SERPENTINE ACCELERATION IN SCALING FFAG

E. Yamakawa*, J-B. Lagrange, R. Nakano, T. Planche†
Graduate school of Engineering, Kyoto University, Japan
Y.Ishi, T. Uesugi, Y. Kuriyama, B. Qin, Y. Mori,
Kyoto University Research Reactor Institute, Kumatori, Japan
K.Okabe, I. Sakai, Fukui University, Fukui, Japan

Abstract

A serpentine acceleration in scaling FFAG accelerator has been examined. In this scheme, high-energy and high-current beam can be obtained in non-relativistic energy region. Longitudinal hamiltonian is also derived analytically.

INTRODUCTION

Recently, particle accelerators have been widely used not only for particle physics [1] but also for many applications such as nuclear power engineering [2]. Requests for high beam power accelerator to produce intense secondary particle beams, in particular, are increasing. Linear accelerators have been considered as a proper candidate so far. However it is expensive to construct and power comsumption. An Alternative candidate is an FFAG (fixed-field alternating gradient) accelerator [3]. Since the guiding magnetic field is static, the acceleration repetition rate only depends on the capability of rf system. Thanks to this advantage, not only producing high-energy and high-intensity beam but also accelerating short-life particles, such as muons, can be achieved with high repetition rate.

There are two types of FFAG accelerators; the non-scaling type and the scaling type. The scaling FFAG accelerator ring is composed of non-linear magnetic fields so that the betatron tune is constant for every particle momentam, contrary to the non-scaling FFAG accelerator.

Various methods of beam acceleration have been proposed. In scaling FFAG accelerators, the beam acceleration is usually realized with frequency modulation of the rf system. With this acceleration scheme, the acceleration repetition rate is limited by rf voltage and changing speed of rf frequency. Another scheme is the stationary bucket acceleration [4] where rf frequency is fixed. Since cw operation becomes possible in this scheme, a large current beam can be obtained. In stationary bucket acceleration, however, the total acceleration energy gain is limited by the bucket height, which limits this acceleration to the relativistic energy region [5]. On the other hand, in non-scaling FFAG accelerator, beam acceleration with fixed rf frequency, called serpentine acceleration [6], has been considered. The beam has to cross the transition energy during serpentine acceleration, that is to say the slippage factor has to change sign. However, in order to minimize orbit shift during acceleration, momentum compaction is chosen very small in non-scaling FFAGs. Consequence, only relativistic particles ($\gamma \gg 1$) can be accelerated in this scheme.

Relativistic energy is suited for both type of FFAG accelerators in acceleration with fixed rf frequency. However, serpentine acceleration can be also applied to the scaling FFAG accelerator. In this case, high-energy and high-current beam can be obtained in the non-relativistic energy region as well.

In this paper, the longitudinal hamiltonian in scaling FFAG accelerator is derived analytically, and the features of serpentine acceleration and some applications based on serpentine acceleration are also presented.

LONGITUDINAL HAMILTONIAN IN SCALING FFAG

In cylindrical coordinates, the magnetic field in scaling FFAG accelerator has the form:

$$B_z(r, z=0) = B_0 \left(\frac{r}{r_0}\right)^k, \tag{1}$$

where r is the radial coordinate with respect to the center of the ring, B_0 is the magnetic field at r_0, k is the geometric field index, and z is the vertical coordinate. The closed orbits for different momenta are given by

$$r = r_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}, \tag{2}$$

where r_0 is the radius of the closed orbit at the momentum of P_0.

