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RESONANT SLOW EXTRACTION IN SYNCHROTRONS BY USING ANTI-SYMMETRIC SEXTUPOLE FIELDS

Ye Zou#, Jingyu Tang, Jianquan Yang
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049

Abstract

This paper proposes a novel method for resonant slow extraction in synchrotrons by using special anti-symmetric sextupole field, which can be produced by a special magnet structure and was proposed earlier [1]. The method has the potential in applications asking for very stable slow extraction from synchrotrons. Our studies show that the slow extraction at the half-integer resonance by using anti-symmetric sextupole field has some advantages compared to the normal sextupole field which is used in the usual third-order resonant extraction method. One of them is that it can work at a more distant tune from the resonance, so that it can weaken significantly the problem of the intensity variation of the extracted beam mainly caused by the ripples of magnet power supplies. The studies by both Hamiltonian theory and numerical simulations show that the stable region at the proximity of the half-integer resonance by anti-symmetric sextupole field is much smaller and flatter than the one by standard sextupole field at the third-order resonance, and the particles outside the region will be driven out in two possible directions in quite short transit time but with spiral steps similar as in the third-order resonance extraction. By gradually increasing the field strength, the beam can be extracted in a more homogeneous intensity than the usual third-order resonant method, in the means of both smaller intensity variation and spike in the beginning spill.

INTRODUCTION

Slow extraction in synchrotrons is the main extraction method for external target experiments in particle physics and nuclear physics and for proton or heavy ion therapy. It should provide stable beam in long time duration. Third-order resonant slow extraction [2] is usually used, and the principle is that one can intentionally excite the third-order resonance by controlling the distance between the working point and the resonance line and the sextupole strength to peel off gradually the particles from outer to inner in the beam emittance. This method can produce high quality stable beam and quite high extraction efficiency, if it is applied properly. However, this method also has some disadvantages, e.g. the tune should be moved very close to the resonance line before extraction and the stable region is very sensitive to the tune's stability. If we want to obtain the high stability of the beam intensity, very small ripple for the power supplies of magnets is required, especially when the inner core of the beam is extracted. The method that we propose here is also a resonant extraction method, but it uses a special sextupole magnet called anti-symmetric sextupole to extract beam slowly at the half-integer resonance. The anti-symmetric sextupole can produce anti-symmetric second-order field, which can decrease the stable region area of the accelerator effectively near the half-integer resonance and can be used to extract particles which are out of the stable region. Our studies show that anti-symmetric sextupole could produce quite strong perturbation to the beam dynamics in a larger tune range than the standard sextupole, which could benefit the slow extraction. The beam intensity extracted by this method is more stable and less affected by working point jitters than that by the usual third-order method. The septum is easier to set due to two-turn separation. Because the stable region by anti-symmetric sextupole is similar in shape with the initial distribution in phase space, this method has a potential application in the halo collimation of very large machines.

RESONANT SLOW EXTRACTION BY ANTI-SYMMETRIC SEXTUPOLE

In the current-free region of a magnet gap, the field can be derived from the scalar potential. For the normal sextupole, the magnetic field can be expressed as [3],

$$B_x = \left(\frac{d^2B_y}{dx^2}\right)_0 xy\ , \quad B_y = \frac{1}{2}\left(\frac{d^2B_y}{dx^2}\right)_0 (x^2 - y^2). \quad (1)$$

Under normal circumstances, the distribution of magnetic field produced by standard multipole magnets is fixed, e.g. magnetic field of even order is a symmetric distribution and magnetic field of odd order is an anti-symmetric distribution. However, if a pair of magnetic field shielding plates is placed at the centre of two pairs of poles to decouple the two halves of the magnets [1], one can obtain any order either symmetric or anti-symmetric multipole field distribution with relative field errors of about 1% in a good field region of rectangular shape. Figure 1 shows the layout and magnetic field distribution of the anti-symmetric sextupole.

The magnetic field produced by anti-symmetric sextupole magnet can be expressed as,

$$B_x = \left(\frac{d^2B_y}{dx^2}\right)_0 |x|y\ , \quad B_y = \frac{1}{2}\left(\frac{d^2B_y}{dx^2}\right)_0 (x^2 - y^2)\frac{|x|}{x}. \quad (2)$$

The effect of an anti-symmetric sextupole on a particle trajectory can be described in a simple way just by treating the magnet as a thin lens. For a positively charged particle in an anticlockwise ring in the normalized co-ordinates, we can obtain,

#zouye@ihep.ac.cn

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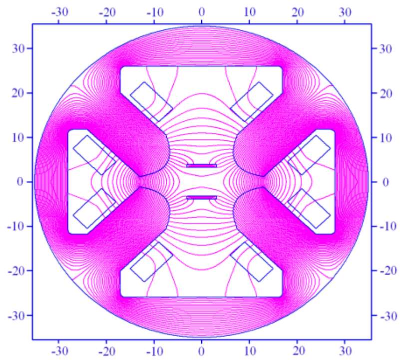

