INFLUENCE OF TRANSITION RADIATION ON FORMATION OF A BUNCH WAKEFIELD IN A CIRCULAR WAVEGUIDE*

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Abstract

We investigate the field of a small bunch crossing a boundary between two dielectrics in a circular waveguide. It includes a "forced" field and a "free" one. The "forced" field is the field of the charge in the regular waveguide (it can contain the wakefield). The "free" field is connected with influence of the boundary (it includes transition radiation). Two cases are studied in detail: the bunch flies from vacuum into dielectric and from dielectric into vacuum. The behavior of the field depending on distance and time is explored analytically and numerically. Influence of the border on formation of the wakefield in the dielectric region is analyzed. It is found as well that a large quasi monochromatic radiation is generated in the vacuum region when the charge flies from dielectric into vacuum with certain velocity.

INTRODUCTION

Investigation of a field of a particle bunch in a waveguide loaded with a dielectric is important for the wakefield acceleration technique and for other problems in the accelerator physics. One of questions consists in the influence of boundary on the wave field when the bunch flies into (or from) the dielectric structure.

In present paper we consider the electromagnetic field (EMF) generated by a point charge particle q moving in a metal circular waveguide of radius a along its axis (z-axis) through the interface ($z = 0$) between homogeneous isotropic dielectrics with permittivity ε_1 ($z < 0$) and ε_2 ($z > 0$). The media have no dispersion but small losses are taken into account. The charge moves uniformly with a velocity $\vec{V} = c\beta\vec{e}_z$ and intersects the boundary at the moment $t = 0$.

Note that analogues problem with a border between vacuum and a cold plasma was analysed in papers [1–3]. However, Cherenkov radiation (CR) is absent in such situation. Therefore this problem and the one considered here vary radically. The case of semi-infinite waveguide with dielectric was studied in [4]. But such problem differs essentially from our problem as well because some important physical effects cannot exist in the model with metallic wall at the waveguide end [4].

METHODS OF ANALYSIS

The analytical solution of the problem is found traditionally [1] as an expansion into a series of eigenfunctions of the transversal operator. This expression is a decomposition in an infinite series of normal modes. It has two summands. The first one gives the field in the regular waveguide with homogeneous filling. V.L. Ginzburg [5] called it the 'forced' field. It contains CR if the charge velocity exceeds the Cherenkov threshold. The second summand is connected with the influence of the boundary and gives so-called 'free' field. It includes transition radiation (TR).

We investigate the exact solution with analytical and computational methods. Analytical research is an asymptotic investigation using the steepest descent technique. Computations are based on original algorithm using certain transformation of the integration path. Such approaches were applied as well in some papers concerning both boundless media [6] and problems with interface between two media [7, 8] including the case waveguide partially filled with cold plasma [3].

Here we omit all cumbersome analytical transformations. We give only some results of computations and physical summary for two cases: the particle is flying from vacuum ($\varepsilon_1 = 1$) into dielectric ($\varepsilon_2 > 1$) and the inverse case of flying from dielectric ($\varepsilon_1 > 1$) into vacuum ($\varepsilon_2 = 1$).

Next we present the behaviour of the first mode of the longitudinal component E_z of the whole field and the forced one in vacuum and dielectric for different velocities of the charge motion β and at different moments in two cases mentioned above.

THE CASE OF FLYING FROM VACUUM INTO DIELECTRIC

Figure 1 illustrates the case of flying from vacuum into dielectric ($\varepsilon_1 = 1, \varepsilon_2 = 5$, the Cherenkov threshold is $\beta_{C2} = 1/\sqrt{\varepsilon_2} \approx 0.447$). If $\beta < \beta_{C2}$ the whole field is TR everywhere except some small neighbourhood of the charge (Fig. 1 *a, b, c*). That is also true for all velocities in the vacuum area. If $\beta > \beta_{C2}$ Cherenkov radiation is generated in dielectric, and the whole field here is a combination of CR and TR.

