A MULTI-CONDUCTOR TRANSMISSION LINE MODEL FOR THE BPMS

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Abstract

A multi-conductor transmission line model for beam position monitors (BPMs) is developed for an accurate and efficient analysis. This method combines the two-dimensional electrostatic analysis including beams in the transverse plane and the transmission line analysis in the longitudinal direction. The loads are also included in the boundary condition of the transmission line analysis. A problem can be solved self-consistently. Calculation of 2D electrostatic fields can be easily performed with the boundary element method. The BPM response to a beam is compared with that to a stretched wire in this framework.

INTRODUCTION

When measuring the position sensitivity of BPMs with a stretched wire, a fundamental question might be whether it gives the same result as the beam will do. A simplified BPM response to a beam is usually explained with induced charges on the BPM wall where no discrepancy is assumed between the pipe wall and the electrodes. The beam at (x, y) induces a charge on the wall. The charge density $\sigma(a, \theta)$ is expressed as

$$\sigma(a,\theta,r,\phi) = \frac{\lambda_B}{2\pi a} \frac{a^2 - r^2}{a^2 + r^2 - 2a\, r\cos(\theta - \phi)}.$$

The beam position is obtained by calculating, for example,

$$x = \frac{1}{\kappa} \cdot \frac{\sigma(a,0,r,\phi) - \sigma(a,\pi/2,r,\phi)}{\sigma(a,0,r,\phi) + \sigma(a,\pi/2,r,\phi)},$$

by Δ/Σ calculation with $\theta = 0$ and $\pi/2$.

A typical stretched wire method is the setup where the beam is replaced by the wire. A conductor wire of the radius r_W is stretched in the pipe. The power supply of a voltage V is connected at one end and the matching resistor at the other end. The charge on the wire in this case is

$$\lambda_W = C_W(x,y)V,$$

$$C_W = \frac{2\pi\varepsilon_0}{\cosh^{-1}\left(\frac{r_W^2 + a^2 - \left(x^2 + y^2\right)}{2r_W a}\right)}.$$

The induced charge density in this case is

$$\sigma'(a,\theta,r,\phi) = \frac{C_w(x,y)V}{2\pi a} \frac{a^2 - r^2}{a^2 + r^2 - 2a\, r\cos(\theta - \phi)}.$$

Therefore the induced charge density on the wall is different from that due to the beam. The estimated positions with Δ/Σ methods, however, agree with each other because the terms, C_w, in the numerator and denominator vanish by division. In real BPMs the structure is not simple as the above example. Such a practical structure is analyzed with multiconductor transmission line model for both the beam and the wire case.

MODELING OF BPMS

The system is defined as follows. Pickups in a beam pipe, both of which are longitudinally uniform and made of a perfect conductor, are terminated by some load impedances at some longitudinal points. A beam runs longitudinally with ultra-relativistic velocity. We neglect higher order modes, which assumes only TEM modes in the pipe. The analysis of the wire case is an application of [1], whereas we need to re-examine for the above configuration with the beam.

Solution of the Maxwell Equation with a Beam

Using the coordinate system, x, y, z where the z-direction coincides the beam and pipe axis, x and y are in the transverse plane. Under the above conditions the Maxwell's equations result in the Poisson equation for the transverse plane and a multi-conductor transmission line equation for the longitudinal plane. The electric and magnetic fields have the relation:

$$\mathbf{E}(x,y,z) = \mathbf{c} \times \mathbf{B}(x,y,z). \tag{1}$$

The electric fields can be expressed as

$$\mathbf{E}(x,y,z) = \tilde{\mathbf{E}}(x,y) \cdot e^{\mp jkz}. \tag{2}$$

and obey the 2D Poisson equation:

$$\Delta V(x,y) = -\frac{\rho(x,y)}{\varepsilon_0}. \tag{3}$$

Using the potential coefficients and the line charge densities l_n and l_0 of n-th conductor and the beam,

$$\begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} + \begin{bmatrix} p_{10} \\ \vdots \\ p_{n0} \end{bmatrix} \lambda_0 \tag{4}$$

$$= \mathbf{p}\lambda + \mathbf{p}_0\lambda_0,$$

the traveling waves are expressed as [2]