In longitudinal particle dynamics with constant rf frequency acceleration in the scaling FFAG accelerator, the phase discrepancy per revolution $\Delta\phi$ is written by

$$\Delta\phi = 2\pi(f_{rf} \cdot T - h), \tag{3}$$

where h is the harmonic number, f_{rf} is the rf frequency and T is the revolution period of a non-synchronous particle. Equation3 becomes

$$\frac{T}{T_s} = \left(\frac{r}{r_s}\right) \Big/ \frac{P/E}{P_s/E_s} = P_s^{1-\alpha} \frac{E}{E_s} P^{\alpha-1}, \tag{4}$$

where T_s is the revolution period of a synchronous particle, r_s is the reference radius, α is the momentum compaction factor and E_s is the stationary energy. Below the transition

* yamakawa@post3.rri.kyoto-u.ac.jp
† Moved to Triumph, Vancouver, Canada

energy, there is a stationary energy E_{s1}. As the particle energy increases across the transition energy, there is another stationary energy E_{s2}. The phase difference becomes

$$\Delta\phi = 2\pi h\Big(\frac{P_s^{1-\alpha}}{E_s} E P^{\alpha-1} - 1\Big). \quad (5)$$

Now we exchange $\Delta\phi/2\pi$ and $d\phi/d\theta$ to derive the phase and energy equation of longitudinal motion,

$$\frac{d\phi}{d\theta} = h\Big(\frac{P_s^{1-\alpha}}{E_s} E P^{\alpha-1} - 1\Big) \quad (6)$$

$$\frac{dE}{d\theta} = \frac{eV_{rf}}{2\pi}\sin\phi, \quad (7)$$

where V_{rf} is the rf voltage per turn and θ is an azimuthal angle in the machine. We introduce the energy variable E canonically conjugate to the coordinate variable ϕ. Equation.6 and 7 derive the longitudinal hamiltonian:

$$H(E,\phi;\theta) = h\Big(\frac{1}{\alpha+1}\frac{\sqrt{E^2-m^2}^{\alpha+1}}{E_s\sqrt{E^2-m^2_s}^{\alpha-1}} - E\Big) + \frac{eV_{rf}}{2\pi}\cos\phi, \quad (8)$$

where m is rest mass of the beam particle.

Phase space in scaling FFAG

When both stationary energies E_{s1} and E_{s2} are far from each other, two stationary buckets are separated as shown in Fig.1. Approaching the two stationary energies each other, a channel between two stationary buckets appears as shown in Fig.2.

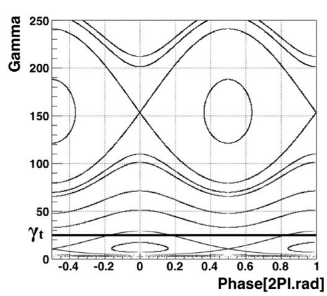

Figure 1: Longitudinal phase space. Two stationary buckets are separated from each other.

Minimum RF voltage to make serpentine channel

The minimum rf voltage to make the acceleration channels is derived from Eq.8. As shown by Fig.3, the limiting serpentine channel goes through two unstable fixed points where $H(E_{s1},\pi)$ equals $H(E_{s2},0)$.

$$H(E_{s1},\pi) = H(E_{s2},0). \quad (9)$$

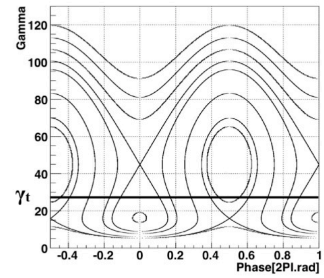

Figure 2: Longitudinal phase space near the transition energy. There are two stationary buckets which are close to each other.

From Eq.9 and the relation between E_{s1} and E_{s2};

$$E_{s1}P_{s1}^{\alpha-1} = E_{s2}P_{s2}^{\alpha-1}, \quad (10)$$

the minimum rf voltage is derived;

$$V_{rf} = \pi h\left[\frac{1}{\alpha+1}\left(\frac{P_{s1}^2}{E_{s1}} - \frac{P_{s2}^2}{E_{s2}}\right) + (E_{s2}-E_{s1})\right]. \quad (11)$$

Equation11 shows that once the k value and the two stationary energies are given, the minimum rf voltage to realize the serpentine acceleration can be calculated.

