omega\sqrt{1-\beta^2}\right), \quad a \ll 2\pi c/\omega, \quad r_0 \ll a. \qquad (4)$$

Using conditions (2) we obtain the exact solution in the form of three-dimensional Fourier integral, and the approximations for the volume radiation and the surface wave are derived.

VOLUME RADIATION

Firstly, we examine the volume field of transition radiation. Using the saddle-point method and spherical coordinate system ($x = R\sin\theta\cos\varphi$, $y = R\sin\theta\sin\varphi$, and $z = R\cos\theta$), the following expressions for the non-zero components of the field in the far-field zone are obtained

$$\begin{Bmatrix} E^{tr}_{\theta\omega} \\ E^{tr}_{\varphi\omega} \end{Bmatrix} = \begin{Bmatrix} H^{tr}_{\varphi\omega} \\ -H^{tr}_{\theta\omega} \end{Bmatrix} = \frac{2\beta}{c} \begin{Bmatrix} \cos\theta\ \cos\varphi \\ -\sin\varphi \end{Bmatrix} \times \\ \times \frac{\exp\left\{i\frac{\omega}{c}R\right\}}{R} \frac{1}{(1-\beta^2\cos^2\theta)} \times \\ \times \frac{\sin\theta\ |\cos\theta|\ \cos\varphi\ \tilde{\eta}\left(\frac{\omega}{c\beta}\right)}{\left[\left(1-\sin^2\theta\ \cos^2\varphi\right)(1-i\kappa\,|\cos\theta|)+\delta\,|\cos\theta|\right]}, \qquad (5)$$

where

$$\tilde{\eta}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \eta(\zeta)\exp\{-ik\zeta\}\,d\zeta,$$

$\kappa = \tilde{\kappa}\omega/c$, $\tilde{\kappa} = \frac{a}{\pi}\ln\frac{a}{2\pi r_0}$, $\delta = \frac{acZ}{2\pi}$,

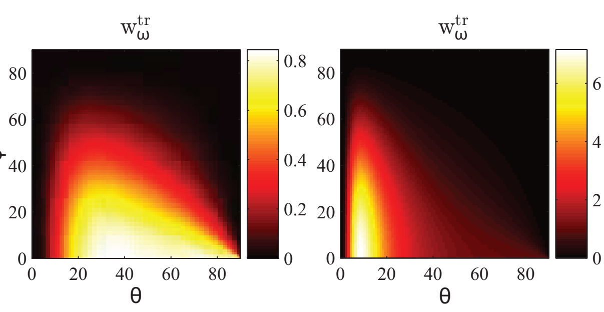

Figure 2: Total angular spectral density of volume radiation w^{tr}_ω depending on θ and φ for ideal wires. The energy density is measured in $4|\tilde{\eta}|^2/c$ units; $a = 10$ mm; $r_0 = 0.02$ mm ($\kappa \approx 0.87$); $\omega = \pi c/(5a)$. The charge velocity $\beta = 0.9$ for the left image, $\beta = 0.99$ for the right image.

Further we analyze the properties of radiation with help of the angular spectral density $w^{tr}_\omega = cR^2\,\mathrm{Re}\left[\vec{E}^{tr}_\omega \times \vec{H}^{tr*}_\omega\right]$ (sign * denotes the complex conjugation). Analytical investigation of the expression for w^{tr}_ω show, that the maximum of radiation lies in the plane xz, but the radiation is absent in the planes yz (Fig. 2). For the low velocities this maximum is directed under the small angle to the wires or along them, if wires are assumed to be perfectly conducting ($\delta = 0$). Transition volume radiation of highly relativistic bunches have the maximum directed close to the direction of bunch motion. Moreover, in some range of κ and β there exist two local maximums of radiation if $\delta = 0$. This fact allows obtaining radiation with almost uniform angular distribution (Fig. 2). Note that the volume transition radiation is symmetrical in the respect to xy, xz and yz planes. Numerical result for different grid properties are shown in Fig. 3.

SURFACE WAVE

The exact expression for the field induced by the boundary has a pole that determines the surface wave. When the wires are perfect conductors ($\delta = 0$), this poles are $k_x = \pm\omega/c$. Here we present the expression for the surface wave, when the bunch has a rectangular profile, i.e. $\eta(\zeta) = \frac{q}{2\sigma}\Theta(\sigma - |\zeta|)$, where q is the bunch's total charge, 2σ is the bunch's length and Θ is the step-function

