THE MOTION OF AN ELECTRON IN THE PERIODIC CUSPED MAGNETIC FIELDS*

Guangxing Du#, Baoliang Qian, Honggang Wang
College of Optoelectric Science and Engineering, National University of Defense Technology, Changsha, Hunan, 410073, P. R. China

Abstract

The motion and its stability of an electron in the periodic cusped magnetic fields have been analyzed theoretically and calculated numerically, as the stability could not be well predicted by the Mathieu's Equation to guide the design of the magnetic focusing system for the propagation of the sheet electron beams in the waveguides. The precise solution to the motion equations of the electron has been obtained by iteration. To validate the analytical solution and to evaluate the stability of the motion, numerical calculations have been carried out. And the results show that the analytical solution is reliable, and there is only one stable region in the (p_0, B_0) space, where the parameter p_0 is the period of the magnetic fields, and B_0 is the magnitude of the magnetic fields. Besides, the stability of the electron motion would become weaker while the initial distance between the electron and the axis becomes larger. These results are interesting to the area of the sheet-electron-beam microwave sources focused by the periodical cusped magnetic fields.

INTRODUCTION

Sheet electron beams for microwave generators have been of academic interest for several years for its potential carrying the larger current with the larger width [1-4]. As important issues, the motion and its stability of an electron in the periodic cusped magnetic (PCM) fields have been investigated theoretically and numerically in the past, and useful results have been obtained [1], [4]. However, most of the results are approximate, and is difficult to be used to guide the design of the magnetic focusing system for the propagation of the sheet electron beams in the waveguides.

To achieve the exact analytical solution to the motion equation of electron in the PCM fields, the iteration method is proposed here. Just after the first iteration, the analytical results become almost the same with the numerical results, showing that the electron in the PCM fields oscillate at two frequencies at the same velocity in the horizontal plane, different with that in the wiggler magnetic fields. Consequently, the coupling between the motions in the two transverse directions in the PCM fields is weaker than that in the wiggler fields. It is the real reason for that the motion of electron in the PCM fields is stable than that in the wiggler fields.

With numerical calculations, the stable region of the motion of electron in the PCM fields in the (p_0, B_0) space has been obtained and shows that there is only one stable region which is no larger than the first stable region predicted by the Mathieu' Equation, where the parameter p_0 is the period of the magnetic fields, and B_0 is the magnitude of the magnetic fields. Besides, the stable region becomes smaller while the initial vertical distance between the electron and the axis are increasing.

*Work supported by the National High Technology Research and Development Program of China
guangxingdu@yahoo.com.cn

THEORETICAL ANALYSIS

It was generally considered [1] that the electron starting from the initial point (x_0, y_0, z_0) in the "semi-infinite" PCM fields,

$$B_{y,PCM}(y,z) = -B_0 \sinh(k_0 y)\cos(k_0 z) \tag{1}$$

$$B_{z,PCM}(y,z) = B_0 \cosh(k_0 y)\sin(k_0 z) \tag{2}$$

with the initial velocity (v_{x0}, v_{y0}, v_{z0}) and relativistic mass would "wiggle" at the velocity of

$$v_{x,PCM} = -\frac{\sqrt{2}k_\beta v_{z0}}{k_0}\sinh(k_0 y)\sin(k_0 z) + v_{x0} \tag{3}$$

in the horizontal (x) direction, and oscillate at the velocity of

$$v_{y,\beta} \approx k_\beta v_{z0} y_{\beta 0}\cos(k_\beta z + \theta) \tag{4}$$

in the orbit of the betatron motion,

$$y_\beta \approx y_{\beta 0}\sin(k_\beta z + \theta) \tag{5}$$

in the vertical (y) direction, where "semi-infinite" meant infinitely wide in the x direction and infinitely long in axial (z) with finite dimensions in the y direction, the parameters $k_0 = 2\pi/p_0$, $k_\beta = eB_0/\sqrt{2}\gamma m_e v_{z0}$, $y_{\beta 0} = \sqrt{y_0^2 + v_{y0}^2/k_\beta^2 v_{z0}^2}$ and $\theta = arcsin(y_0/y_{\beta 0})$ were the wave number of the PCM fields, the wave number, the spatial amplitude and the initial phase of the betatron motion, respectively. To obtain Eqs. (3), (4), and (5), the paraxial approximations $k_0 y \ll 1$ and $z \approx v_{z0}t$, and the relations $k_\beta \ll k_0$ had been assumed.

In the same way, the velocity components of electron in the wiggler fields with the same parameters could be written in the following forms,

$$v_{x,WIGG} = -\frac{\sqrt{2}k_\beta v_{z0}}{k_0}\cosh(k_0 y)\sin(k_0 z) + v_{x0} \tag{6}$$

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$$v_{y,\beta} \approx k_\beta v_{z0} y_{\beta 0} \cos(k_\beta z + \theta) \quad (7)$$

It would be concluded from Eq. (3)-(7) that the electrons in the both periodic magnetic fields should wiggle at the same frequency but at the different velocities. And consequently, the motion of the electron in the PCM fields should be more stable than that in the wiggler fields, for $\cosh(k_0 y) > \sinh(k_0 y)$,while $k_0 y \ll 1$, according to the paraxial approximation [1].

