SYMMETRY–BASED DESIGN FOR BEAM LINES*

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Abstract

In this paper we present the symmetry–design concept based on symbolic computations for the corresponding beam line propagator. The suggested approach can be realized in both exact and approximate forms of the symmetry terms.

INTRODUCTION

Last years there appear some papers devoted to group-theoretical approach for the design of magnetic optical systems. It is known that many problems of similar systems can be formulated in the terms of corresponding symmetry conditions (see,i e. [1, 2, 3]. The modern trend in accelerator physics research requires advanced beam facilities based on accurate elaboration of similar facilities projects. Thus and so the main attention is paid to two the following problems. The first of them is connected with selection of optimal structures implementing desired goals and the second — with engineering development. On this step the designer rejects incorrect structures, for example, sensitive to different kind of unwanted effects. In the present paper we suggest an uniform approach based on symmetry theory methods. On the first step this approach can be applied for working out the desired beam line structure in detail. On the second step this approach can be used for identification of those structures, which "guarantee" (with a given degree of accuracy for required restrictions) the desired properties. One can separate accelerator facilities on two groups. The first family consists on sufficiently "small" systems, supporting some particular problems: matching channels, focusing systems, "invisible" insertions and so on. The facilities of the second family can be presented as complex systems. In this case one can present such type of the system as a sequence of subsystems compatible with each other. It is well known that there is a wide class of symmetries, which support the process of control systems for particle beam facilities. The first class of the symmetries (the class of intrinsic symmetries) are generated by physical principles. As an example of similar symmetry it should be denoted the symplectic property for Hamiltonian systems. This property brings about necessity of special integration methods (numerical and/or analytical). The second class of symmetries fulfills two following requirements: the constrained conditions can be formulated in the form of some symmetries and these conditions can be varied according to a concrete problem without mathematical tools changing.

In the present paper we attend exactly to the second type of symmetries, because the first class of symmetries leads only to limitations on parameters of corresponding integration methods (see, for example, [2]). The second class of symmetries leads to some control parameters restrictions. In particular, this can reduce to control parameters number. It is necessary also to mention that appropriate symmetries can remove some aberrations of higher order (see, i. e. [1, 2, 4]). The suggested approach is based on the matrix formalism [2], which allows to obtain necessary conditions in a sufficiently simple form.

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SYMMETRIES APPLIED TO THE BEAM LINE DESIGN

The particle motion in a beam line can be written in the form of a vector ordinary differential equation

$$\frac{d\mathbf{X}}{ds} = \mathbb{P}^{\text{syst}}(s)\mathbf{X} + \text{nonlinear terms}.$$

As an example let us consider the well known problem of "Russian quadruple" (or "rotation quadruple") construction, which implements conversion a circular section (in coordinate or impulse subspaces) to circular section correspondingly. Using symmetry concept one can say about conservation of rotating symmetry for a beam "portrait". This condition can be written (for example, in coordinates subspace) in the following form

$$\mathcal{T}_\alpha \circ \mathfrak{N}_0 = \mathfrak{N}_0 \overset{\mathcal{M}(s_t|s_0)}{\Longrightarrow} \mathcal{T}_\alpha \circ \mathfrak{N}_t = \mathfrak{N}_t, \tag{1}$$

where $\mathfrak{N}_0$ — an initial beam portrait in the configuration space $\{x, y\}$, $\mathfrak{N}_t$ — the corresponding image on the target under the following map $\mathcal{M}(s_t|s_0)$, generated by the system under study, $\mathcal{T}_\alpha$ — the rotation map in the transverse configuration space under an arbitrary α around the optical axis of the beam. Using (1) one can write the commutating equality for $\mathcal{T}_\alpha$ and $\mathcal{M}$

$$\mathcal{T}_\alpha \circ \mathcal{M} \circ \mathcal{T}_\alpha^{-1} = \mathcal{M}. \tag{2}$$