In the domain $0 < z < z_2 = ct/(\beta\varepsilon_2)$ the forced wave field (so-called "wakefield") is compensated by some part of the free field which is equal to the wakefield taken with an opposite sign. Note that the point z_2 is determined with the group velocity $v_{g2} = c/\beta\varepsilon_2$ in a regular waveguide. The compensation domain is large for the velocities closed to the Cherenkov threshold (Fig. 1 *d,e, f*), and it is reduced with increase in β (Fig. 1 *g,h,i*).

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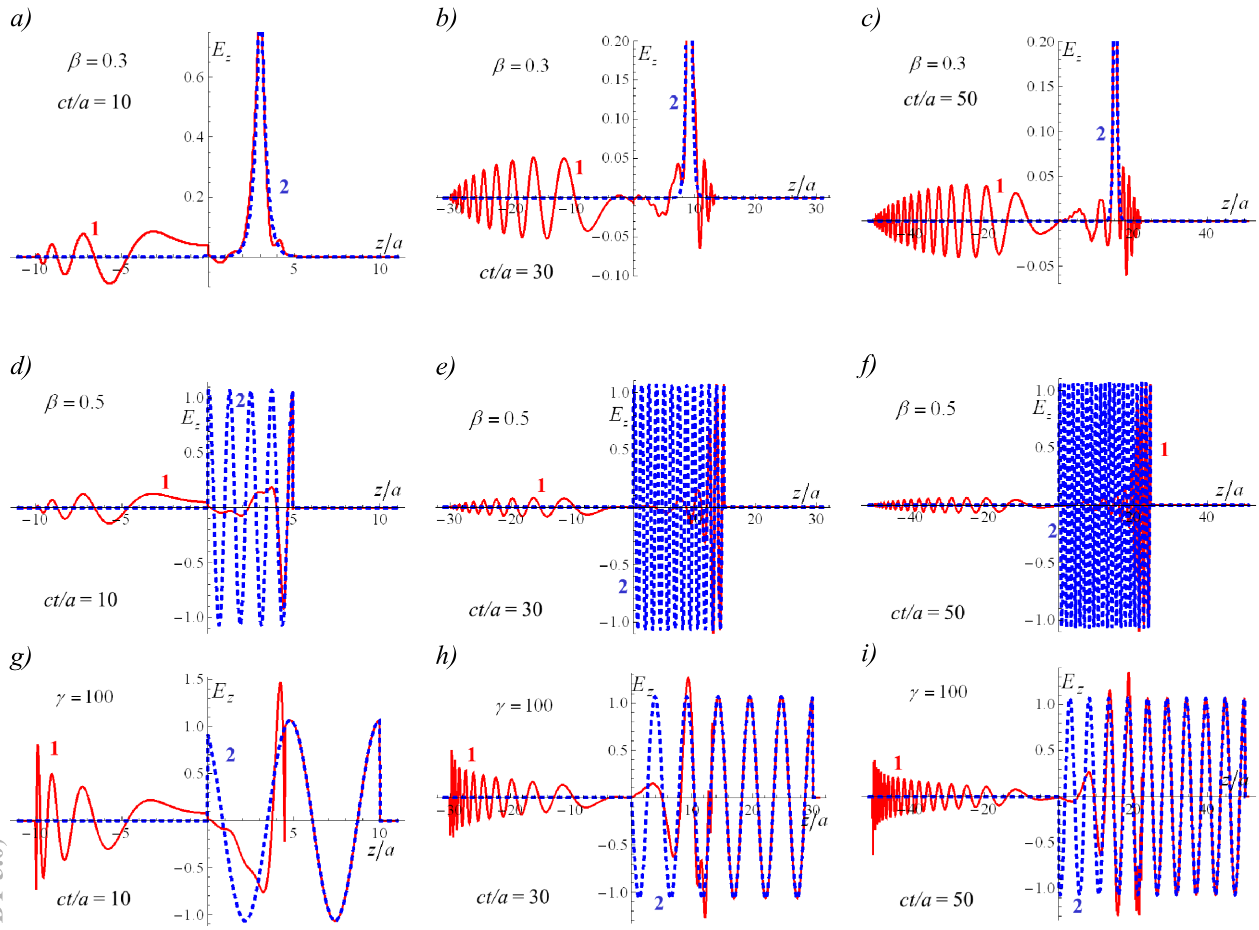