However, the betatron motion is going to be coupled to the wiggling motion more intensively for the electron in the PCM fields than that in the wiggler fields, leading to different wiggling motions of the two electrons. To obtain relatively accurate expressions of the horizontal motion, Eq. (5) should be substituted into Eq. (3), then one can get,

$$v_{x,PCM} \approx -\sqrt{2} k_\beta v_{z0} y_{\beta 0} \sin(k_\beta z + \theta) \sin(k_0 z) + v_{x0} \quad (8)$$

With the similar iteration, the wiggling motion of the electron in wiggler fields can be written as,

$$v_{x,WIGG} \approx -\frac{\sqrt{2} k_\beta v_{z0}}{k_0} \sin(k_0 z) + v_{x0} \quad (9)$$

Equations (8) and (9) imply that the wiggling motion of the electron in the PCM fields will mainly contain velocity components at two frequencies $f_{1,2} = f_w \pm f_\beta$ with the same amplitude, while the wiggling motion of the electron in wiggler fields will be almost monochromatic, at frequency f_w, with the amplitude twice as large as that in the PCM fields, where $f_w = k_0 v_{z0}/2\pi$ and $f_\beta = k_\beta v_{z0}/2\pi$. It is the real reason for that the motion of electron in the PCM fields is stabler than that in the wiggler fields.

To achieve more exact solutions, the iteration should be continued.

NUMERICAL CALCULATIONS

The Motion of an Electron

To validate the analytical results above, the numerical calculations have been performed. As an example, the motion of electrons initiating at the point $(x_0, y_0, z_0) = (0, 0.05p_0, 0)$ with the velocity $(v_{x0}, v_{y0}, v_{z0}) = (0, 0, 0.776c)$ in the two periodic magnetic fields with the same amplitude $B_0 = 0.2T$ and period $p_0 = 2$cm has been calculated, respectively, where $v_{z0} = 0.776c$ corresponds to an energy of 300 keV, $y_0 = 0.05p_0$ is a typical value of the initial vertical displacement from the xoz plane. In this situation, $k_\beta/k_0 \approx 0.2$ and $k_0 y_0 \approx 0.314$, hence the conditions $k_\beta \ll k_0$ and $k_w y_0 \ll 1$ used to obtain Eqs. (3) - (9) could be basically satisfied.

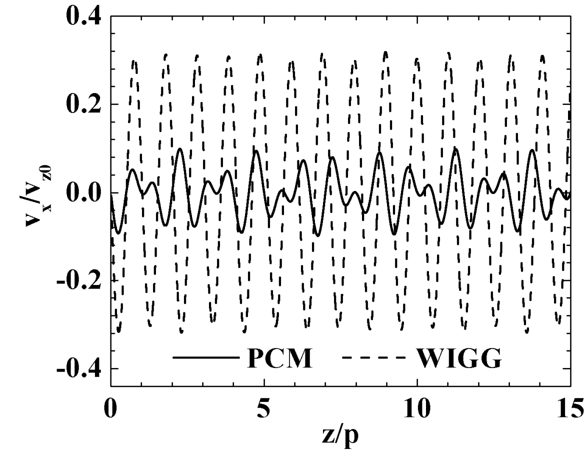

Figure 1: Normalized transverse velocities $v_{x,PCM}/v_{z0}$ and $v_{x,WIGG}/v_{z0}$ versus axial position z/p_0 for the case $B_0 = 0.2T$ and $p_0 = 2cm$.

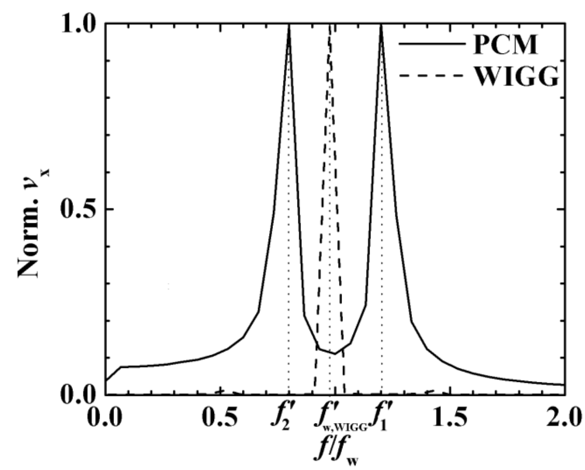

Figure 2: FFT of the curves of the transverse velocities $v_{x,PCM}/v_{z0}$ and $v_{x,WIGG}/v_{z0}$ for the case $B_0 = 0.2T$ and $p_0 = 2cm$.

