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SPACE CHARGE SIMULATION IN COSY USING THE FAST MULTIPOLE METHOD

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Abstract

A method is implemented in COSY Infinity that allows the computation of space charge effects of arbitrary and large distributions of particles in an efficient and accurate way based on a variant of the Fast Multipole Method (FMM). It relies on an automatic multigrid-based decomposition of charges in near and far regions and the use of high-order differential algebra methods to obtain decompositions of far fields that lead to an error that scales with a high power of the order. Given an ensemble of N particles, the method allows the computation of the self-fields of all particles on each other with a computational expense that scales as $O(N)$. Furthermore, the method allows the computation of all high-order multipoles of the space charge fields that are necessary for the computation of high-order transfer maps and all resulting aberrations. Space charge effects are crucial in modeling the latter stages of the six-dimensional (6D) cooling channel for the Muon Collider. Results of simulating the 6D cooling channel for the Muon Collider using the FMM method and other tools and improvements implemented for ionization cooling lattices are presented.

6D MUON COOLING

The demonstration of muon ionization cooling is one of the key challenges for the Muon Accelerator Program hosted at Fermilab [1]. For a muon collider, the 6D phase space volume of the muon beam must be reduced in order to accelerate it further for injection into a storage ring. The muon beam is produced by sending protons through a target, producing pions which in turn decay into muons with a large momentum spread. Ionization coolling is currently the only feasible method for cooling the beam within a muon lifetime of 2.2 μs. In order for a full 6D ionization cooling experiment to be constructed, a baseline lattice design has to be studied and selected based on detailed simulations.

To reduce the transverse emittance, the beam is strongly focused with high magnetic fields and subsequently sent through an absorber material to reduce overall momentum. The beam regains longitudinal momentum in RF cavities, resulting in an overall loss in transverse emittance. Longitudinal emittance reduction is achieved by shaping the absorbers into wedges and providing a bending magnetic field, generating a dispersion such that particles with higher energy are sent through more material. This results in an overall reduction of the energy spread [2, 3].

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The cooling channel is composed of many identical cells, containing: tilted alternating solenoids, wedge absorbers, and pillbox RF cavities running in TM010 mode, pictured in Figure 1.

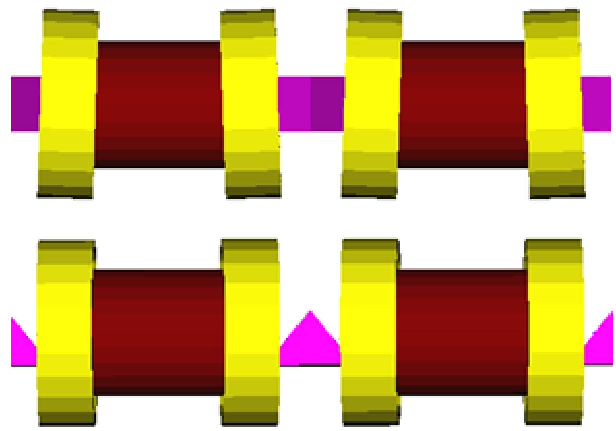

Figure 1: Side and top view of a cooling cell; yellow: tilted solenoids; red: rf cavities; magenta: absorber wedges.

IMPLEMENTATION IN COSY INFINITY

As the size of the muon beam decreases, the density of particles increases. This leads to Coulomb repulsion (space charge effects) in the beam becoming significant during the late stages of the cooling channel, when muon densities are of the order of 10^{12} particles per bunch. A method to achieve efficient and accurate calculation of these space charge effects based on variants of the Fast Multipole Method (FMM) has been implemented in COSY Infinity [4].

In addition, we have used techniques to continuously vary the density of beam elements to avoid small stepping and intropolation difficulties at boundaries.

FMM Algorithm

The fast multipole method for the Coulomb interaction between many charged particles divides an arbitrary charge distribution into small boxes with a hierarchical structure. It then computes the multipole expansions and local expansions of charges far from the observer to achieve a computation efficiency that scales with the number of particles, N, and computational errors scaling with a high power of the differential algebra order. The FMM algorithm is especially suited for beam dynamics simulations because of the efficiency and low computational error compared to other space charge algorithms.