One can represent the turn transformation in the form $\mathcal{T}_\alpha = \exp\{\alpha \cdot \mathcal{L}_{\text{turn}}\}$ (here $\mathcal{L}_{\text{turn}} = = \mathbf{X}^*\mathbb{T}^*\partial/\partial\mathbf{X}$ — a generator for turn transformation in the plane $\{x, y\}$, $\mathbf{X} = (x, x', y, y')^*$, and $\mathbb{T}$ is the matrix of the following form

$$\mathbb{T} = \begin{pmatrix} \mathbb{O} & -\mathbb{E} \\ \mathbb{E} & \mathbb{O} \end{pmatrix}.$$

Introducing the Lie operator of the forming system $\mathcal{L}^{\rm syst}(s_t|s_0)$, we can rewrite eq. (2):

$$\exp\left\{\exp\left\{\alpha\,\mathcal{L}_{\rm turn}\right\}\circ\mathcal{L}^{\rm syst}(s_t|s_0)\right\} = \\ = \exp\mathcal{L}^{\rm syst}(s_t|s_0).$$

The Lie operator $\mathcal{L}^{\rm syst}$ generated by some function $G^{\rm syst}(\mathbf{X}) = \sum\limits_{k=1}^{\infty}\mathbb{G}_k^{\rm syst}(s_t|s_0)\mathbf{X}^{[k]}$ can be written as

$$\exp\left\{\alpha\,\mathcal{L}_{\rm turn}\right\}\circ\mathcal{L}^{\rm syst} = \tilde{\mathcal{L}}^{\rm syst},$$

$$\tilde{\mathcal{L}}^{\rm syst} = \sum_{k=1}^{\infty}\left(\mathbf{X}^{[k]}\right)^*\left(\tilde{\mathbb{G}}_k^{\rm syst}\right)^*\frac{\partial}{\partial\mathbf{X}},$$

where $\oplus$ is the Kronecker sum. Using the matrix equality for matrices generating corresponding Lie operators (see [2]) we can write

$$\mathbb{G}_k^{\rm syst}\mathbb{T}_\alpha^{\oplus k} - \mathbb{T}_\alpha\mathbb{G}_k^{{\rm syst}\,\oplus k} = 0.$$

Let consider the linear case (for $k=1$):

$$\mathbb{G}_1^{\rm syst}\mathbb{T}_\alpha - \mathbb{T}_\alpha\mathbb{G}_1^{\rm syst} = 0.$$

In our case can be presented as the block matrix ($\mathbb{G}_1^{\rm syst} = \mathbb{G}^{\rm syst}(s_t|s_0)$):

$$\mathbb{G}_1^{\rm syst} = \begin{pmatrix}\mathbb{G}^{11} & \mathbb{G}^{12}\\ \mathbb{G}^{21} & \mathbb{G}^{22}\end{pmatrix}.$$

Using the presentation for $\mathbb{T}$ we obtain following equalities for $\mathbb{G}^{ik}$:

$$\mathbb{G}^{11} = \mathbb{G}^{22}, \quad \mathbb{G}^{12} = -\mathbb{G}^{21}. \tag{3}$$

For the next calculations we should use the well known Magnus presentation for the matrix of the beam transport system under study $\mathbb{G}^{\rm syst}(s_t|s_0)$ (see [2]):

$$\mathbb{G}^{\rm syst}(s_t|s_0) = \int\limits_{s_0}^{s_t}\mathbb{P}^{\rm syst}(\tau)d\tau - \\ -\frac{1}{2}\int\limits_{s_0}^{s_t}\int\limits_{s_0}^{\tau}\left\{\mathbb{P}^{\rm syst}(\tau),\mathbb{P}(\tau')\right\}d\tau' d\tau + \\ + \text{nested commutators integrals.} \tag{4}$$