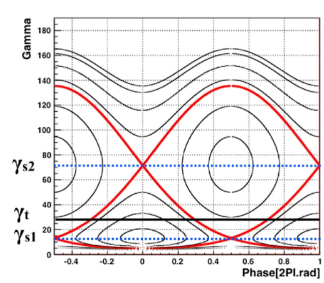

Figure 3: Two separatrix are close to each other, where γ_{s1} corresponds to E_{s1} and γ_{s2} corresponds to E_{s2}.

APPLICATION

With serpentine acceleration in scaling FFAG accelerator, a high current beam is generated by cw operations. And also, total energy gain is larger compared with the single stationary bucket acceleration. Furthermore, a fast acceleration can be achieved. This is desirable for unstable particles such as muons. In this section, some examples of applications are shown; proton, muon and electron accelerator.

Parameters of a proton accelerator are summarized in Table.1 where h is harmonic number. As shown in Fig.4, injection kinetic energy is 380 MeV in longitudinal tracking. Injection phase is 0 rad. Final kinetic energy is 1.1 GeV.

Table 1: Proton case

Stationary Kinetic Energy	891 MeV
Mean radius (Stationary energy)	10 m
k value	2
rf voltage/turn	6 (h=1) MV
rf frequency	8.58 (h=1) MHz

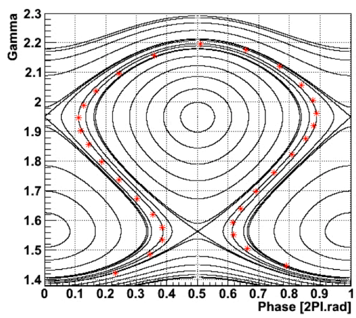

Figure 4: Proton beam tracking in longitudinal phase space. Hamiltonian contours are superimposed.

Parameters of a muon accelerator are summarized in Table.2. As shown in Fig.5, injection kinetic energy region is from 45 to 46 MeV in longitudinal tracking. Injection phase range is from 2.2 to 2.5 rad. Final kinetic energy region is from 1755 to 1955 MeV. In this scheme, a low energy muon beam can be accelerated to high energy within a few turns. It shows a good expectation for the neutrino factory or muon collider, in the future.

Table 2: Muon case

Stationary Kinetic Energy	845 MeV
Mean radius (Stationary energy)	10 m
k value	6
rf voltage/turn	250 (h=1) MV
rf frequency	4.7 (h=1) MHz

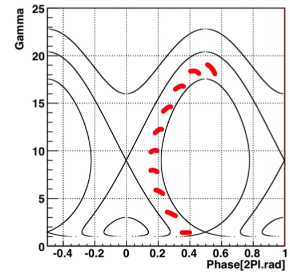

Figure 5: Muon beam tracking in longitudinal phase space. Hamiltonian contours are superimposed.

Parameters of an electron accelerator is summarized in Table.3. As shown in Fig.6, injection phase range is from 2.5 to 2.6 rad in longitudinal tracking. Final kinetic energy is over 10 MeV.

Table 3: Electron case

Stationary Kinetic Energy	4.69 MeV
Injection energy	200 keV
Mean radius (Injection energy)	0.37 m
k value	4.45
rf voltage/turn	680 (h=1) kV
rf frequency	75 (h=1) MHz

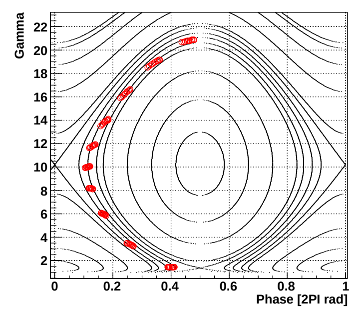

Figure 6: Electron beam tracking in longitudinal phase space. Hamiltonian contours are superimposed.

SUMMARY

In order to obtain high power beam with high repetition rate, serpentine acceleration with constant rf frequency has been proposed for the scaling FFAG accelerator. Longitudinal hamiltonian has been derived analytically. In order to demonstrate the serpentine acceleration in scaling FFAG, the experiments will be taken soon.

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