Figure 1: Layout and magnetic field distribution of the anti-symmetric sextupole.

$$\Delta X = \Delta Y = 0,$$

$$\Delta X' = S\left(X^2 - \frac{\beta_y}{\beta_x}Y^2\right)\frac{|X|}{X}, \quad (3)$$

$$\Delta Y' = -2S\frac{\beta_y}{\beta_x}|X|Y,$$

where S is the normalised sextupole strength,

$$S = \frac{1}{2}{\beta_x}^{3/2}\frac{l_s}{|B\rho|}\left(\frac{d^2B_y}{dx^2}\right)_0. \quad (4)$$

According to Eq. (3), we can see that an anti-symmetric sextupole couples the horizontal and vertical motions unless Y=0 and the strength of the coupling is proportional to the ratio of the vertical and horizontal betatron amplitude functions (β_y/β_x). For a horizontal extraction, one places the magnet where Y is much smaller than X, so we can ignore the influence of the vertical motion. Eq. (3) can be rewritten with a simplified form,

$$\Delta X = \Delta Y = \Delta Y' = 0, \Delta X' = S|X|X. \quad (5)$$

As we know, the standard sextupole may cause third-order resonance when the betatron tune is close to a third-integer. The theory based on Hamiltonian and numerical simulations show that the anti-symmetric sextupole will cause any order resonance but only second-order resonance can be used as resonant slow extraction. Now we will deduce the Kobayashi Hamiltonian [4] close to a half-integer.

In synchrotrons, from the general transfer matrix for normalized co-ordinates,

$$M_n = \begin{pmatrix} \cos 2\pi(nQ_x) & \sin 2\pi(nQ_x) \\ -\sin 2\pi(nQ_x) & \cos 2\pi(nQ_x) \end{pmatrix}, \quad (6)$$

we can obtain the transfer matrix of the particle after one and two turns, with a horizontal betatron tune close to a half-integer, i.e. Q_x=m+1/2+δQ, where m is integer and $|\delta Q|$<<1/2,

$$M_1 = \begin{pmatrix} -1 & -\varepsilon \\ \varepsilon & -1 \end{pmatrix}, \; M_2 = \begin{pmatrix} 1 & 2\varepsilon \\ -2\varepsilon & 1 \end{pmatrix}, \quad (7)$$

where ε replaces $2\pi\delta Q$ for brevity. From Eq. (7) it can be seen that a particle with exactly the half-integer resonant tune will return to its initial position every two turns. Now we will calculate the change of the position and divergence of the particle after two turns with the anti-symmetric sextupole as a perturbation by the linear addition and finally we can obtain the expressions, known as the spiral step and spiral kick:

$$\Delta X_2 = X_2 - X_0 = 2\varepsilon X_0{}'$$

$$\Delta X_2' = X_2' - X_0' = -2\varepsilon X_0 + 2SX_0|X_0|, \quad (8)$$

The time needed for two revolutions in the machine is very short compared to the spill time, so it can be safely used as the basic time unit. Thus the subscripts are no longer needed and Eq. (8) could be treated as a continuous function that is derived from a Hamiltonian. Finally we can get the Kobayashi Hamiltonian of the anti-symmetric sextupole close to half-integer resonant line by dealing with the absolute value sign and integrating the partial differentials,

$$X \geq 0 \quad H = \varepsilon(X^2 + X'^2) - \frac{2}{3}SX^3$$

$$X < 0 \quad H = \varepsilon(X^2 + X'^2) + \frac{2}{3}SX^3 \quad (9)$$

From the Kobayashi Hamiltonian above, we could calculate the phase-space map, see Figure 2. From Figure 2, we can see that the particle trajectories in the phase space with the anti-symmetric sextupole have a stable region and the motions of the particles out of it are unstable. The phase space area of the stable region is the acceptance of the accelerator and could also be regarded as the dynamic aperture. As the tune moves closer to the half-integer resonance or increasing the strength of the anti-symmetric sextupole, the area of stable region shrinks gradually just as in the case of the standard sextupole at the third-order resonance.

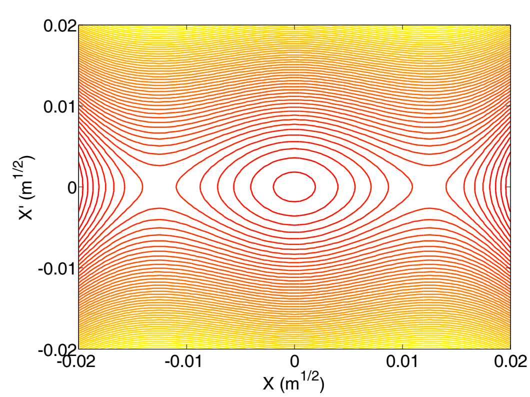

Figure 2: Phase-space map calculated from the Kobayashi Hamiltonian ($\varepsilon/S > 0$).

