ESTIMATION OF THE DYNAMIC APERTURE BY TRANSVERSE BEAM EXCITATION WITH NOISE CLOSE TO A RESONANCE

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Abstract

The present heavy ion synchrotron SIS-18 will be upgraded to be used as a booster for further synchrotrons being part of the FAIR project underway at GSI. Recently, a method was developed to measure the machine acceptance of SIS-18 using transverse rf noise. This method is based on the transverse expansion of the beam with noise beyond the limiting aperture generating beam loss. The acceptance was determined from the comparison of the resulting measured time evolution of the beam current with that obtained from a numerical simulation. It would be desirable to extend this method in order to determine the dynamic aperture. We present in this paper as the first step a numerical study on the time evolution of the beam current affected by the dynamic aperture generated with sextupoles.

INTRODUCTION

In this work we present a numerical study on the time evolution of the beam current in the present GSI heavy ion synchrotron SIS-18 affected by noise driven beam loss. The study is the first step to determine the dynamic aperture of SIS-18 which is limited due to the influence of non-linear elements in the lattice. The method to determine the machine acceptance of a circular accelerator from noise driven beam loss was recently developed and applied to measure the vertical acceptance of SIS-18 [1]. Generally, the necessity to measure the transverse acceptance arises from possible deviations of the real acceptance from the design acceptance due to closed orbit distortions. The opportunity to measure the transverse acceptance will also be important for the future synchrotron SIS-100 because its horizontal aperture will be only about 5σ of the beam width. Further restrictions to the acceptance arise from several septa installed for injection and slow extraction of the beam. Therefore, a small reduction of the aperture can lead to significantly increased beam loss, in particular during injection.

The aim of our study is to compare the time evolution of the beam current determined by the dynamic aperture to that given by a physical aperture of the same size. Here, we restrict ourselves to the dynamic aperture caused by the 3rd order resonance given by $13 = 3\nu_x$ so that the dynamic aperture appears in the two dimensions of the horizontal phase space plane. In order to change the size of the dynamic aperture, different horizontal tunes close to the resonance tune $\nu_{x,res} = 13/3$ are used.

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MODEL AND PARAMETERS

Table 1: Main simulation parameters

Variable	Size
Circumference of SIS-18, C	216.72 m
Horizontal tunes, ν_x	$4.27, \ldots, 4.32$
Vertical tune, ν_y	3.23
Ion	Ta^{61+}
Kinetic energy	100 MeV/u
RMS momentum spread, σ_p	5×10^{-4}
Chromaticities, ξ_x, ξ_y	$-1.3, -1.5$
Revolution time, T_0	1.683 μs
Simulation time interval, t_{fin}	up to 30 s
Exciter voltage amplitude, U_a	250 V
Number of test particles N_0	2000

To compute the time evolution of the particle number in the beam we performed particle tracking simulations using the MAD-X code [2]. The main simulation parameters are presented in Table 1. To simulate horizontal particle diffusion, we defined a horizontal kicker element in the lattice to excite the beam. The momentum kick corresponds to the electrostatic voltage of an exciter implemented in the real machine. The voltage has the time behaviour [3]

$$U(t) = U_a \sin\left[2\pi f_C t + \phi(t)\right] \qquad (1)$$

with the voltage amplitude $U_a = 250$ V and the carrier frequency $f_C = \nu_{x,frac}/T_0$ of the signal. The phase $\phi(t)$ is determined by a pseudorandom bit sequence, where $\phi = \pi$ for the bit status 1 and $\phi = 0$ otherwise. This leads to the noise power spectrum

$$P(f) \propto \frac{\sin^2\left[\pi(f - f_C)/f_S\right]}{\left[(f - f_C)/f_S\right]^2}. \qquad (2)$$

The width of the central peak of the spectrum, f_S, is the bit rate. f_S has to be large enough to cover the spread in the particle tunes arising from momentum spread and amplitude dependence of the betatron motion in a lattice with sextupoles. In our simulations, $f_S = 0.05/T_0$ was used.