Figure 1: The case of flying from vacuum into dielectric. Dependence of longitudinal component E_z (kV/m) of the first mode of the whole field (continuous line 1) and the forced one (dashed line 2) on distance z/a for different dimensionless times *ct/a* and velocities β (or $\gamma = 1\big/\sqrt{1-\beta^2}$); $q=-1\,\mathrm{pC}$, $\varepsilon_1 = 1$, $\varepsilon_2 = 5$, $a = 5\,\mathrm{mm}$. Sighting point is on the waveguide axis.

As a result, there is the area where the wave field practically coincides with the wakefield in the regular waveguide (at least, at $z_2 < z < c\beta t$) and the area where the boundary influence is principal ($z < z_2$). It is important that the first area is much more than the second one if ε_2 takes on a large value.

Note as well that there is some increase and concentration of TR near the "wave front" $z_f = ct$ in vacuum for the case of the ultra-relativistic particle (Fig.1*g,h,i*), and the fields in the vacuum and dielectric are getting comparable.

THE CASE OF FLYING FROM DIELECTRIC INTO VACUUM

The case of flying from dielectric into vacuum ($\varepsilon_1 = 5$, $\varepsilon_2 = 1$) is presented in Figure 2. If the charge velocity does not exceed the Cherenkov threshold $\beta_{C1} = 1\big/\sqrt{\varepsilon_1}$ the whole field is TR in dielectric and in vacuum everywhere except the charge vicinity (Fig. 2 *a,b,c*).

If $\beta > \beta_{C1}$ CR emerges in dielectric. The whole wave field here consists of CR, reflected wave of CR (in the area $|z| < z_1 = ct/(\beta\varepsilon_1)$) and TR. One can find that reflected wave of CR is relatively small in comparison with CR (wakefield) at

$$\beta_{C1} < \beta < \beta_{CT1} = 1\big/\sqrt{\varepsilon_1 - 1}$$

(Fig. 2,*d,e,f*). At $\beta > \beta_{CT1}$ the reflected field is significant in some domain near the border (Fig. 2 *g, h, i*).

In vacuum, the whole wave field consists of TR and transmitted wave connected with CR. Note that reflected and transmitted wave of CR can be called Cherenkov transition radiation (CTR) [7, 8]. CTR exists at $\beta_{C1} < \beta < \beta_{CT1}$ in the area

$$z < z_3 = ct\sqrt{\left|1 - \beta^2(\varepsilon_1 - 1)\right|}\Big/\beta$$

(Fig. 2 *d,e,f*). At $\beta > \beta_{CT1}$ this field decreases exponentially with the distance from the boundary.

So, Cherenkov radiation escapes from dielectric into a vacuum at $\beta_{C1} < \beta < \beta_{CT1}$. CTR is dominant for these velocities in some large area. It should be noted that similar effect takes place at the boundary between vacuum and "left-handed medium" where the CTR has "reversed" directivity [7, 8].