$$\mathbf{V}_+ \, e^{-jkz} = \frac{1}{c}\mathbf{p}\,\mathbf{I}_+ \, e^{-jkz} + \mathbf{p}_0\lambda_0 \, e^{-jkz}, \tag{5}$$

$$\mathbf{V}_- \, e^{+jkz} = -\frac{1}{c}\mathbf{p}\,\mathbf{I}_- \, e^{+jkz}, \tag{6}$$

where l and l_0 are the charge densities of the conductors and beam, $\mathbf{V}_+$ and $\mathbf{V}_-$ ($\mathbf{I}_+$ and $\mathbf{I}_-$) are the voltages (currents) of forward and backward waves, k the wave number (=w/c), c the light velocity. The time variation is assumed to be $e^{j\omega t}$. The beam propagating forward z-direction is

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06 Instrumentation, Controls, Feedback & Operational Aspects

T03 Beam Diagnostics and Instrumentation

expressed as e^{-jkz}. The beam affects only the forward wave as Eq. 5. There is subtlety that concerns the integral on the boundaries in the transverse plane. The boundary conditions are

$$\mathbf{V}_+ + \mathbf{V}_- = -\mathbf{R}_0(\mathbf{I}_+ + \mathbf{I}_-) , \quad (7)$$

$$\mathbf{V}_+ e^{-jk\ell} + \mathbf{V}_- e^{+jk\ell} = \mathbf{R}_\ell(\mathbf{I}_+ e^{-jk\ell} + \mathbf{I}_- e^{+jk\ell}) , \quad (8)$$

if the terminations are done with the impedances, $\mathbf{R}_0$ and $\mathbf{R}_\ell$, at z = 0 and ℓ. As an example the geometry of one electrode and the beam is shown in Fig. 1. Other configurations as the center termination and so on can be also included in the boundary conditions in a similar manner.

When the upstream termination is an open circuit as the 3-50 BT BPM (Fig. 3), and $\mathbf{R}_\ell = R_\ell \mathbf{E}$, the voltage at z = ℓ is

$$V(\ell)$$
$$= \left[\cos k\ell \cdot \mathbf{E} + j\sin k\ell \cdot cR_\ell \mathbf{q}\right]^{-1} j e^{-jk\ell} \sin k\ell \cdot R_\ell \mathbf{q} \cdot \mathbf{p}_0 I_0 \quad (9)$$

where $\mathbf{E}$ is the unit matrix, $\mathbf{q} = \mathbf{p}^{-1}$, and I_0 the beam current.

As described above the boundary conditions are included in the formulation without artificial manipulations or non-physical assumption as grounded electrodes.
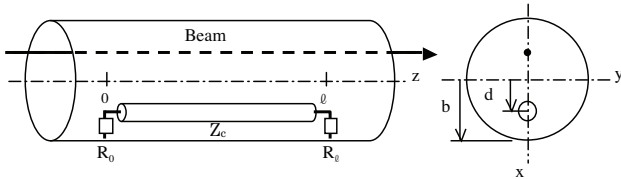

Figure 1: Schematic of one conductor and a beam.

Low Frequency Limit

At the low frequency ($k\ell = \omega\ell/c \ll 1$) Eq. 9 can be approximated up to the first order of $k\ell$. For the pickup with one conductor the voltage reads

$$V(\ell) = \frac{j\omega\ell C}{j\omega\ell C + \dfrac{1}{R_\ell}} \frac{p_0}{c} I_0 . \quad (10)$$

For the pickup with multi-conductor the voltage reads

$$\mathbf{V}(\ell) = \left[j\omega\ell \mathbf{q} + \mathbf{R}_\ell^{-1}\right]^{-1} j\omega\ell \mathbf{q} \frac{\mathbf{p}_0}{c} \mathbf{I}_0 . \quad (11)$$

The capacitance C in Eq. 10 corresponds to the coefficients of capacitances and inductions $\mathbf{q}$ in Eq. 11, the termination load R_ℓ to the matrix $\mathbf{R}_\ell$, the coefficient of potential p_0 to $\mathbf{p}_0$, and division to matrix inversion.