To generate a horizontal dynamic aperture we applied six sextupoles equidistantly distributed in the lattice. Their total focussing strengths are shown in Table 2. The sextupoles excite a third order resonance and, hence, cause the formation of a triangular phase space region of stable particle motion. Its size depends on the difference between resonance tune and machine tune. Fig. 1 shows the stable phase space regions for the smallest and largest tunes used

Table 2: Focussing strengths of the sextupoles

Name of Sextupole	Focussing strength k_2L (m^{-2})
S01KS1C	0.07071
S03KS1C	0.10833
S05KS1C	0.03762
S07KS1C	-0.07071
S09KS1C	-0.10833
S11KS1C	-0.03762

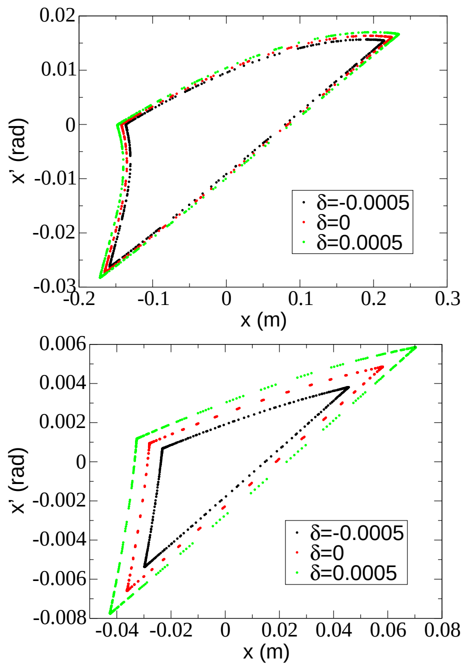

Figure 1: Stable phase areas for $\delta = -\sigma_p, 0, +\sigma_p$ with $\sigma_p = 5\times10^{-4}$ from Table 1 at $\nu_x = 4.27$ (graph above) and $\nu_x = 4.32$ (graph below). Note the different axis scales.

in this study. Realistic values used for slow extraction from the real machine are above 4.31. In Fig. 1, also the change of the triangular stable phase space area due to a relative momentum deviation $\delta = \Delta p/p = \pm5 \times 10^{-4}$ resulting in the tune shift $\Delta\nu_x = \xi_x\delta\nu_x$ are shown.

The dynamic aperture yields a nonlinear horizontal acceptance which is related to the stable phase space area by

$$\epsilon_{lim} = \frac{\text{area}}{\pi}. \tag{3}$$

The nonlinear acceptances for all horizontal tunes considered are presented in Table 3, where the areas of the stable phase space regions were determined from the corners of the triangles assuming that the edges are straight lines.

Table 3: Acceptances ϵ_{lim} for $\delta = 0$ determined from the area of the stable phase space regions. ϵ_{lim} for $\nu_x = 4.27$ and $\nu_x = 4.32$ correspond to the red graphs in Fig. 1.

ν_x	ϵ_{lim}(mm mrad)
4.27	1540
4.28	1144
4.29	799
4.30	499
4.31	263
4.32	98

RESULTS

Beam Loss with a Linear Lattice

In a linear lattice, horizontal noise drives diffusion leading to a growth of the averaged or beam emittance linear in time [1, 4]. When the beam width starts to exceed the machine acceptance ϵ_{lim}, beam loss will start leading to a reduction of the beam current. It can be shown that an asymptotic beam profile will be reached which is independent of its initial shape. The resulting number of remaining particles approaches an exponentially decreasing behaviour,

$$N(t) \propto \mathrm{e}^{-\frac{t}{\tau_{loss}}}, \tag{4}$$

where the loss rate is

$$\tau_{loss}^{-1} \propto \frac{\beta_x \sigma_{\Delta x'}^2}{\epsilon_{lim} T_0}, \tag{5}$$

where $\sigma_{\Delta x'}$ and T_0 are the rms size of the momentum kicks of the noise and the revolution time T_0.

To compute the time evolution of the particle number $N(t)$ with the linear SIS-18 lattice we implemented a machine acceptance according to the tune from Table 3. In doing so, the results are comparable to those obtained with sextupoles which will be presented in the next subsection. Fig. 2 shows $N(t)$ due to a physical aperture and a dynamic aperture, respectively, for $\nu_x = 4.27$ and $\nu_x = 4.32$. According to equation (4) the loss times are given by

$$\tau_{loss} = (t_2 - t_1)\left[\ln\frac{N(t_1)}{N(t_2)}\right]^{-1}. \tag{6}$$

Here, we chose t_1, t_2, so that $N(t_1)/N_0 = 0.3$ and $N(t_1)/N_0 = 0.7$. According to equation (5) the loss times should fulfill the condition $\tau_{loss}/\epsilon_{lim} = \text{const}$ because all parameters except ϵ_{lim} were kept constant. We found that fulfilled in good approximation, see the black curve in the graph above of Fig. 3. There, at least, one does not find a tendency for tunes close or far from the resonance tune.