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Other methods, such as the particle-particle interaction (PPI) and the particle in cell (PIC) method, while computationally efficient, incur excess error due to modeling and statistics. The PPI method uses macroparticles and assumes a particular distribution. The PIC method places the charge distribution onto a mesh, solves the Poisson equation on mesh points and interpolates between mesh points to find the field on each particle. Both of these methods suffer from an inability to precisely handle complicated charge distributions. This difficulty is overcome in the FMM by decomposing the charge distribution into boxes according to the charge density such that there are a pre-specified number of particles in each box to efficiently and accurately compute the multipole expansions [5,6].

Continuous Beam Elements

To track the beam, COSY uses an 8th order Runge-Kutta integrator with automatic step size control. Based on a pre-specified error bound, the integrator automatically selects the time step size necessary to accurately include all forces on the particles. This allows the user to select large initial timesteps, as the integrator will reduce the stepping at difficult areas in the cooling cell, which typically occur at the boundaries of elements. However, tracking particles through elements behaving as step functions makes things unnecessarily difficult for the integrator, resulting in very small timestepping at boundaries in order to reach the desired error bound. To improve the efficiency of the integrator, it is advantageous to continuously vary the density of the elements.

To accomplish this, we apply a function of the form:

$$D(z,a) = \frac{1}{1+e^{a(z_s-z)}} * \frac{1}{1+e^{a(z-z_e)}}, \tag{1}$$

where z_s and z_e are the start and end of the element along the axis of the cooling channel, and a is a constant that determines the sharpness of the element falloff. A plot of (1) is provided in Figure 2 for various fallof constants. This function has the property of being zero for positions outside and unity for positions inside the element while varying smoothly at the boundary [7].

For more complicated objects, such as the wedge absorbers in Figure 1, we cannot use (1) and simply define the elements start and end position in (x,y,z). Instead, we can recreate the object using immense overlapping spheres. To do so, we find a hypothetical centerpoint for a sphere of large radius (10^3m) such that the surface of the sphere will coincide with one of the elements' faces. For a large enough radius, the curvature of the sphere becomes negligible compared to the size of the beam element. If we have an object with n faces, with the centerpoint of the sphere of radius R describing the ith face centered at r_i, the density function is of the form (pictured in Figure 3):

$$D(\vec{r},a) = \Pi_{i=1}^{n} \frac{1}{1+e^{a(|\vec{r}-\vec{r_i}|-R)}}. \tag{2}$$

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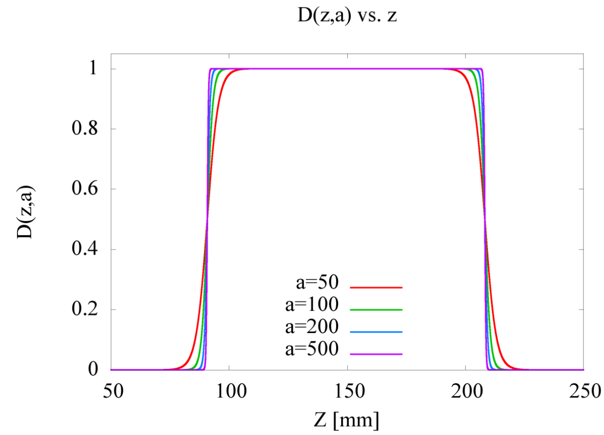

Figure 2: Plot of arbitrary rectangular beam element with smooth boundaries.

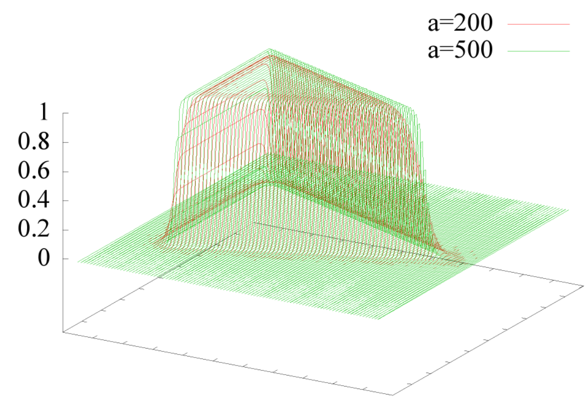

Figure 3: Plot of the profile of a wedge absorber for two different density falloffs.