Solution for the Stretched Wire Method

Applying the ordinary MTL theory to the stretched wire method as Fig. 2, the following equation is reduced,

$$\mathbf{V}_\pm(z) = \hat{\mathbf{V}}_\pm e^{\mp jkz} , \quad (12)$$

$$\mathbf{I}_\pm(z) = \hat{\mathbf{I}}_\pm e^{\mp jkz} = \pm \mathbf{Z}^{-1} \hat{\mathbf{V}}_\pm e^{\mp jkz} \quad (13)$$

with the boundary condition

$$\hat{\mathbf{V}}_+ + \hat{\mathbf{V}}_- = \mathbf{V}_S - \mathbf{Z}_0(\hat{\mathbf{I}}_+ + \hat{\mathbf{I}}_-) , \quad (14)$$

$$\hat{\mathbf{V}}_+ e^{-jk\ell} + \hat{\mathbf{V}}_- e^{+jk\ell} = \mathbf{Z}_\ell(\hat{\mathbf{I}}_+ e^{-jk\ell} + \hat{\mathbf{I}}_- e^{+jk\ell}) . \quad (15)$$

For example, the case of a two-conductor transmission line with one electrode and one wire, the impedance, $\mathbf{Z}$, obeys

$$\mathbf{Z}^{-1} = c\mathbf{C} = c\begin{bmatrix} C_{BPM} & -\Delta C \\ -\Delta C & C_W \end{bmatrix}, \quad (16)$$

where c, $\mathbf{C}$, C_{BPM}, C_W and $-\Delta C$ are the light velocity, the capacitance matrix, the electrode capacitance, the wire capacitance and the coupling between the wire and the electrode, respectively. At the terminals $\mathbf{Z}_0$, $\mathbf{Z}_\ell$ and $\mathbf{V}_S$ are expressed as

$$\mathbf{Z}_0 = \begin{bmatrix} R_0 & 0 \\ 0 & Z_C \end{bmatrix}, \mathbf{Z}_\ell = \begin{bmatrix} R_\ell & 0 \\ 0 & Z_C \end{bmatrix},$$

$$\mathbf{V}_S = \begin{bmatrix} 0 \\ V_0 \end{bmatrix},$$

with R_0, R_ℓ, Z_C and V_0 defined in Fig. 2. The limiting case of $\Delta C \ll C_{BPM}$, $\omega\ell/c \ll 1$, $R_0 \gg 1$, the output voltage of the electrode is obtained as

$$V_{BPM}(\ell) \approx \frac{j\omega\ell C_{BPM}}{j\omega\ell C_{BPM} + 1/R_\ell} \cdot \frac{Z_C}{C_{BPM}} \Delta C I_W(\ell), \quad (17)$$

where $I_W(\ell)$ is the wire current at $z = \ell$. This equation is the wire-method counterpart of Eq. (10). It should be noted that $I_W(\ell)$ is position dependent whereas I_0 is the constant.
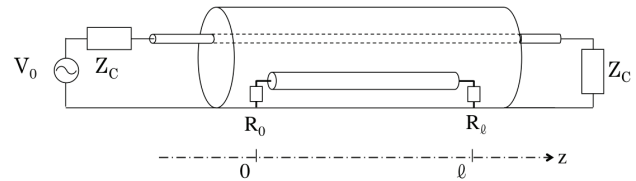

Figure 2: Schematic of one electrode and one wire.

MODELING AND ANALYSIS OF THE 3-50BT BPM

There are 14 BPMs in the "3-50BT", the beam transport line from the 3 GeV RCS to the 50 GeV main ring [3] [4]. The beam from the RCS is collimated by the collimator situated upstream of the 3-50 BT. Then the beam is bended down to fit the MR tunnel level.