Beam Loss with Sextupoles

The insertion of sextupoles in the lattice yields a dependence of the particle tune on the betatron amplitude. The

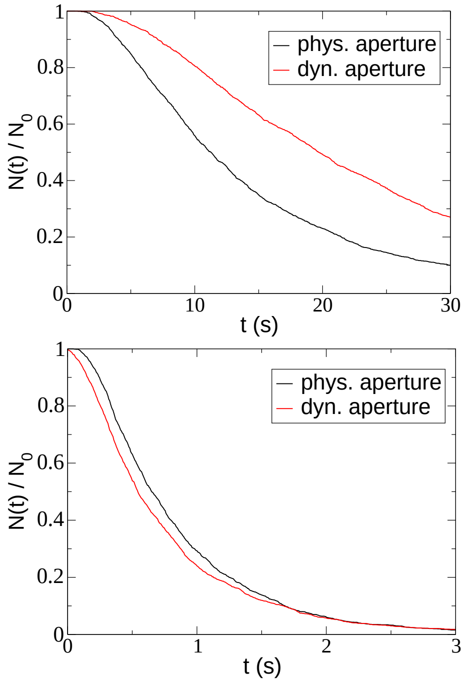

Figure 2: $N(t)/N_0$ for $\nu_x = 4.27$ (graph above) and $\nu_x = 4.32$ (graph below) where the corresponding acceptances ϵ_{lim} from Table 3 were used.

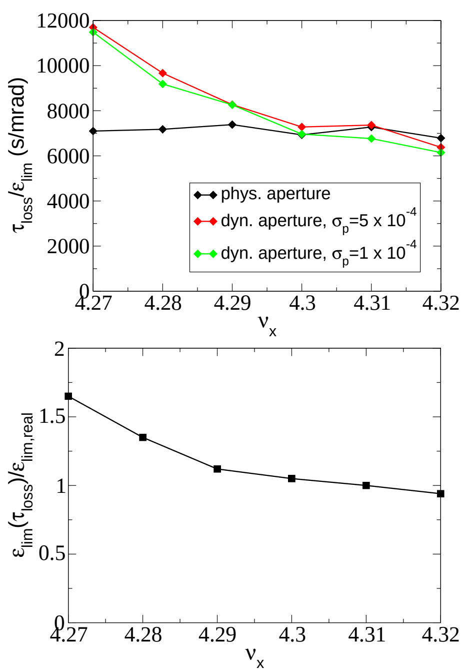

Figure 3: Loss times τ_{loss} normalised to the acceptance ϵ_{lim} for physical and dynamic aperture (graph above) and dynamic aperture calculated from the loss times according to Eq. (5) (graph below).

tune varies from the machine tune ν_x in the beam centre to the resonance tune $\nu_{x,res} = 4.33333$ at the corners of the stable phase space region, which are unstable fixed points of the horizontal betatron motion. On the other hand, the carrier frequency of the noise f_C was adjusted to the fractional machine tune. Therefore, particles with a tune close to the resonance tune are less excited by the noise when the difference between machine tune and resonance tune approaches or even exceeds the width of the noise power spectrum $f_S \cdot T_0 = 0.05$. The reduced excitation causes slower beam loss which one can see in Fig. 2. The loss times normalised to the acceptance, $\tau_{loss}/\epsilon_{lim}$, start to deviate strongly from the almost constant value obtained with the linear lattice and the physical aperture when the difference between resonance tune and machine tune exceeds about $2 \cdot f_S \cdot T_0/3$, see graph above of Fig. 3. Consequently, the assumption that, according to Eq. (5), a physical and a dynamic aperture have the same size if the beam currents decrease with the same loss time, would lead to an overestimate of the dynamic aperture. The relative deviation from the values given in Table 3 would be approximately given by the factor $\tau_{loss,dynap}/\tau_{loss,physap}$ which one can see by comparing both graphs in Fig. 3.

For small deviations of the machine tune from the resonance tune, i.e. when $\nu_x > 4.3$, Fig. 3 shows that the loss times are not dominated by the nonuniform power density. Instead, there is a strong dependence of the dynamic aperture on the particle momenta, which is shown in Fig. 1. To study the influence of the dependence of the dynamic aperture on the momentum spread, we performed tracking calculations with reduced momentum spread represented by the green curve in Fig. 3. Obviously, the influence of the momentum spread is small and the loss times are slightly below those obtained with the linear lattice for both values of the rms momentum spread. Possibly, this slight reduction of the loss time is a consequence of enhanced diffusion arising from the influence of the sextupoles. However, our simulations suggest that the particle loss determined by a dynamic aperture is similar to that determined with the linear lattice and a physical aperture of the same size, if the machine tune is sufficiently close to the resonance tune. This can be fulfilled in SIS-18. Therefore, the measurement of the dynamic aperture from noise driven beam loss should be as possible as the measurement of the machine acceptance.

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[4] Chap. 7 in D. A. Edwards and M. Syphers, "An Introduction to the Physics of High Energy Accelerators", (Wiley, New York, 1993).