$$\begin{Bmatrix} E^s_z \\ E^s_y \end{Bmatrix}_{rect} = -\frac{q\beta}{2\sigma}\int_0^\infty \begin{Bmatrix} \cos\left(k_y y\right)\,\mathrm{sgn}\,z \\ \sin\left(k_y y\right) \end{Bmatrix} \times \\ \times \frac{e^{-k_y|z|}}{\left(1+\tilde{\kappa}k_y\right)}\left[e^{-k_y|\beta\xi+\sigma|} - e^{-k_y|\beta\xi-\sigma|}\right]dk_y, \qquad (6)$$

the component $E^s_x = 0$. In case when $\tilde{\kappa} = 0$, i.e. the grid represents the plane ideally conducting in the only direction, then (6) can be simplified

$$\begin{Bmatrix} E^s_z \\ E^s_y \end{Bmatrix}_{rect} = -\frac{q\beta}{2\sigma}\times \\ \times \begin{Bmatrix} \mathrm{sgn}\,z\left[\frac{|z|+|\beta\xi+\sigma|}{y^2+(|z|+|\beta\xi+\sigma|)^2} - \frac{|z|+|\beta\xi-\sigma|}{y^2+(|z|+|\beta\xi-\sigma|)^2}\right] \\ \frac{y}{y^2+(|z|+|\beta\xi+\sigma|)^2} - \frac{y}{y^2+(|z|+|\beta\xi-\sigma|)^2} \end{Bmatrix}. \qquad (7)$$

This surface wave propagates strictly along the wires with speed of light in vacuum. If $\delta = 0$, then its shape doesn't change with time, it shifts as a whole along the wires. Numerical results are shown in Fig. 4, where along with both components, the energy flow density $\vec{S}^s = \frac{c}{4\pi}\left(\vec{E}^s\right)^2\mathrm{sgn}(x)\,\vec{\mathbf{e}}_x$ is presented. One can see that maximums of $|E^s_z|$ and S^s as well as zeros of E^s_y correspond to the ends of the bunch. Thus, this wave can be used for measurement of length of charged bunches.

Finally we note, that consideration of finite conductivity of wires in the first approximation affects attenuation of Fourier harmonics of the surface wave by the following way

$$E^s_{z\omega}\Big|_{y=z=0} \sim \frac{-4iq\omega}{\pi c^2\beta}\exp\left\{i\frac{\omega}{c}|x|\right\}\frac{\tilde{\kappa}^2}{(\mathrm{Re}\,\delta)^2x^2} \qquad (8)$$

for $|x|\,\mathrm{Re}\,\delta/(2\tilde{\kappa}) \gg 1$.

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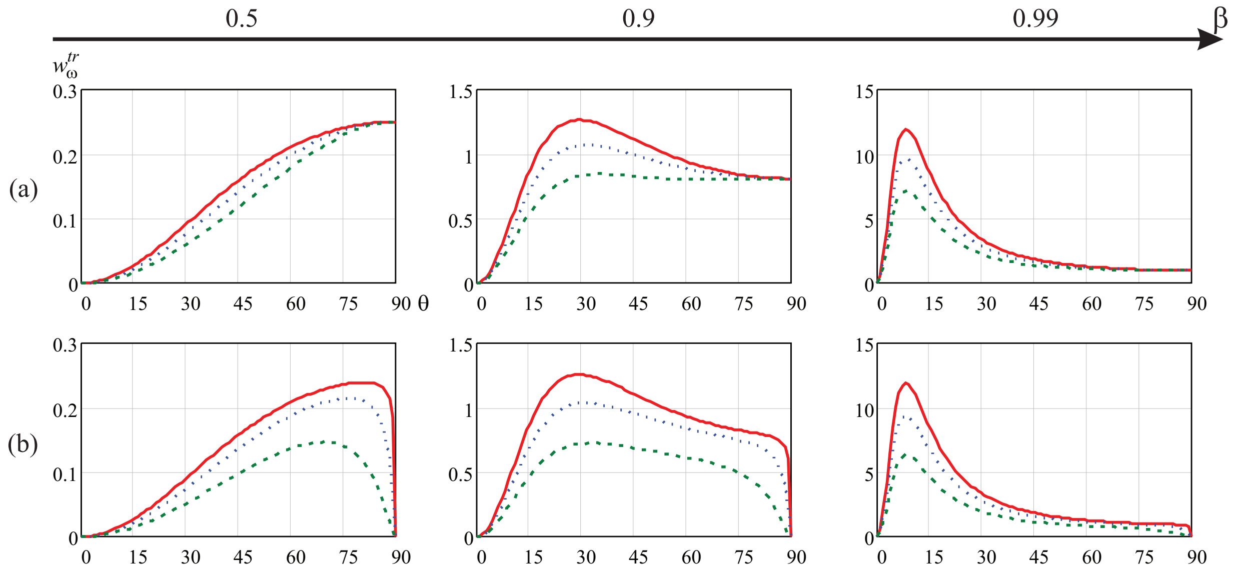