Figure 1 illustrates the evolution of the horizontal velocities of the electrons in the PCM and wiggler fields, using solid and dashed lines, respectively. It is obvious that the electron in the wiggler fields oscillates at a higher speed than that in the PCM fields. In addition, the curve of $v_{x,WIGG}$ looks like a monochromatic sinusoid while another curve looks like multi-frequency sinusoid. The Fast Fourier Transforms (FFT) of the curves of $v_{x,PCM}$ and $v_{x,WIGG}$ in Fig. 1 are performed and plotted in Fig. 2. In this figure, one can find out that the numerically calculated wiggling frequency $f'_{w,WIGG}/f_w \approx 0.98$ of $v_{x,WIGG}$ is slightly less than the predicted value of 1.00. This is caused by the slight drop of the axial velocity component $v_{z,WIGG}$ due to the transverse motion, and the wiggling frequency should be written in the form $f_w \approx k_0 \langle v_{z,WIGG} \rangle / 2\pi$ to be more exact, where $\langle v_{z,WIGG} \rangle$ is the periodical average of $v_{z,WIGG}$, and in the numerical calculation, $\langle v_{z,WIGG} \rangle / v_{z0} \approx 0.98$, which is consistent with the slightly drop of the wiggling frequency.

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However, the wiggling frequency of the electrons in the horizontal direction in the PCM fields $f'_{w,PCM} = (f'_1 + f'_2)/2 \approx f_w$ since the calculated $\langle v_{z,PCM} \rangle / v_{z0} \approx 1.00$, implying that the transverse motion of the electrons in the PCM fields provides much smaller perturbation to the axial velocity than that in the wiggler fields. In addition, the numerically calculated betatron frequency $f'_{\beta,PCM} = (f'_1 - f'_2)/2 \approx f_\beta$.

In conclusion, the analysis in the previous section 2 has been validated by the numerical calculations under the conditions $k_\beta \ll k_0$ and $k_0 y_0 \ll 1$.

The Stability Criteria for the Motion of an Electron

To evaluate the stability criteria for the motion of an electron in the periodic magnetic fields, the Mathieu's Equation [4], [5] based on the paraxial approximation was widely used [3], [4]. According to the Equation, the motion of electrons would be stable in the discrete regions $\alpha < 0.66$, $1.75 < \alpha < 3.70$, and so on, where $\alpha = k_\beta^2 / k_0^2$ was a factor in the Equation. However, when the instability grows up, the paraxial approximation becomes inapplicable. And consequently, the Mathieu's Equation and the stability evaluation may become unbelievable. So it is necessary to perform numerical calculations to evaluate reliability of the discrete regions predicted by the Mathieu's Equation.

The stable regions predicted by the numerical calculations as well as the Mathieu's Equation in the (p_0, B_0) space are plotted in Fig. 3. In the calculations, the parameters of the electron, except the initial vertical location y_0, are almost the same as those in the previous section. It is obvious that there is only one stable region located in and being smaller than the first stable region predicted by the Mathieu' Equation, wherever the electron initiates from. Besides, the stable region becomes smaller while the initial vertical distance between the electron and the axis are increasing.

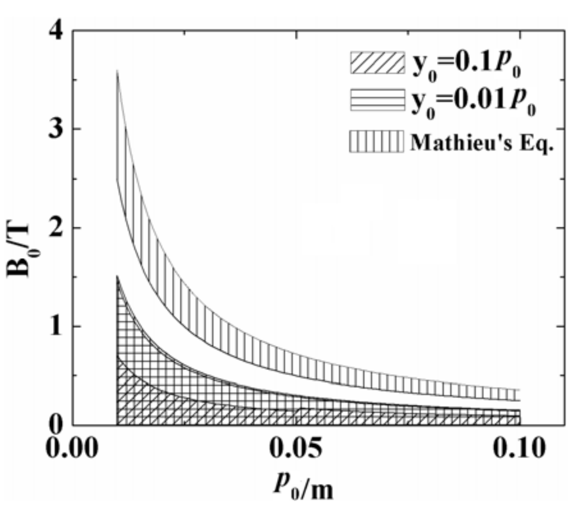

Figure 3: The stable regions predicted by the Mathieu's Equation and the numerical calculations.

CONCLUSIONS

The motion and its stability of an electron in the PCM fields have been analyzed theoretically and calculated numerically, as the stability could not be well predicted by the Mathieu's equation to guide the design of the magnetic focusing system for the propagation of the sheet electron beams in the waveguides. The precise solution to the motion equations of the electron has been obtained by iteration. To validate the analytical solution and to evaluate the stability of the motion, numerical calculations have been carried out. And the results show that the analytical solution is reliable, and there is only one stable region in the (p_0, B_0) space. Besides, the stability of the electron motion would become weaker while the initial distance between the electron and the axis becomes larger. These results are interesting to the area of the sheet-electron-beam microwave sources focused by the PCM fields.

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