# STRAIGHT SCALING FFAG

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#### Abstract

Recent developments in scaling fixed field alternating gradient (FFAG) accelerators have opened new ways for lattice design, with straight sections. An experiment to study zero-chromatic straight sections is under progress at KURRI.

### **INTRODUCTION**

Until today, scaling FFAG accelerators were only designed in a ring shape. But new criterion of the magnetic field configuration satisfying the scaling condition even for straight FFAG beam line has been recently found. It is indeed possible to guide particles with no overall bend in scaling FFAGs, with a field law in the mid-plane following [1, 2]

$$B_z = B_0 e^{m(x-x_0)} \mathcal{F}(s), \tag{1}$$

with x and s the horizontal and longitudinal coordinates, respectively,  $B_0 = B_z(x_0)$ ,  $\mathcal{F}$  an arbitrary function of s and m the normalized field gradient

$$m = \frac{1}{B} \frac{dB}{dx}.$$
 (2)

To verify and study this new field law, an experiment is under progress at Kyoto University Research Reactor Institute (KURRI).

### **GOAL OF THE EXPERIMENT**

The goal of the Straight Experiment is to measure the phase advance for different energies in a straight scaling FFAG cell. In order to do it, the Twiss parameters as well as position and angle of the beam will be measured at the exit of the Straight Cell when it is launched off its reference trajectory. The linear transfer matrix of the line is

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}} \left(\cos\psi + \alpha_0 \sin\psi\right) & a_{12} \\ \frac{(\alpha_0 - \alpha_1)\cos\psi - (1 + \alpha_0\alpha_1)\sin\psi}{\sqrt{\beta_1\beta_0}} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ 0 \end{pmatrix},$$

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with  $x_1$  and  $x'_1$  the distance and the angle to the reference trajectory at the exit of the cell, respectively.  $x_0$  is the distance to the reference trajectory at the entrance of the cell. No angle with the reference trajectory is added at the entrance of the cell.  $\alpha_0$  and  $\beta_0$  are the Twiss parameters at the entrance of the cell, while  $\alpha_1$  and  $\beta_1$  are the Twiss parameters at the exit of the cell.  $\psi$  is the linear phase advance. The system of equations to be solved is then

$$\begin{cases} \sqrt{\frac{\beta_1}{\beta_0}} \left(\cos\psi + \alpha_0 \sin\psi\right) = \frac{x_1}{x_0} \\ \frac{(\alpha_0 - \alpha_1)\cos\psi - (1 + \alpha_0\alpha_1)\sin\psi}{\sqrt{\beta_1\beta_0}} = \frac{x_1'}{x_0} \end{cases},$$
(3)

and we get

$$\begin{cases} \sin\psi = \sqrt{\frac{\beta_0}{\beta_1}} \frac{1}{x_0(1+\alpha_0^2)} \left( (\alpha_0 - \alpha_1) x_1 - \beta_1 x_1' \right) \\ \cos\psi = \sqrt{\frac{\beta_0}{\beta_1}} \frac{1}{x_0(1+\alpha_0^2)} \left( (1+\alpha_0\alpha_1) x_1 + \alpha_0\beta_1 x_1' \right) \end{cases}$$
(4)

So we can calculate the linear phase advance  $\psi$  by:

$$\tan \psi = \frac{(\alpha_0 - \alpha_1)x_1 - \beta_1 x_1'}{(1 + \alpha_0 \alpha_1)x_1 + \alpha_0 \beta_1 x_1'}.$$
 (5)



Figure 1: scheme of the emittance measurement system to be installed after the Straight Cell.

#### LAYOUT OF THE EXPERIMENT

For the Straight Experiment, the injector of the 150 MeV FFAG complex is used, a Linac delivering H- particles of 7 MeV and 11 MeV in the KURRI FFAG complex. The prototype of the straight scaling FFAG cell is installed in

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Figure 2: Layout of the Straight Experiment.

the beam line between the Linac and the FFAG Main Ring (see Fig. 2). Two collimators will be installed before the Straight Cell to set the Twiss parameters at the entrance. An emittance measurement system will be added after the Straight Cell to determine the exit parameters of the beam. This system is similar to the "slit-grid method" [3], but the grid is replaced by a fluorescent screen (see Fig. 1). This system has been successfully checked both for horizontal [4] and vertical [5] emittance in the H- beam line at the exit of the Linac.

Since each energy has a different reference trajectory (corresponding term to a closed orbit in terms of circular machines), the cell is able to move horizontally to match the different trajectories without changing the magnetic field. Bellows before and after the cell allows the vacuum chamber to move along with the magnets without breaking the vacuum.

## STRAIGHT CELL DESIGN

The prototype for the straight scaling FFAG cell is a FDF triplet. The parameters of the cell are summarized in Tab. 1. The pole shape has been designed with TOSCA code [6] (see Fig. 3 and 4). A "C" type magnet has been chosen to have an easier access to the pole for field measurement.

Table 1		Parameters	of	the	Stra	iş	ght	Cel	1
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Туре	FDF
<i>m</i> -value	$11 {\rm ~m^{-1}}$
Total length	2.4 m
Length of F magnet	15 cm
Length of D magnet	30 cm
Max. B Field	0.35 T
Horizontal phase advance	72 deg.
Vertical phase advance	103 deg.



Figure 3: Magnetic field in F magnet (Tosca model).



Figure 4: Magnetic field in D magnet (Tosca model).

# TRACKING IN FIELD MAP

Tracking step by step using Runge Kutta integration has been done in field maps from TOSCA [6]. The reference trajectories of 7 MeV and 11 MeV are shown in Fig. 5. The local phase advances in the Straight Cell are plotted in Fig. 6. The results show a constant horizontal phase advance, since the local *m*-value in the field map is constant (see Fig. 7) and a slightly changing vertical phase advance, typical of the scaling FFAGs, because the fringe fields do not scale due to the pole shape. It would be possible to adjust the length of the fringe field by adjusting the effective field boundaries [7], but rectangular magnets have been kept for economical reasons. The betafunctions in the system are shown Fig. 8.



Figure 5: Reference trajectories in the Straight FFAG line.



Figure 6: Horizontal (plain) and vertical (dotted) local phase advances versus kinetic energy in the Straight FFAG line.



Figure 7: Local m value versus horizontal abscissa in the Straight FFAG line.

# SUMMARY

To verify the straight scaling field law, an experiment is in progress at KURRI. Design and tracking in field maps from TOSCA [6] has been done, the magnets have been built and delivered (see Fig. 9). Field measurement of the magnets is underway and the results of the experiment are expected before the end of this year.

#### REFERENCES

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Figure 8: Horizontal (plain red) and vertical (dotted purple) betafunctions in the Straight FFAG line.



Figure 9: Picture of the straight scaling FFAG magnets prototype.

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