COOLING CHANNEL SIMULATIONS

We have performed simulations of the late stages of a proposed cooling channel design in both COSY Infinity and G4Beamline, with and without space charge effects. G4Beamline is one of the de-facto codes for muon beam analysis and provides a PIC space charge calculation algorithm [8]. Stochastic effects and particle decays have yet to be implemented in COSY, so this functionality has been disabled to ensure a clear comparison between the codes.

This late-stage cooling channel consists of four 26 MV/m 650 MHz RF pillboxes, two 120 degree Lithium Hydride (LiH) wedge absorbers, and a solenoid arrangement producing a maximum on-axis field of 12.9 T. The simulations were run using a bunch of 10^4 muons and Figures 4 and 5 compare transverse emittance as well as transmission rate over approximately 60 cells. For the purposes of space charge, the bunch was given a total charge of 10^{12}.

The most significant difference in the two codes becomes evident in Figure 5. In the first few cells, COSY is showing a higher rate of particle loss. While this difference is only on the order of a few percent, it is due to a discrepancy in

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the codes' longitudinal behavior. There are known issues in COSY regarding longitudinal cooling that do not match G4Beamline and we are working to resolve.

Despite the differences in transmission, the behavior of the transverse emittance agrees well between the two codes. Here, the effect of the space charge is evident: As the beam size is reduced, the coulomb repulsion in the beam limits the reduction of the transverse emittance for both codes as well as the transmission rate, with COSY showing a much more drastic difference. COSY seems to predict a much larger space charge effect, leading to higher transverse emittance and lower transmission.

SUMMARY

The effect of coulomb repulsion on dense muon beams in late stage muon cooling channels is an important factor that limits emittance reduction. A method for the computation of space charge effects in an efficient and accurate way based on a variant of the Fast Multipole Method has been implemented in COSY Infinity. Simulations of these later stages in COSY show space charge is indeed a limiting factor for transverse emittance and transmission rate, an effect which is noticeably less significant in G4Beamline.

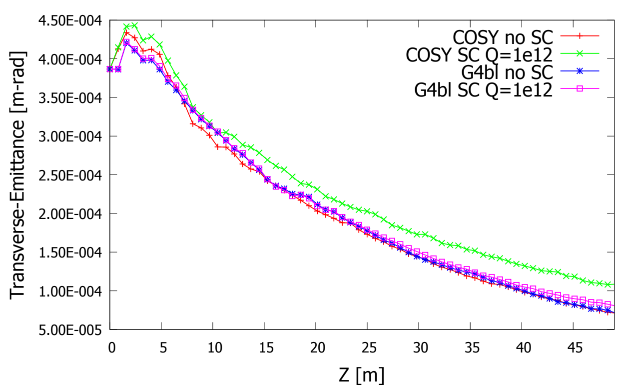

Figure 4: Plot of Transverse Emittance from G4Beamline and COSY Infinity with and without space charge. COSY is showing a much more significant effect of space charge when compared with G4beamline.

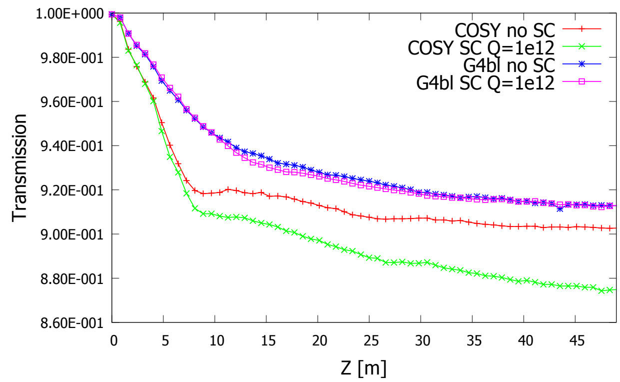

Figure 5: Plot of transmission rate through 60 cells of late stage muon cooling channel, with and without space charge. The difference in transmision is due to issues with longitudinal dynamics, as well as a larger space charge effect in COSY.

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