Figure 3: Total angular spectral density of volume radiation w_ω^{tr} depending on θ. The energy density is measured in $4|\tilde{\eta}|^2/c$ units; $a = 10$ mm; $r_0 = 0.5$ mm ($\kappa \approx 0.23$, solid line), $r_0 = 0.1$ mm ($\kappa \approx 0.55$, dotted line), and $r_0 = 0.02$ mm ($\kappa \approx 0.87$, dashed line); (a) ideal conductors, (b) copper ($\sigma_e = 5.8 \cdot 10^7$ S/m); $\omega = \pi c/(5a)$. The charge velocity is shown in the external axis, $\varphi = 0$.

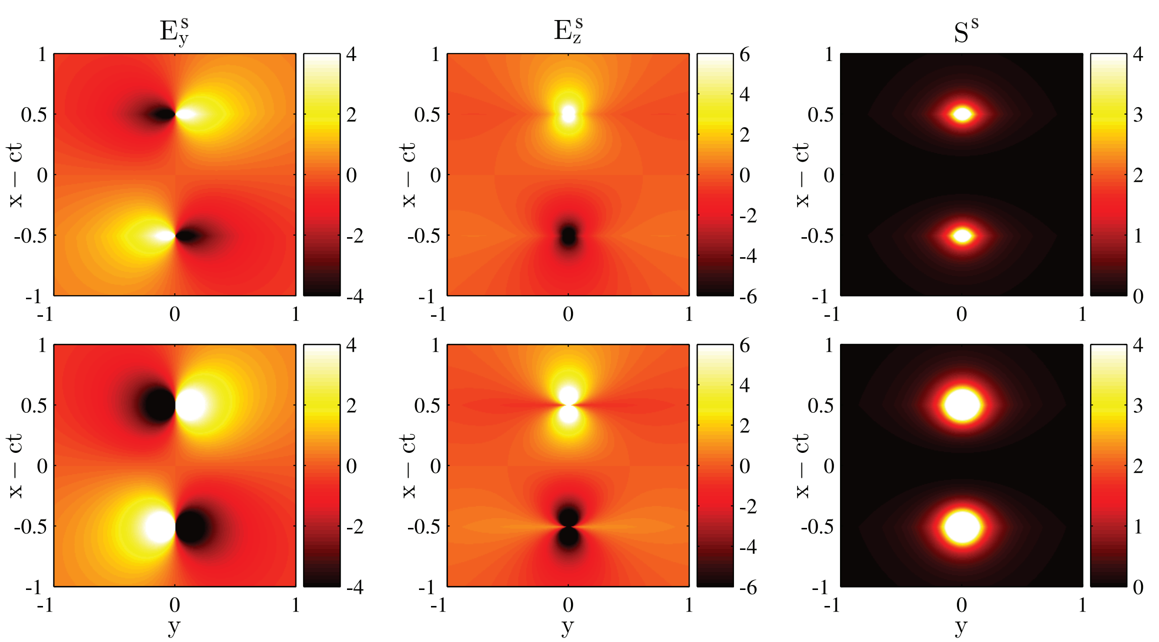

Figure 4: The field components and the energy flow density of the surface wave from the rectangular bunch with $q = 1$ esu. Gaussian units are used. The bunch length is $\sigma = 0.5$ cm for all pictures; $\tilde{\kappa} = 0.148$ cm ($a = 1$ cm, $r_0 = 0.1$ cm) for the first row, and $\tilde{\kappa} = 0$ for the second row; $\delta = 0$.

REFERENCES

[1] M. I. Kontorovich et al., *Electrodynamics of grid structures* [in Russian] (Radio Svyaz, Moscow, 1987).

[2] V.V. Vorobev, A.V. Tyukhtin, Phys. Rev. Lett. 108, 184801 (2012).

[3] S. I. Maslovski, S. A. Tretyakov, and P. A. Belov, Microw. Opt. Technol. Lett. 35, 47 (2002).

[4] A. V. Tyukhtin and V. V. Vorobev, J. Opt. Soc. Am. B 30, 1524 (2013).

[5] A. V. Tyukhtin and V. V. Vorobev, Phys. Rev. E 89, 013202 (2014).

[6] K. A. Barsukov and L. G. Naryshkina, Soviet Radiophysics 8, 667 (1965).

[7] K. A. Barsukov and L. G. Naryshkina, Radiophysics and Quantum Electronics 10, 276 (1967).

[8] V. N. Krasil'nikov and A. V. Tyukhtin, Radiophysics and Quantum Electronics 33, 945 (1990).

[9] B. M. Bolotovsky and A. V. Serov, JETP 125, 1269 (2004).