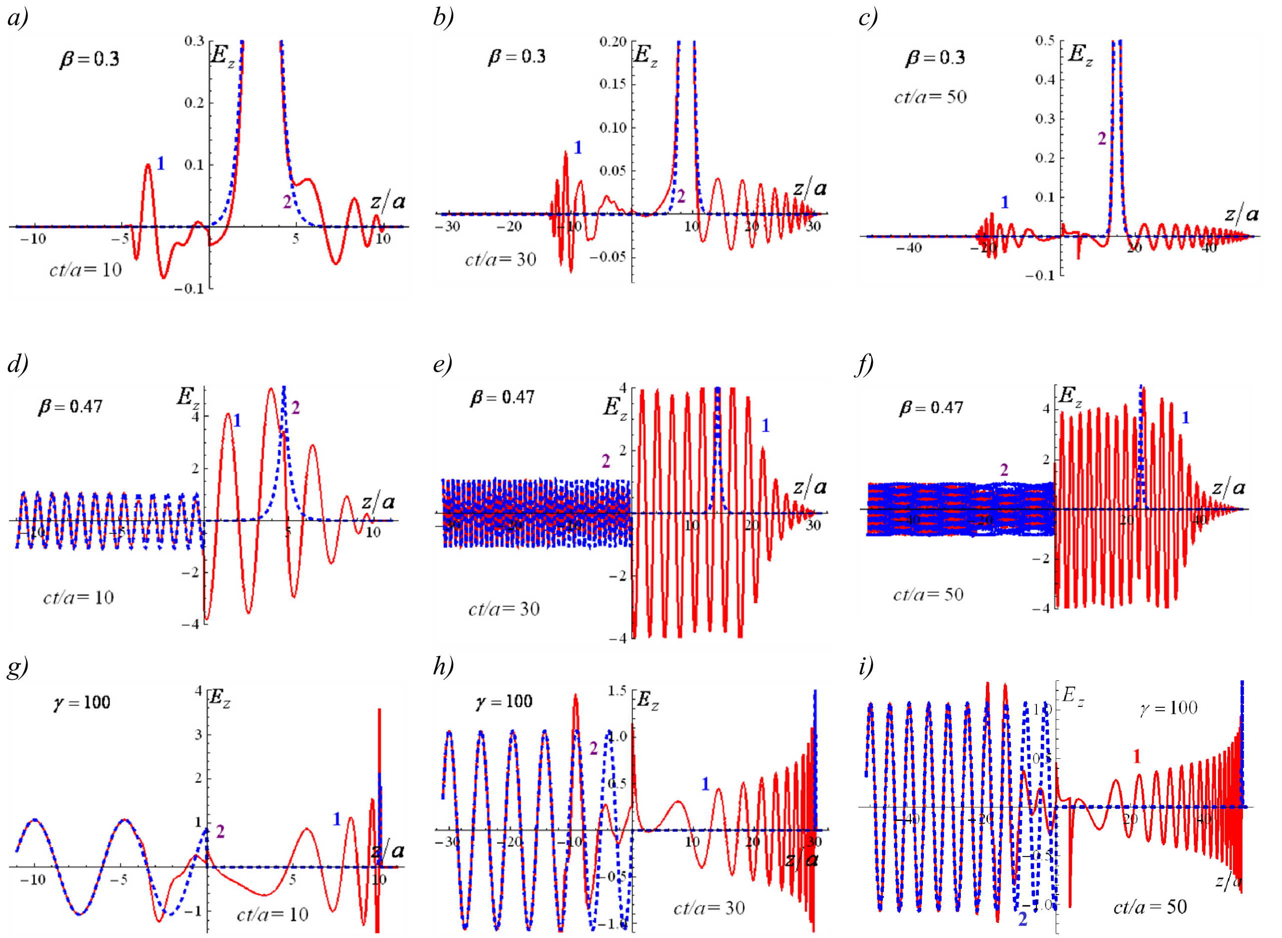

Figure 2: The case of flying from dielectric into vacuum. The same as in Fig.1 for $\varepsilon_1 = 1, \varepsilon_2 = 5$.

CONCLUSION

We can see that when the charge flies from vacuum into dielectric there is the area where the wave field practically coincides with the wakefield in the regular waveguide and the area where the boundary influence is principal. The first area is large if the dielectric permittivity takes on a large value. This conclusion is important for the wakefield acceleration technique.

When the charge flies from dielectric into vacuum with certain velocity a large quasi monochromatic radiation is generated in the vacuum region. This conclusion is of interest for development of new methods of generation of electromagnetic radiation which is similar to maser one.

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