Here $\{\mathbb{A},\mathbb{B}\}$ is the matrix commutator. The matrix $\mathbb{P}^{\rm syst}(s)$ depends on the vector of control functions $\mathbf{U}(s)$ (in the case of a system consisting on quadrupole lenses only the vector function $\mathbf{U}(s)$ degenerates in a scalar function $u(s)$). It is not difficult to evaluate the following equalities for submatrices $\mathbb{P}^{ik}$ for $\mathbb{P}$.

$$\mathbb{P}^{11}(\mathbf{U}(s),s) = \mathbb{P}^{22}(\mathbf{U}(s_t-s),s_t-s),$$
$$\mathbb{P}^{12}(\mathbf{U}(s),s) = -\mathbb{P}^{21}(\mathbf{U}(s_t-s),s_t-s).$$

We should note that these equalities are non-unique feasible solutions of the similar problem. For receiving of additional restrictions one can should use the structure of Magnus representation (starting from the second term of the expansion (4) all terms consist commutators of corresponding matrices only). In the case of the quadrupole symmetry the matrices $\mathbb{P}^{kk}$ have the following form $\mathbb{P}^{ii} = \begin{pmatrix}0 & 1\\ \pm k(s) & 0\end{pmatrix}$. It is not difficult to show, that under integrals one can receive only two types of matrices:

$$f(k(t_1),\ldots,k(t_m))\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$
$$g(k(t_1),\ldots,k(t_{m-1}))\begin{pmatrix}0 & 1\\ \pm k(t_m) & 0\end{pmatrix}.$$

Here t_i, $i\geq 1$ are integration variables for "inner integrals".

Using some matrix properties and the exponential presentation of Magnus one can evaluate an additional condition for $\mathbb{R}^{11}$:

$$\mathbb{U}_2\mathbb{R}^{11}\mathbb{U}_2 = \left(\mathbb{R}^{11}\right)^*, \tag{5}$$

which reduces to

$$\left\{\mathbb{R}^{11}\right\}_{11} = \left\{\mathbb{R}^{11}\right\}_{22}. \tag{6}$$

This condition results to a following condition

$$k(s) = -k(s_t - s). \tag{7}$$

We should note that these conditions were received from the natural restrictions on a beam form and algebraic and functional constraints for the corresponding matrix functions. The condition (6) defines so called load curves (for the "Russian quadruplet") and load surfaces (for $2n$ quadrupole lenses, $n\geq 3$).

COMPUTER EXPERIMENTS

The above written approach is the first step of optimal parameters evaluation. This approach allows us to constraint several types of so called "load curves" (for four quadrupoles) or "load surfaces" (for $2n$ quadrupole lenses, $n\geq 3$). As one can see on the fig. 1 the corresponding curves have very complete forms and several branches. Besides for different geometrical parameters the relative positions of these curves are modified. The optimization procedures (see, i.e. [6]) make it possible to find appropriate solutions and investigate them for more relevant solutions.

In the case of six or more lenses we obtain surfaces of the corresponding dimension. On the fig. 2 one can see an example of similar surface We should note that the above described approach accepts both symbolic and numerical evaluations. The computer algebra packages (for example, well known Mathematica, Maple os Maxima) give us a very power tools for necessary investigation procedures. In

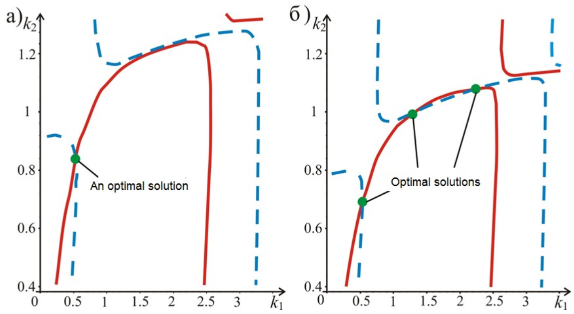

Figure 1: Two types of optimal solutions for the “Russian quadruplet”.