BPM Structure

The schematic of the BPMs with inner diameter of 200 mm is shown in Fig. 3. The opening angle is 60 degree and the gap between the electrode and pipe wall is 1 mm. The loads, impedance matching transformers, are set at the downstream end of the electrodes.
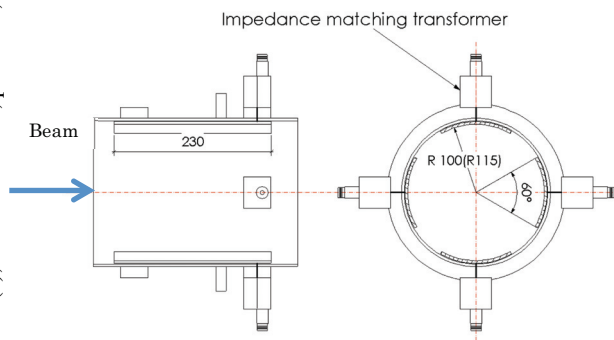
Figure 3: Schematic of the 3-50 BPM.

Numerical Calculation with BEM

The coefficients of capacitance and induction $\mathbf{q}$, and the coefficient of potential $\mathbf{p}_0$ are obtained using the boundary element method (BEM) with two-dimensional BPM geometry and the beam point charge [5]. These quantities are independent on a conductor's specific boundary condition. Using Eq. 9 we obtain the BPM response $V_{BPM}(\ell)/I_0$. In this calculation we use the same $\mathbf{q}$, while varying $\mathbf{p}_0$ depending on the beam position (x,y).

Comparison between the Beam and the Stretched-wire Measurement

The stretched-wire measurement described in [1] is modeled according to Figs. 2 and 3. In this case the capacitance matrix $\mathbf{C}$ is calculated with BEM including the wire. Therefore $\mathbf{C}$ for each wire position should be recalculated in contrast to the beam case where $\mathbf{q}$ does not depend on the beam position. The numerical calculation of four electrodes and the wire was performed without approximation used for Eq. (17).

The results are shown in Figs. 4 and 5. Red curves show the responses to the beam and the black dots the responses to the wire. The responses of the left and upper electrodes, $V_{BPM}(\ell)/I_W(\ell)$, are depicted in Fig 4 (a) and (b), respectively. The values are normalized to 1 at x = 0. The plots with the beam and the wire disagree at large x. On the other hand normalized results with Δ/Z in Fig. 5 agree with each other, within a few 10 μm in this calculation. This is with in our precision requirement.
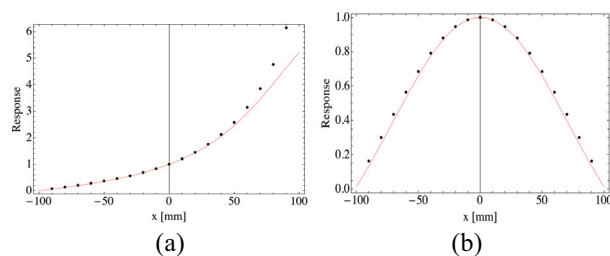

(a)   (b)

Figure 4: BPM responses to the beam or wire. (a) Left electrode response. (b) Upper electrode response. Red lines: the response to the beam. Black dots: the response to the wire.
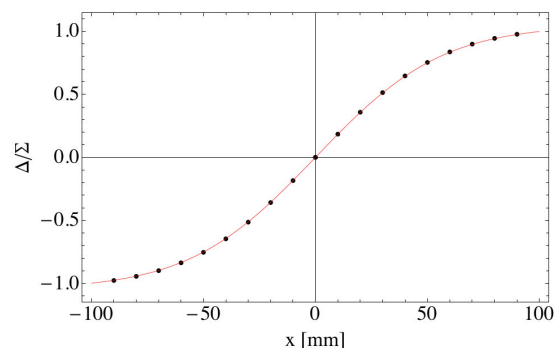

Figure 5: Δ/Σ response to the beam or wire. Red lines: the response to the beam. Black dots: the response to the wire.

CONCLUSION

The analysis method for multi-conductor pickups, in which a 2D electrostatic analysis including beams is performed in the transverse plane and the transmission line analysis is performed in the longitudinal direction, is applied to the BPM with four electrodes at the 3-50BT in J-PARC. The BPM responses to the beam and the wire are compared using this method. As expected with the simple electrostatic model, even in a practical structure as four electrodes the situation dose not change within a precision, in which Δ/Σ calculations agree although each electrode response differs each other. This result supports the validity of the stretched wire method for BPM calibration in this case.

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