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As we know the area of the stable region produced by standard sextupole (SX) near the third-order resonance can be expressed as follows,

$$A_{SX} = 48\sqrt{3}\pi^2 \frac{(\delta Q)^2}{S^2} \quad (10)$$

Similarly, we found that the area of the stable region produced by anti-symmetric sextupole (ASX) can also be expressed by an empirical formula,

$$A_{ASX} = 5.91\pi^2 \frac{(\delta Q)^2}{S^2} \quad (11)$$

From Eq. (10) and Eq. (11) we can see that the area of the stable region produced by anti-symmetric sextupole is also proportional to $(\delta Q/S)^2$. So we can conclude that with the same sextupole strength and the same tune distance to the resonance, the area of stable region caused by anti-symmetric sextupole is about 1/14 of the one by standard sextupole. Or in other word, to have the same area of stable region and same sextupole strength, the tune distance to resonance in the case of anti-symmetric sextupole is $\sqrt{14}$ times the one in the case of standard sextupole, and this represents the advantage of less sensitivity to the tune variation due to the ripple of power supplies of magnets.

SIMULATION RESULTS

Multi-particle simulations with the self-made code by Matlab have been carried to show the feasibility of this method in realistic cases. A synchrotron lattice for proton therapy is used here and the initial beam distribution is called 2D Gaussian distribution which means both X and X' are Gaussian distributions. The strength of sextupole is 10 $m^{-1/2}$ and the initial rms emittance is 10 πmm mrad. Figure 3 shows the phase space distributions in the course of and after the resonance with the same strength and same tune distance to resonance. Figure 4 shows the difference of stable region area between simulation results and Hamiltonian method. Figure 5 shows the numbers of extracted particles in different conditions.

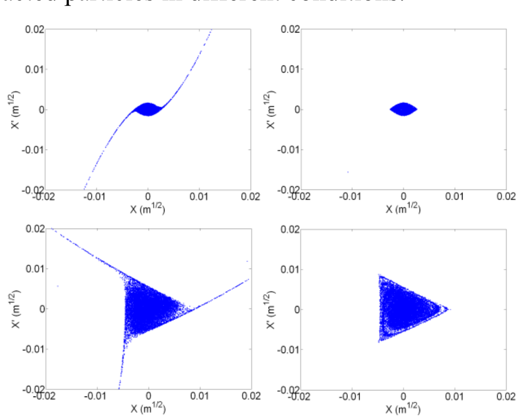

Figure 3: Phase space distributions in the course of and after the resonance (Upper: caused by anti-symmetric sextupole; lower: caused by standard sextupole).

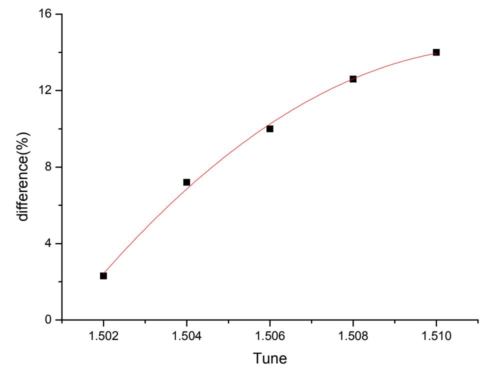

Figure 4: The difference of stable region area between simulation results and Hamiltonian method with respect to tunes.

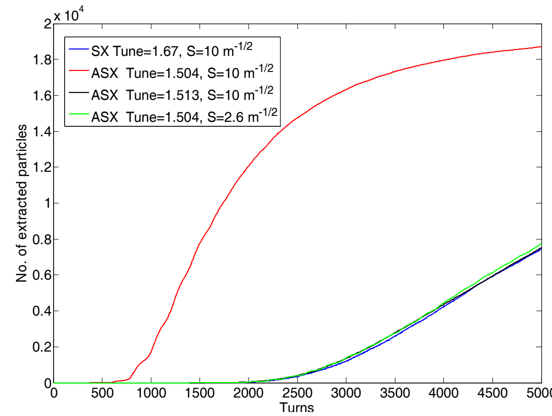

Figure 5: Numbers of extracted particles with respect to turns in different conditions.

From Fig. 3 to Fig. 5, one can find that the simulation results are consistent to the one defined by Eq. (11) or by the Hamiltonian method.

CONCLUSIONS

The simulation studies show that resonant extraction by using anti-symmetric sextupole has some very important advantages compared to the standard sextupole, e.g. more distant tune from the resonance, less sensitivity to the tune variation and better phase space distribution, which fit very well with the phase-space map calculated by Kobayashi Hamiltonian.

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