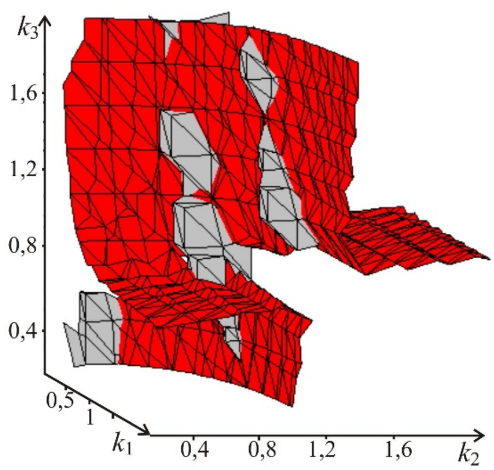

Figure 2: Load surfaces for six quadrupoles.

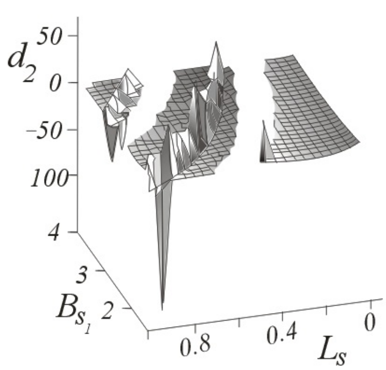

Figure 3: Load surfaces for six quadrupoles.

particular, a researcher can combine symbolic and numerical presentation for more convenient research organization.

The similar approach was applied for modeling of high solid angle mass-separator [2] (see fig. 3). The corresponding computational experiments lead us to a set of feasible solutions.

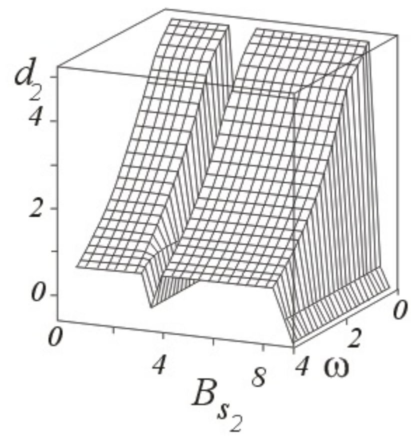

Figure 4: Load surfaces for six quadrupoles.

The corresponding surfaces of admissible working points are presented on fig. 4.

CONCLUSION

In this paper we show that a based on symmetry-based approach realized in matrix formalism terms can provide a powerful tool in the conceptual design of charged particle optical devices. The corresponding tools can be extend for beam line systems with different symmetries including linear and nonlinear abberations (for example for a fragment mass analyzer [2, 4]). This approach allows us not only reducing number of control parameters, but also simplify corresponding optimization procedures [5]).

REFERENCES

[1] W. Wan, M. Berz: Analytical theory of arbitrary-order achromats Phys. Rev. E, Vol. 54. Iss. 3, P. 2870–2883, 1996.

[2] Andrianov, S.N.: Dynamical Modeling of Control Systems for Particle Beams. SPbSU, Saint Petersburg. (2004) (in Russian)

[3] Balandin V., Brinkmann R., Decking W., Golubeva N. Two Cell Repetitive Achromats and Four Cell Achromats Based on Mirror Symmetry. Proc. of IPAC’10, Kyoto, Japan, 2010. P. 4479–4481.

[4] Andrianov S.N. Some Problems of Nonlinear Aberration Correction. Beam Stability and Nonlinear Dynamics, Santa Barbara, California, 1996. AIP Conf. Proc. N 405. N.-Y., 1997. P. 103–116.

[5] Andrianov S.N., Ivanov A.N., Kosovtsov M.E., Podzyvalov Eu.A. A Global Optimization Approach Based on Symbolic Presentation of a Beam Propagator. In this Proc.

[6] Chernyshev A. Multi-objective optimization for beam lines. From Physics to Control Through an Emergent View. World Scientific Series on Nonlinear Science, Ser. B, Vol. 15, 2010. P. 365–370.